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EVALUATION ON POST-BUCKLING BEHAVIOR OF STEEL BRACES WITH ECCENTRIC THRUST AND END-RESTRAINT

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SUMMARY

The post-buckling behavior of eccentric thrust braces with or without end-restraint is investigated. For structural design, the equivalent length factor Keq is introduced as a factor that indicates the quantitative relation between the eccentricity of the thrust and the post-buckling behavior of the braces. The Keq-factor is obtained as a function of eccentricity of the thrust, bending stiffness of the end-restraining member and ductility of the brace. The post-buckling strength of the eccentric thrust brace can be evaluated on the basis of the Keq-factor and the axial force of the pin-ended and centrally compressed brace.

INTRODUCTION

Bending moments as well as axial forces are applied to braces in actual steel frames, because the braces are surrounded by beams and columns through elastic junction elements such as gusset plates and, in some cases, the axis of the braces does not meet with junctions of the beam-to-column. The effect of the former case on post-buckling behavior of the braces was discussed in Ref.1. The latter case is on the problem of the eccentric thrust applied to the braces. Mitani analyzed the end-restrained braces with the eccentric thrust and got the qualitative relation between the eccentric thrust and the post-buckling behavior of braces(Ref.2). His report did not quantify the relation of the post-buckling load-displacement curves between the eccentric thrust and perfect compression. The braces with no eccentricity of the thrust have already been analyzed by many investigators(Refs.3, 4). If the quantitative relation between the eccentric thrust and the post-buckling behavior of the braces would be made clear, the post-buckling strength of the braces with the eccentric thrust could be evaluated on the basis of this quantitative relation and the results investigated about the centrally compressed braces.

The objective of this study is to make clear the quantitative relation between the amount of eccentricity of the thrust and the post-buckling behavior of the braces. Fig.1 schematically shows possible eccentricity in steel frames. Braces in the cases b) and c) of this figure are subjected to less bending moment induced by the eccentric thrust than the brace of the case a). Therefore, the effect of eccentric thrust on the post-buckling load-displacement relation of braces becomes maximum in the case of a). Braces with the slenderness ratio such as 40 to 130 reduce their strength in post-buckling domain. It is significant in these braces to investigate the relation between eccentricity of the thrust and reduction of the strength in post-buckling. Thus, braces with slenderness ratio of 40 to 130 and eccentricity presented in the case a) of Fig.1 are studied.

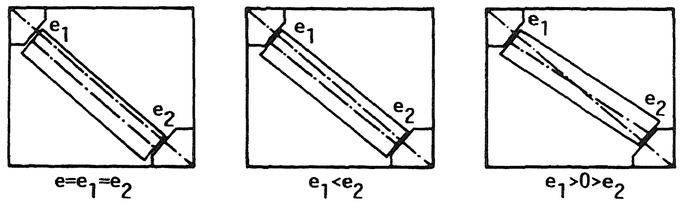


Fig.1 Eccentricity of Thrust

ANALYTICAL METHOD

The analytical method used in this paper has almost the same base as that proposed by Kato et al.(Ref.5) and was extensively modified in Ref.1.

Analytical Model The brace considered in this paper has no initial imperfections such as in-plane and/or out-of-plane deflections and residual stresses as one of assumptions. At the ends of A and A' in Fig.2, the brace has the rigid cantilever with the length of e, so that the thrust and the bending moment Mec = P•e are applied to it. When the brace is welded to the surrounding beams or columns, it is also subjected to the bending moment Mr = Kr•θ. Here, Kr is a bending stiffness of the surrounding members. θ is an end rotation of the brace.

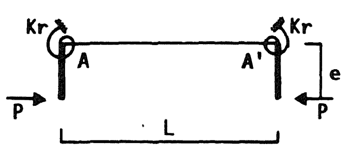


Fig.2 Analyzed Model

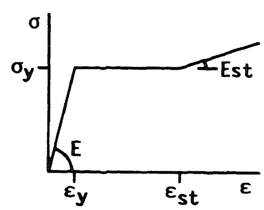


Fig.3 Stress-Strain Relation of Material Used in Analysis

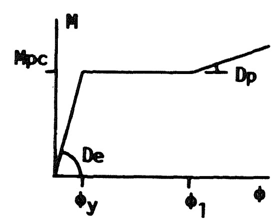


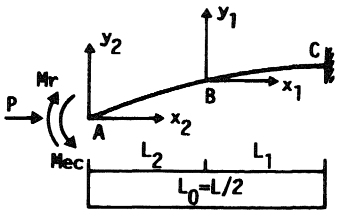
Fig.4 M - ϕ Relation of Section Used in Analysis

