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ESTIMATION OF IMPEDANCE AND TRANSFER FUNCTIONS FOR END BEARING AND FLOATING PILES

Hisashi NOZOE[1], Yoshihisa GYŌTEN[2], Kōji MIZUHATA[3], and Tadahiro FUKUSUMI[3]

[1]Graduate School of Science and Technology, Kobe University,
Nada-ku, Kobe, Japan
[2]Professor Emeritus, Kobe University,
Nada-ku, Kobe, Japan
[3]Department of Architecture, Kobe University,
Nada-ku, Kobe, Japan

SUMMARY

The impedance and transfer functions referred to the pile head are theoretically estimated in a wide range of parameters for both end bearing and floating piles. The effects of the secondary displacements in vertical and horizontal vibrations, the shear deformation of piles, the rotational resistance of the surrounding soil layer, and the compressibility and flexibility of piles on the impedance and transfer functions are clarified by a parametric study. Physically approximate formulas of impedance functions are proposed in the explicit and simple expressions corresponding to each problem parameter.

INTRODUCTION

It is necessary that the dynamic response of structures with pile foundations exposed to seismic excitation is predicted and estimated in consideration of the soil-pile-structure interaction. In recent years, a significant amount of work has been done on the dynamic behavior of single pile foundations by several analytical approaches and simplified procedures. However, the dynamic characteristics of floating piles in comparison with that of end bearing piles has not yet been fully understood.

In this paper, the impedance and transfer functions, which play essentially important roles in coupling to superstructures, are theoretically estimated in a wide range of parameters for both end bearing and floating piles. A parametric study is made to investigate mainly the effects of the secondary displacements in vertical and horizontal vibrations, the shear deformation of piles, the rotational resistance of the surrounding soil layer, and the compressibility and flexibility of piles on the impedance and transfer functions. Approximate formulas of the impedance functions are derived physically and consistently for the static and dynamic cases. The accuracy of these formulas is investigated by comparison with the exact results for a wide parametric range.

DESCRIPTIONS OF MODEL AND PARAMETRIC STUDY

The problem of a soil-pile system is sketched in Fig. 1. The analytical model consists of a floating pile, its surrounding soil layer on a rigid bedrock and a soil column beneath the pile tip. The pile is dealt with as elastic Timoshenko's beam considered the shear deformation. Both the surrounding soil layer and the soil column are dealt with as three-dimensional continua to be elastic, homogeneous and isotropic with the linear hysteretic damping. The governing equations of motion in vertical and horizontal vibrations are based on the elastic wave propagation theory in cylindrical coordinates, as shown in Fig. 1.

For the pile assumed to be perfectly in contact with the soil during the motion, the equations of motion and constitutive relationships with respect to the vertical and horizontal total displacements W_P and U_P, respectively, and the rotational angle R_P are expressed by

$$\frac{dN_P}{dz} - P_V = -\rho_P A_P \omega^2 W_P \quad ; \quad \frac{N_P}{E_P A_P} = \frac{dW_P}{dz}$$
$$\frac{dQ_P}{dz} - P_H = -\rho_P A_P \omega^2 U_P \quad ; \quad \frac{Q_P}{\kappa G_P A_P} = \frac{dU_P}{dz} + R_P \qquad (1)$$
$$\frac{dM_P}{dz} - Q_P = -\rho_P I_P \omega^2 R_P + r_0 P_M ; \quad \frac{M_P}{E_P I_P} = \frac{dR_P}{dz}$$