If the stress-strain diagram of the material is assumed to be tri-linear as shown in Fig.3, the moment-curvature relation of the section of the brace is represented by the model illustrated in Fig.4. After the buckling, the braces studied here lose more than a half of their maximum resisting axial force by the small increase of the axial displacement, namely, the stress level is not so high relative to the yield stress. This is reflected to estimation of the axial displacement, i.e., the plastic shortening of the center of the section of the brace due to the direct stress can be neglected because the center of the section remains in elastic under such a low stress level of the thrust(These were discussed in detail in Ref.1). Therefore, assuming that the section of the brace is modelled by the two-flange-section of which sectional area and moment inertia are identical to the original ones respectively, then De, Dp, ϕ_1 and full plastic moment Mpc of the section under the thrust are described as Eq.1.

$$\left.\begin{aligned} &De = E\,I,\ Dp = Est\,I,\ \phi_1 = (\varepsilon st - \varepsilon y)/i,\ i^2 = I/A \\ &Mpc = Mpo\ (1 - P/Py) \end{aligned}\right\} \quad (1)$$

where E is Young's modulus, Est is the modulus in the strain hardening range, εy is the yield strain, εst is the strain at beginning of the strain hardening, I is the moment inertia of the section, A is the sectional area of the brace, Mpo is the full-plastic moment of the section without the thrust, and Py is the yield strength of the section.

<u>Axial</u> <u>Displacement-1</u> <u>after</u> <u>Maximum</u> <u>Thrust</u> As a reasonable and acceptable assumption, the brace has a symmetric deflection profile about the axis perpendicular to the brace axis at its mid-length so that the behavior of only half portion of the brace is considered hereinafter(Figs.5 and 6). The applied axial force is assumed to be less than the yield axial force of the section. In Fig.5, the portion between A and B is elastic and the rest portion is plastic.



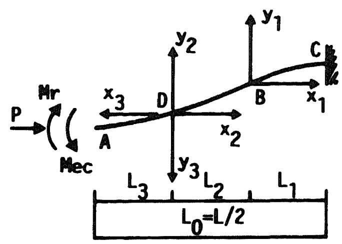


Table 1 Numerals Used in Analysis

A(cm²)	20.12
I(cm⁴)	356.6
Mpo(tonf·cm)	231.2
σy(tonf/cm²)	2.9
E(tonf/cm²)	2100
Est(tonf/cm²)	40.0
εst/εy	12.3

Fig.5 Analytical Model-1 Fig.6 Analytical Model-2

(i) Deflection y_1(BC) and y_2(AB) In each portion, the equilibrium on the bending moment is solved under the boundary conditions. Then, the deflection is obtained as follows.

$$y_1 = \phi_1 \;(\cos(kp\, x_1) + \tan(kp\, L_1) \sin(kp\, x_1) - 1)/kp^2 \tag{2}$$

$$y_2 = [\; e - Kr\; ke\; C_1/P \;][\; \cos(ke\, x_2) - 1 \;] + C_1 \sin(ke\, x_2) \tag{3}$$

where $C_1 = P[\phi_1 \tan(kp\, L_1)/kp/ke + e \sin(ke\, L_2)]/[Kr\; ke \sin(ke\, L_2) + P \cos(ke\, L_2)]$

$$kp^2 = P/Dp,\; ke^2 = P/De$$

(ii) Axial Displacement The following equations are derived from the equilibrium on the moment at the points B and A, and the length of the two portions.

$$Mpc = P\, y_2(L_2) + P\, e - Kr\, y_2'(0) \tag{4}$$

$$Mpc > |P\, y_2(0) - Mec - Mr| \tag{5}$$

$$L_1 + L_2 = L_0 = L/2 \tag{6}$$

The axial displacement(Δ) of the brace is described by the summation of the axial displacement due to the flexure and the shortening caused by the direct stress in the two portions:

$$\Delta/\Delta y = 2[\; 0.5\int_0^{L1}(y_1')^2 dx_1 + 0.5\int_0^{L2}(y_2')^2 dx_2 + \varepsilon y\, L_0\, P/Py \;]/\Delta y \tag{7}$$

where Δy is the yield axial displacement. The first and the second terms in Eq.7 indicate the shortening caused by the flexural deflection in the two portions. The third term is the elastic shortening. By using Eqs.4 and 6, L_1 and L_2 under the thrust P are calculated by iteration. Putting P, L_1 and L_2 into Eq.7, then the axial displacement at the thrust P is given. If P, L_1 and L_2 do not satisf Eq.5, the results should be obtained by the equation in the next section

<u>Axial</u> <u>Displacement-2</u> <u>after</u> <u>Maximum</u> <u>Thrust</u> After the maximum thrust, the develops its deflection and has the plastic portions in its mid-span and The portions B to C and A to D are plastic and intermediate part D to B i elastic(Fig.6).