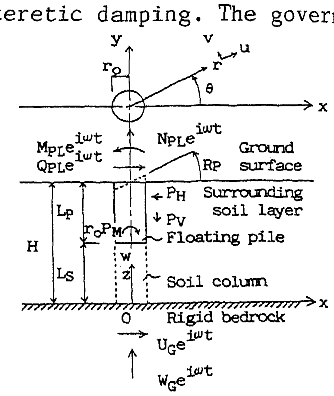


Fig. 1 Model of a soil-pile system

where the time factor $\exp(i\omega t)$ in the harmonic excitation is abbreviated for convenience. W_G and U_G are amplitudes of the uniform bedrock motion. N_P and Q_P are the axial and shear forces, respectively, and M_P is the bending moment. ρ_P= mass density, E_P=Young's modulus, G_P= shear modulus, A_P= cross-sectional area, I_P= moment of inertia of the cross section, and κ = form factor for shear. P_V, P_H and P_M are the vertical, horizontal and rotational forces per unit length of the surrounding soil layer acting on the pile circumference, respectively. The pile is also subjected to the harmonic excitations N_{PL}, Q_{PL} and M_{PL} at the pile head, and N_{PO}, Q_{PO} and M_{PO} of the soil column at the pile tip. The authors have presented theoretical solutions of Eqs. 1 by employing the finite Fourier-Hankel transforms and the Fourier-Bessel integral (Refs. 1 and 2). This approach has such advantages that the solutions are involving the effect of the generalized Rayleigh wave and the equations of boundary condition are expressed as the complex simultaneous linear equations without using integral equations. However, the above exact resistances of the soil on the pile circumference and at the pile tip are expressed in the implicit form in order to include the effect of the complicated soil-pile interaction.

The impedance and transfer functions, which are of importance in predicting the response of superstructures in consideration of the inertial and kinematic interaction effects, respectively, are defined as [K] referred to the pile head and [S] at the free pile head in the following forms.

$$\begin{Bmatrix} N_{PL} \\ Q_{PL} \\ M_{PL} \end{Bmatrix} = \begin{bmatrix} K_{VV}, & 0, & 0 \\ 0, & K_{HH}, & K_{HR} \\ 0, & K_{RH}, & K_{RR} \end{bmatrix} \begin{Bmatrix} W_{PL} \\ U_{PL} \\ R_{PL} \end{Bmatrix}, \quad \begin{Bmatrix} W_{PL} \\ U_{PL} \\ R_{PL} \end{Bmatrix} = \begin{bmatrix} S_{VG}, & 0 \\ 0, & S_{HG} \\ 0, & S_{RG} \end{bmatrix} \begin{Bmatrix} W_G \\ U_G \end{Bmatrix} \qquad (2)$$

where the coupling terms K_{HR} and K_{RH} are equal. K or S can be expressed as the function consisting of ten parameters in the form:

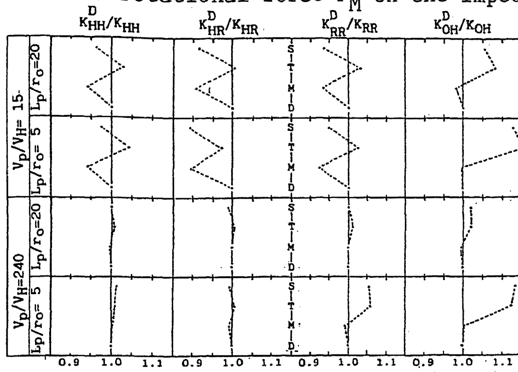
$$K \text{ or } S = F\left(\frac{L_P}{r_0}, \frac{L_S}{r_0}, \frac{V_P}{V_H}, \frac{A_P}{\pi r_0^2}, \frac{I_P}{\pi r_0^4}, \frac{E_P}{\kappa G_P}, \frac{\rho_P}{\rho}, \frac{V_V}{V_H}, \xi, b_1\right) \qquad (3)$$

where $V_P=\sqrt{E_P/\rho_P}$, $V_H=\sqrt{\mu/\rho}$, $V_V/V_H=\sqrt{2(1-\nu)/(1-2\nu)}$, μ=shear modulus of soil. ρ=soil density. ξ is the ratio of hysteretic damping for soil and defined in the form of the complex shear modulus $\mu^*=\mu(1+2i\xi)$. $b_1=\omega/\omega_{H1}$ and the fundamental frequency of the soil layer due to the distortional wave: $\omega_{H1}=\pi V_H/(2H)$.

A parametric study is performed with the numerical analysis. In the range of problem parameters, a soft soil is considered, whereas Poisson's ratio ν=1/3 to 0.479 (V_V/V_H=2.0 to 5.0), and the ratio of hysteretic damping ξ=0.01 to 0.1. A concrete pile with the solid circular cross section is considered, whereas $\kappa=6(1+\nu_P)/(7+6\nu_P)$, Poisson's ratio ν_P=1/6 and $E_P=2.1\times10^5\,kg/cm^2$. The density ratio of pile to soil ρ_P/ρ=4/3 typically. The velocity ratio of pile to soil, which reflects the stiffness ratio of pile to soil E_P/μ, is widely selected to comprise the stiffness existing in common surface stratum: i.e., V_P/V_H=7.5 to 240. The slenderness ratio of the pile L_P/r_o=5 to 80, L_S/r_o=5 to 80, and $H/L_P\leqq2$.

APPROXIMATION OF SOIL RESISTANCES, IMPEDANCE AND TRANSFER FUNCTIONS

For the sake of a physical approximation of the impedance and transfer functions, Eqs. 1 of motion must be solved in a simplified model. First, the effects of the shear deformation of the pile and the rotational resistance on the impedance functions are investigated. The ratio of the real parts of impedance function by the rigorous method D to those by four different methods are shown in Fig. 2, where K_{OH} is the real part of impedance function at the rotational free pile head. The effect of the shear deformation of the pile on the impedances is considerable as the soil becomes stiffer irrelevantly to the slenderness ratio $L_P/r_0 \geqq 5$. The effect of the rotational force P_M on the impedances is little except K_{RR} for the short-rigid pile, though its effect on K_{OH} is remarkable. Since the floating pile tip is nearly pinned, the rotational resistances of the soil on the pile circumference and at the pile tip are neglected herein.



Method	Shear deformation	Rotational inertia and force; P_M
S	neglected	neglected
T	considered	neglected
M	neglected	considered
D	considered	considered

H/L_P= 2, V_V/V_H= 2.5, ξ = 0.05

Fig. 2 Comparison of impedance functions by methods of S, T, M and D at b_1= 0.2

Second, the Winkler type medium is assumed for the soil in order to simplify the vertical and horizontal resistances of the soil. That is

$$P_V = K^*_{CV}(W_P - W_S);\ K^*_{CV} = K_{CV}(1+2i\xi),\ P_H = K^*_{CH}(U_P - U_S);\ K^*_{CH} = K_{CH}(1+2i\xi)$$
$$N_{P0} = K^*_{SV}(W_{P0}-W_{S0});\ K^*_{SV} = K_{SV}(1+2i\xi),\ Q_{P0} = K^*_{SH}(U_{P0}-U_{S0});\ K^*_{SH} = K_{SH}(1+2i\xi) \quad (4)$$

where K_{CV}* and K_{CH}* are the local impedances and assumed to be constant along depth, and K_{SV}* and K_{SH}* are the complex stiffnesses. W_S and U_S are the vertical and horizontal total displacements of the soil layer, respectively, when the pile is absent. The dynamic coefficients of the vertical and horizontal acting force k_V and k_H, respectively, are derived from approximate solutions for a tentative floating pile in a half medium. These solutions are obtained for the acting force distributed along depth triangularly and uniformly by applying the dynamic Kelvin's solution. An approximation of the average displacements and stresses W and τ in vertical loading ,and U and σ in horizontal loading, respectively, at the ground surface is performed under $L_P/r_0 \geqq 10$ in the explicit form. That is