(i) Deflection y_1(BC), y_2(DB) and y_3(AD) The deflection of each portion is given by the same way as in the former section. y_1 is the same as Eq.2.

$$y_2 = C_2 \sin(ke\ x_2) + Mpc\ [\ 1 - \cos(ke\ x_2)]/P \tag{8}$$

$$y_3 = [\ \phi_1 \cos(kp\ x_3) + kp\ ke\ C_2 \sin(kp\ x_3) - \phi_1\ ]/kp^2 \tag{9}$$

where, $C_2 = [\ kp\ Dp\ \phi_1 \tan(kp\ L_1) - ke\ Mpc \sin(ke\ L_2)]/[\ P\ ke \cos(ke\ L_2)]$

(ii) Axial Displacement The following equations are derived from the equilibrium on the bending moment at the points B and A, and the length of the three portions.

$$P\ y_2(L_2) = 2Mpc \tag{10}$$

$$P\ y_3(L_3) + Mpc = Kr\ y_3'(L_3) - P\ e \tag{11}$$

$$L_1 + L_2 + L_3 = L_0 = L/2 \tag{12}$$

The axial displacement of the brace is described as the same expression as Eq.7, and is also calculated by the same procedure as described in the former section.

$$\Delta/\Delta y = 2[\ 0.5\int_0^{L1}(y_1')^2 dx_1 + 0.5\int_0^{L2}(y_2')^2 dx_2 + 0.5\int_0^{L3}(y_3')^2 dx_3 + \varepsilon y\ L_0\ P/Py\ ]/\Delta y \tag{13}$$

Maximum Axial Force When the moment at the mid-span of the brace reaches to Mpc, the brace has the maximum axial force. Eq.14 gives the maximum axial force.

$$P\ y_2(L_0) - Kr\ y_2'(0) + P\ e = Mpc \tag{14}$$

ANALYTICAL RESULTS AND DISCUSSIONS

Axial Force vs. Axial Displacement Relation First, we analyzed the axial force-axial displacement relation of the brace with the eccentric thrust and without the end-restraint. Then, the stiffness of the end-restraining spring Kr was equal to zero. The parameters were the slenderness ratio of the brace and the thrust eccentricity represented by the factor e/L. The slenderness ratios adopted here ranged in 40, 60, 80, 100 and 130. The factors of e/L were 0.0, 0.0001, 0.0005 and those from 0.001 to 0.01 by a division of 0.001. The other numerals were kept constant as tabulated in Table 1. The factor e/L=0 means that the thrust applies to the center of the section of the brace in the direction of the brace axis. This brace becomes a measure for evaluating the behavior of the eccentric thrust braces. Hereinafter, it is called "pin-ended brace".

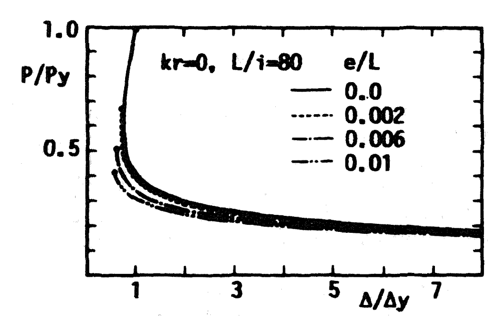


Fig.7 Axial Force-Axial Displacement Relation of Eccentric Thrust Braces without End-Restraint

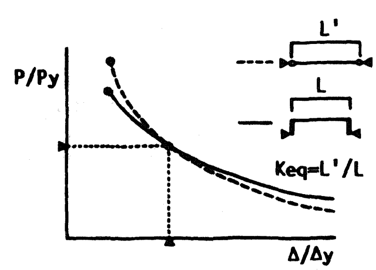


Fig.8 Definition of Equivalent Length Factor Keq

Representative results are drawn in Fig.7. The eccentricity of the thrust affects the maximum strength of the brace; however, it does not dominate its strength in the large deflection range.

Equivalent Length Factor(Keq-Factor)

(i)Keq-Factor of Eccentric Thrust Brace without End-Restraint In Ref.1, the Keq-factor was introduced to quantitatively relate the load-displacement curves of the end-restrained brace with those of pin-ended braces. In the sense of structure design, it would be convenient to relate the strength and ductility of the eccentric thrust brace with those of the pin-ended braces. Therefore, we apply the Keq-factor to those braces for the quantitative evaluation of their post-yielding behavior. The definition of the Keq-factor is written here: "Supposing that the pin-ended brace has the identical sectional properties but the different member length from the eccentric thrust brace, the equivalent length ratio Keq at a certain value of ductility is defined as the ratio of the member length of the pin-ended brace to that of the eccentric thrust brace, where these braces have an identical axial force and the identical ductility as shown in Fig.8."