1) The case of the triangular distribution:

$$\frac{W_1}{A_1 r_0} = -\{\ln(\frac{2L_P}{r_0}) - \frac{1}{4(1-\nu)} - 1\} + \frac{1}{6}(\frac{a_P}{2})^2 + \frac{1}{3}ia_P\{1-\frac{1}{15}(\frac{a_P}{2})^2\} \qquad ;\ a_P \leqq 2$$
$$= \frac{\pi}{2}iH_0^{(2)}(a_0) + \frac{1}{a_P{}^2}[\exp(-ib_P) + \frac{L_P}{r_0}\{(1-ia_0)\exp(-ia_0) - \exp(-ib_0)\}] \qquad ;\ a_P > 2$$
$$\frac{\tau_1}{A_1\mu} = 1 - \frac{3-2\nu}{2(1-\nu)}\cdot\frac{r_0}{L_P} \qquad ;\ a_P \leqq 2$$
$$= -\frac{\pi}{2}ia_0H_1^{(2)}(a_0) - \frac{r_0}{L_P}\exp(-ia_0) - \frac{2}{a_P{}^2}\cdot\frac{L_P}{r_0}\{(1+ia_0)\exp(-ia_0) - (1+ib_0)\exp(-ib_0)\} \qquad ;\ a_P > 2$$
$$\frac{U_1}{B_1 r_0} = -\frac{3-4\nu}{8(1-\nu)}\{\ln(\frac{2L_P}{r_0}) + \frac{1}{2(3-4\nu)} - 1\} + \frac{1}{8}(\frac{a_P}{2})^2\{1-\frac{1}{9}(\frac{a_P}{2})^2\} + \frac{1}{6}ia_P\{1-\frac{2}{15}(\frac{a_P}{2})^2\};\ a_P \leqq 2 \quad (5)$$
$$= \frac{\pi}{8}i\{H_0^{(2)}(a_0) + \frac{1-2\nu}{2(1-\nu)}H_0^{(2)}(b_0)\} + \frac{1}{4a_P{}^2}[2\exp(-ia_P) - \frac{1-ib_P}{2(1-\nu)}\exp(-ib_P)$$
$$-\frac{L_P}{r_0}\{(1+ia_0)\exp(-ia_0) - (1-ib_0)\exp(-ib_0)\}] \qquad ;\ a_P > 2$$
$$\frac{\sigma_1}{B_1\mu} = 1 - \frac{3-4\nu}{4(1-\nu)}\cdot\frac{r_0}{L_P} \qquad ;\ a_P \leqq 2$$
$$= -\frac{\pi}{4}i\{a_0H_1^{(2)}(a_0) + b_0H_1^{(2)}(b_0)\} - \frac{r_0}{2L_P}\{\exp(-ia_0) + \exp(-ib_0)\}$$
$$+ \frac{1}{a_P{}^2}\cdot\frac{L_P}{r_0}\{(1+ia_0)\exp(-ia_0) - (1+ib_0)\exp(-ib_0)\} \qquad ;\ a_P > 2$$

2) the case of the uniform distribution:

$$\frac{W_2}{A_2 r_0} = -\{\ln(\frac{2L_p}{r_0}) - \frac{1}{4(1-\nu)}\} + \frac{1}{2}(\frac{a_p}{2})^2 + \frac{2}{3}ia_p\{1-\frac{2}{15}(\frac{a_p}{2})^2\} \quad ; a_p \leqq 2$$

$$= \frac{\pi}{2}iH_0^{(2)}(a_0) + \frac{1}{a_p^2}\{2\exp(-ia_p) - (1+ib_p)\exp(-ib_p)\} \quad ; a_p > 2$$

$$\frac{\tau_2}{A_2\mu} = 1 \quad ; a_p \leqq 2, \qquad = -\frac{\pi}{2}ia_0H_1^{(2)}(a_0) \quad ; a_p > 2$$

$$\frac{U_2}{B_2 r_0} = -\frac{3-4\nu}{8(1-\nu)}\{\ln(\frac{2L_p}{r_0}) + \frac{1}{2(3-4\nu)}\} + \frac{3}{8}(\frac{a_p}{2})^2\{1-\frac{5}{27}(\frac{a_p}{2})^2\} + \frac{1}{3}ia_p\{1-\frac{4}{15}(\frac{a_p}{2})^2\} \quad ; a_p \leqq 2 \tag{6}$$

$$= \frac{\pi}{8}i\{H_0^{(2)}(a_0) + \frac{1-2\nu}{2(1-\nu)}H_0^{(2)}(b_0)\} + \frac{1}{2a_p^2}[\frac{1}{4(1-\nu)}\{(3-4\nu)+ib_p\}\exp(-ib_p) - ia_p\exp(-ia_p)] \quad ; a_p > 2$$

$$\frac{\sigma_2}{B_2\mu} = 1 \quad ; a_p \leqq 2, \qquad = -\frac{\pi}{4}i\{a_0H_1^{(2)}(a_0) + b_0H_1^{(2)}(b_0)\} \quad ; a_p > 2$$

where A_1, A_2, B_1 and B_2 are arbitrary constants. $K_V=\omega/V_V$, $K_H=\omega/V_H$, $a_o=K_H r_o$, $b_o=K_V r_o$, $a_p=K_H L_p$, $b_p=K_V L_p$, and $H_0^{(2)}$ and $H_1^{(2)}$ are the Hankel functions of the second kind of order zero and one, respectively. When the pile length tends to infinity, the above solutions and those for a plane strain case by Novak et al (Ref. 3) for $\xi=0$ agree precisely in vertical vibration but are different in horizontal vibration. Though the above solutions can not keep the circular condition on the pile circumference in horizontal vibration, those are available at the lower frequencies. The dynamic coefficients of acting force are obtained, and the local impedances for $\xi=0$ can be expressed as follows:

$$k_V = -\frac{\tau_1+\tau_2}{W_1+W_2}, \quad k_H = -\frac{\sigma_1+\sigma_2}{U_1+U_2}, \quad K_{CV} = 2\pi r_0 \cdot k_V, \quad K_{CH} = \pi r_0 \cdot k_H \tag{7}$$

where $A_1=B_1=L_o/H_o$, $A_2=B_2=1-L_o/H_o$, L_p is replaced by L_o in Eqs. 5 and 6, and $L_o=\min(L_p,L_C)$ and $H_o=\min(H,L_C)$. L_C is defined as the effective soil depth in the static state and equal to the effective pile length given later. The static coefficients of acting force obtained by Eqs. 7 can be predicted within 10% error exclusive of the extra short-rigid piles for both the end bearing and floating piles by comparison with this exact result and with the other results for $L_S \to \infty$ (in Refs. 4 and 5). On the other hand, the static stiffness K_{SV} and K_{SH} for $\xi=0$ can be calibrated in the following forms from Kausel's expression for a rigid circular foundation on a stratum over a rigid bedrock.

$$K_{SV} = \frac{2.56\mu r_0}{1-\nu}(1+\frac{2r_0}{L_S}), \quad K_{SH} = \frac{6\mu r_0}{2-\nu} \tag{8}$$

The above simple expressions are valid for $L_S/r_o \geqq 5$, then these error are the order of 5% by comparison with this exact solution. Since the influence of N_{P0} and Q_{P0} on the dynamic impedance functions are small for the slender piles $L_P/r_o \geqq 10$, the dynamic stiffnesses are also estimated by Eqs. 8 approximately.

By neglecting both the rotational inertia of the pile and the rotational resistance of the soil on the pile circumference, and by assuming the pile tip to be pinned for both end bearing and floating piles, Eqs. 1 of motion are solved in the simple expression in consideration of Eqs. 4 to 8. The approximate impedance functions referred to the pile head are reduced to the following forms.

$$K^*_{VV} = \lambda_V E_p A_p \frac{1+\eta_V\tanh(\lambda_V L_p)}{\tanh(\lambda_V L_p)+\eta_V}, \quad K^*_{HH} = \frac{4\lambda_H^2 E_p I_p}{L_p}\cdot\frac{D_{HH}}{D_K}, \quad K^*_{HR} = 2\lambda_H^2 E_p I_p\cdot\frac{D_{HR}}{D_K}, \quad K^*_{RR} = \frac{2E_p I_p}{L_p}\cdot\frac{D_{RR}}{D_K} \tag{9}$$