The Keq-factor of braces without the end-restraint is investigated. Fig.9 shows the relations between the Keq-factor and the e/L ratio under ductility of 5 to 20 by a division of 5. The curves of these relations were fairly represented by a single straight line for the analyzed slenderness ratios, as follows;

$$Keq = 1 + (5 + 65/\mu) e/L \qquad (15)$$

where, μ is ductility in terms of the axial displacement of the brace. The maximum difference between the values given by the analysis and by Eq.15 under ductility of 5, 10, 15 and 20 is only 5.9%, 2.5%, 1.8% and 0.6%, respectively. Therefore, Eq.15 gives a good approximate Keq-factor of the eccentric thrust braces without the end-restraint. When the thrust applies to the brace without the end-restraint in the manner illustrated in Fig.1 a), the Keq-factor is obtained from Eq.15 as a function of L/i-ratio and ductility of the brace, and the axial force of the brace is represented as that of the pin-ended one with the member length of Keq•L.

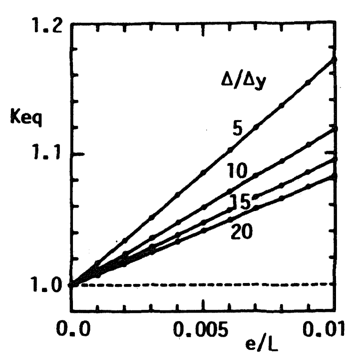


Fig.9 Relation between Keq-Factor and e/L Factor of Eccentric Thrust Braces with no End-Restraint

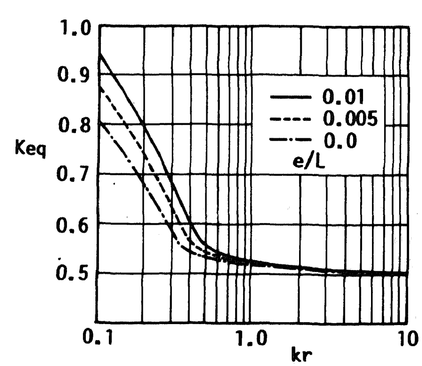


Fig.10 Relation among Keq-Factor, e/L Factor and kr-factor of Eccentric Thrust Braces with End-Restraint (L/i=80, ductility=5)

(ii)Keq-Factor of Eccentric Thrust Brace with End-Restraint In this section, the effect of the end-restraint on the Keq-factor of eccentric thrust braces is evaluated. kr-factors, which represent the bending stiffness ratio of the end-restraining spring to the brace, ranged from 0.1 to 100 in the analysis.

Fig.10 shows the relation between the Keq-factor and the kr-factor in the function of the thrust eccentricity, slenderness ratio and ductility. In the range where the kr-factor is larger than 1.0, the thrust eccentricity does not affect the Keq-factor. This is because the moment due to the eccentric thrust is very small in comparison with the moment induced by the end-restraint. In the light of the structural design, the Keq-factor can be estimated to be 0.5 with $e/L < 0.01$ and $kr > 5$. The analysis dealt with the braces illustrated in Fig.1 a). However, the eccentric thrust as shown in Fig.1 b) and c) has less effect on the bending moment than that in Fig.1 a), and the shear stress would have a negligible effect on the deflection of the braces in Fig.1 b) and c). Therefore, the results of this section are also applicable to the braces as shown in Fig.1 b) and c).

CONCLUSIONS

On the basis of the analytical results presented in this paper, the following conclusions are advanced as to the brace with the slenderness ratio of 40 to 130:

(i) The thrust eccentricity, e/L, affects the maximum strength of the brace, but, scarcely dominates its axial force resistibility in the large deflection range.

(ii) By the equivalent length ratio Keq, the axial force of the eccentric thrust brace is related to that of the pin-ended brace. When the brace without the end-restraint is applied to the eccentric thrust as the manner illustrated in Fig.1 a), the Keq-factor is linear with respect to the e/L ratio as a function of ductility of the brace as written by Eq.15.

(iii) When the brace has an elastic end-restraining spring, whose stiffness is larger than that of the brace, the eccentricity of the thrust is negligible. The Keq-factor of the eccentric thrust and end-restraint brace is 0.5, where the eccentricity is less than 1% of the brace length and the bending stiffness of the end-restraining spring exceeds 5 times as large as that of the brace.

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