where $\alpha=\sqrt{K_{CV}^*/(E_P A_P)}$, $\beta=\sqrt[4]{K_{CH}^*/(4E_P I_P)}$, $\lambda_V=\sqrt{\alpha^2-K_P^2}$, $\lambda_H=\sqrt[4]{\beta^4-K_b^4/4}$, $K_P=\omega/V_P$, $K_b^2=K_P\sqrt{A_P/I_P}$, $\eta_V=\lambda_V E_P A_P/K_{SV}^*$, $\eta_H=2\lambda_H^2 E_P I_P/(K_{SH}^* L_P)$, and $\zeta=\lambda_H^2 E_P I_P/(\kappa G_P A_P)$, which are wholly characteristic parameters. Moreover, it is put beforehand that $\lambda_1=\lambda_H\sqrt{1+\zeta}$, $\lambda_2=\lambda_H\sqrt{1-\zeta}$, $Sh=\sinh(2\lambda_1 L_P)$, $Ch=\cosh(2\lambda_1 L_P)$, $Si=\sin(2\lambda_2 L_P)$, $Co=\cos(2\lambda_2 L_P)$.

$$D_K = [\frac{1+2\zeta}{\lambda_1 L_p}Sh - \frac{1-2\zeta}{\lambda_2 L_p}Si] + \eta_H[\frac{1+2\zeta}{1+\zeta}Ch + \frac{1-2\zeta}{1-\zeta}Co + \frac{2}{1-\zeta^2}], \quad D_{HH} = [Ch + Co] + \eta_H\lambda_H L_p[\frac{Sh}{\sqrt{1+\zeta}} + \frac{Si}{\sqrt{1-\zeta}}] \tag{10}$$

$$D_{HR} = [\frac{Sh}{\lambda_1 L_p} + \frac{Si}{\lambda_2 L_p}] + \eta_H[\frac{Ch}{1+\zeta} - \frac{Co}{1-\zeta} + \frac{2\zeta}{1-\zeta^2}], \quad D_{RR} = [Ch - Co] + \eta_H\lambda_H L_p[\frac{Sh}{\sqrt{1+\zeta}} - \frac{Si}{\sqrt{1-\zeta}}]$$

The characteristic parameters representing the pile behavior, αL_P and βL_P for $\xi=0$, can be classified into three categories: 1) $\alpha L_P \leqq 0.3$ or $\beta L_P \leqq 0.5$ for short-rigid piles, 2) $0.3<\alpha L_P<3$ or $0.5<\beta L_P<3$ for intermediate piles, 3) αL_P or $\beta L_P \geqq 3$ for long-compressible or -flexible piles. Then the effective pile length L_C, which is already defined with respect to Eqs. 7, is determined from $\alpha L_C=3$ or $\beta L_C=3$ and $L_o=H_o=L_C$ with a few iterations. Meanwhile, the approximate transfer functions are easily obtained by using the above approximate soil resistances.

CHARACTERISTICS OF IMPEDANCE AND TRANSFER FUNCTIONS

The numerical results of the exact impedance functions for the floating pile are shown in Fig. 3. Each curve of the impedance functions for Poisson's ratio varies at the frequencies where the group velocity due to the Rayleigh wave is minimum (first: b_1=1.88, 1.98 and 2.07 for V_V/V_H=2.0, 2.5 and 5.0, respectively) and zero (second: b_1=1.95, 2.31 and 2.81, and third: b_1=4.93, 5.00 and 4.96 for V_V/V_H=2.0, 2.5 and 5.0, respectively), and at the resonances of the soil layer due to the distortional wave (b_1=1, 3, 5, 7 and 9). The real parts have valleys and the imaginary parts shift up stepwise at those frequencies. However, the above tendency disappears for K_{VV} at the resonances of the soil layer due to the dilatational wave (b_1=2.0, 2.5 and 5.0 for V_V/V_H=2.0, 2.5 and 5.0, respectively) except for the case of V_V/V_H=5.0 and ξ=0.01. The influence of Poisson's ratio for $\xi \gtrsim 0.05$ in vertical and horizontal vibrations almost focuses at the first and second frequencies due to the Rayleigh wave, respectively.

The results of the exact impedance function K_{HH} and the approximate one K_{HH}^* for both the end bearing and floating piles are shown in Fig. 4 with frequency whose scale is identical, i.e., the value in parentheses of $a_1=b_1 r_o/H$. As the pile approaches the long-flexible pile, K_{HH} becomes independent of the soil depth except the vicinity of resonance. As the pile approaches the short-rigid pile, K_{HH} is affected significantly by the behavior of the soil layer for the floating pile in comparison with the end bearing pile. Such tendency appears similarly for K_{VV} (in Ref. 5), K_{HR} and K_{RR}. In the low frequency range (Ref. 4), the proposed approximate impedances are estimated as follows. The real part of the approximate impedances can be predicted within 5% error for the end bearing piles in vertical vibration and within 10% error for the others. However, the rotational impedance K_{RR}^* for the stocky pile is considerably underestimated, because this approximation neglects the rotational resistance of the soil. The imaginary part only occurs due to the hysteretic damping of the soil and is roughly estimated with the order of 10% error. The above result of K_{HH}^* can be verified in Fig. 4. Above the first resonance, both the real and imaginary parts of K_{HH}^* also nicely agree with K_{HH} for all piles, and such estimation holds good similarly for K_{VV}^* (in Ref. 5), K_{HR}^* and K_{RR}^*. The estimation for the plane strain case is also good except for the long-compressible and -flexible piles. On the other hand, the exact and approximate amplitudes of transfer function are shown in Fig. 5. The transfer functions have peaks at the resonances of soil layer due to the body waves, and are scarcely affected by the Rayleigh wave. At the fundamental resonances, as the pile approaches the short-rigid pile, $|S_{VG}|$ becomes larger but $|S_{HG}|$ becomes smaller in comparison with the corresponding free surface response, and $|S_{RG}|$ depends on $|S_{HG}|$. Such tendency becomes remarkable in vertical vibration. The approximate transfer functions can be predicted satisfactorily at the lower frequencies.

CONCLUSIONS

In estimating the impedance and transfer functions, the authors can clarify the effect of the problem parameters as follows. The influence of Poisson's ratio in vertical and horizontal vibrations for $\xi \gtrsim 0.05$ almost focuses at the first and second frequencies due to the Rayleigh wave, respectively. The characteristic parameters, which depend on the coefficients of the soil resistance and pile stiffness, represent three categories of the pile behavior, i.e., the impedance functions for the long-compressible and -flexible piles are independent of the soil depth and so on. The proposed approximate formulas are valid in practical situation without much computational effort, and can play an auxiliary role in the estimation of the impedance and transfer functions.

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REFERENCES

1. H.Nozoe, Y.Gyōten and T. Fukusumi:"Dynamic Analysis of a Soil-Pile System by the Finite Fourier-Hankel Transformation Method -Case of a Floating Pile in Vertical Vibration-",Mem.Grad.School Sci.& Technol.Kobe Univ., 3-A,35-48,(1985).
2. H.Nozoe, Y.Gyōten and T. Fukusumi:"Dynamic Analysis of a Soil-Pile System by the Finite Fourier-Hankel Transformation Method -Case of a Floating Pile in Horizontal Vibration-",THEORETICAL AND APPLIED MECHANICS,Vol.33, 377-392,(1985).
3. M.Novak, T.Nogami and F. Aboul-Ella:"Dynamic Soil Reaction for Plane Strain Case" ,ASCE,Vol.104, No.EM4, 953-959,(1978).
4. H.Nozoe, Y.Gyōten, K. Mizuhata and T.Fukusumi:"A Physical Approximation for the Pile-Head Impedances of a Soil-Pile System in the Low Frequency Range",Proc. 7th Japan Earthq.Eng.Symp., 997-1002,(1986).
5. H.Nozoe, Y.Gyōten, K. Mizuhata and T.Fukusumi: "Characteristics of Pile Head Impedance and Its Approximate Estimation in Vertical Vibration",Mem. Grad.School Sci. & Technol. ,Kobe Univ., 6-A, 7-22, (1988).

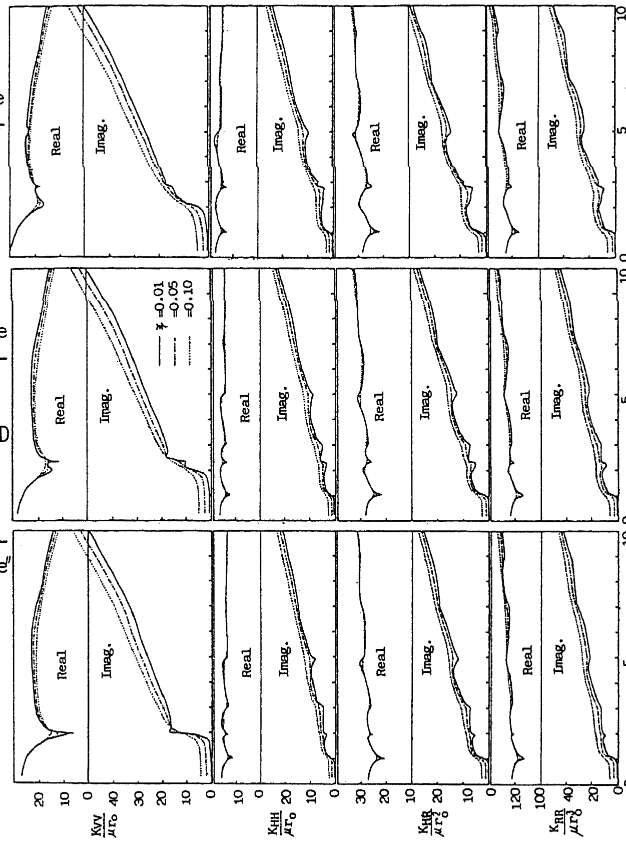

Fig. 3 Impedance functions for floating pile (L_p/r_0=10, H/r_0=20, V_p/V_H=15)

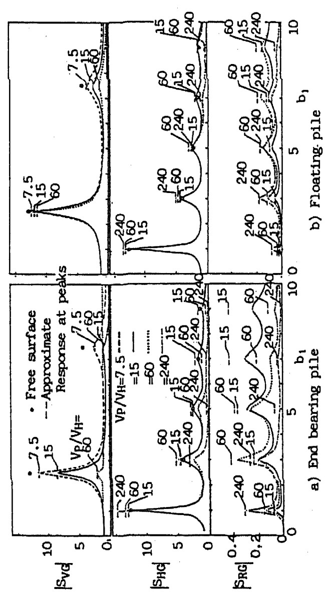

Fig. 5 Exact and approximate transfer functions (L_P/r_0=40, V_V/V_H=2.5, ξ=0.05)

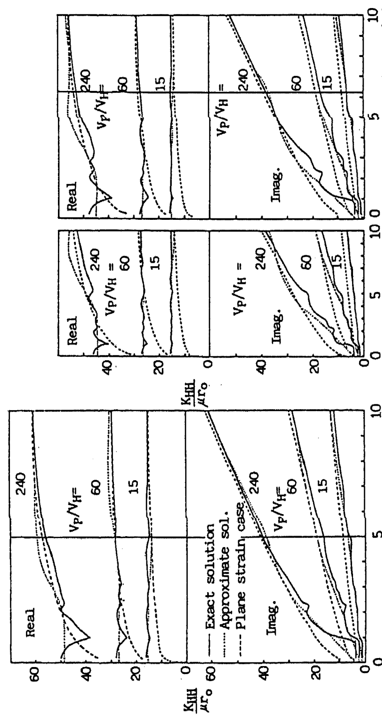

Fig. 4 Exact and approximate impedance functions (L_p/r_0=40, V_V/V_H=2.5, ξ=0.05)