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SHEAR STRENGTH OF REINFORCED CONCRETE COLUMNS WITH SPIRAL HOOPS

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SUMMARY

Test results of an experimental and analytical investigation on the influence of certain factors on shear strength in reinforced concrete columns with spiral hoops are presented. The comparisons of these test results and 118 available data of the most recent shear experiments to computed values given by Architectural Institute of Japan (AIJ) new shear design equations and the other current empirical shear equations are made. Additionally, based on these tests and on tests by previous investigations, modified shear design equations of the AIJ are suggested and are in better agreement with test data.

INTRODUCTION

Recently, there has been a rapid growth of demand for high-rise reinforced concrete structures and interest in high-quality materials in Japan. Understanding of the ultimate strength and ductility of structural frame members is the important requirement for earthquake resistant design of reinforced concrete building.

Now, in the AIJ consideration of a collapse mechanism of a structure shows that the bending yield must be reached at the ends of beams in each story and at bottoms of columns in the first story. Also, the shear design provisions are being developed in the belief that procedures are based on the truss mechanism (proposed by Thürlimann et al.)[Ref. 2] and the arch mechanism (proposed by Nielsen et al.)[Ref. 3], rather than on the current empirical equations.

This paper describes an experimental and analytical investigation into the effects of certain factors on shear strength of octagonal columns with spiral hoops. The shear design equations proposed by the AIJ, the current empirical equations, as well as some countries' code equations for shear strength are compared with 118 available data of the most recent shear experiments. The validity of the new AIJ shear design equations are also examined.

NEW SHEAR DESIGN EQUATIONS IN THE AIJ CODE

In the AIJ, the proposals (A and B methods) for shear strength of reinforced concrete members are taken to be the sum of two terms:

$$Qu = Qut + Qua \qquad (1)$$

where Qut = shear carried by truss mechanisms (ton) = $Pw \cdot \sigma wy \cdot \cot\phi \cdot b \cdot j$ (2)

Qua = shear carried by arch mechanisms (ton) = $\alpha(1-\beta) \cdot b \cdot D \cdot \nu \cdot Fc$ (3)

Pw = ratio of transverse reinforcement (= $a_w/b \cdot S$), a_w = area of shear reinforcement within a distance S (cm^2), b = width of the member (cm), S = spacing of hoops (cm), σ_{wy} = yield strength of hoops ($2400 \leqq \sigma_{wy} \leqq 25Fc$ kg/cm^2), ϕ = angle of inclination of diagonal strut in truss mechanism, j = distance between the compressive and tensile reinforcement (cm), α = factor of the angle of arch action to the longitudinal axis of the member, β = ratio of compressive strength of concrete due to truss action to effective compressive strength of concrete ($\nu \cdot Fc$), D = depth of the member (cm), ν = factor of effective compressive strength of concrete, Fc = compressive strength of concrete ($210 \leqq Fc \leqq 420$ kg/cm^2), θ = angle of inclination of concrete strut in arch mechanism, L = length of the member, M/QD = ratio of shear span. Now, those factors mentioned above are given as follows.

In the A-method

$\alpha = \tan\theta/2 = \{\sqrt{(L/D)^2 + 1} - (L/D)\}/2$ (4a)

$\beta = (1 + \cot^2\phi) Pw \cdot \sigma_{wy}/\nu \cdot Fc$ (5a)

$\nu = 0.7 - Fc/2000$ (6a)

and $\cot\phi$ ($1 \leqq \cot\phi \leqq 2$) is decided by the minimum value among

a) 2.0 ($\phi = 26.6°$)
b) $j/D \cdot \tan\theta$
c) $\sqrt{(\nu \cdot Fc/Pw \cdot \sigma_{wy}) - 1}$ (7a)

In the B-method

$\alpha = \tan\theta/2 = \{\sqrt{(2M/QD)^2 + 1} - (2M/QD)\}/2$ (4b)

$\beta = 2Pw \cdot \sigma_{wy}/\nu \cdot Fc$ (5b)

$\nu = (2M/QD + 1)/4$, ($0.5 \leqq \nu \leqq 1.0$) (6b)

$\cot\phi = 1.0$ ($\phi = 45°$) (7b)

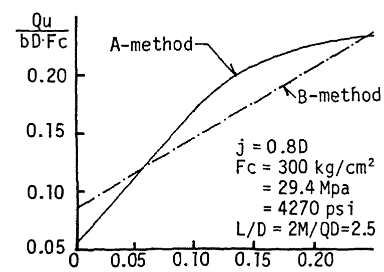

Fig. 1 Comparison Between A and B Method

The comparison between A and B method is shown in Fig. 1 where L/D = 2M/QD=2.5, Fc = 300 kg/cm^2 and j = 0.8 D. As shown in Fig. 1, shear carried by truss mechanism is overestimated, and that carried by arch mechanism is underestimated in the A-method; that in the B-method is just the opposite. Moreover, the influence of the axial compressive stress on shear strength is not being considered in those methods. The adequacy of these methods will be discussed with respect to the test results later.

EXPERIMENTAL RESEARCH ON SHEAR STRENGTH OF OCTAGONAL COLUMNS

Specimens Recent tests on shear strength of reinforced concrete octagonal columns subjected to cyclic load reversals were carried out in Muroran Institute of Technology, Japan [Ref. 4 & 5]. A total of 28 columns divided into two series in each 14, which had the same gross area of section, were prepared. Two of them (Nos.5 & 7) were columns with basic square section of 25 cm x 25 cm, and twenty-six were octagonal columns with the longitudinal reinforcement arrayed in a circular pattern as shown in Fig. 2. The main variables included clear height h_0, Pw, Fc, axial compressive stress ($\sigma_0 = N/bD$) and longitudinal reinforcement described in Table 1.

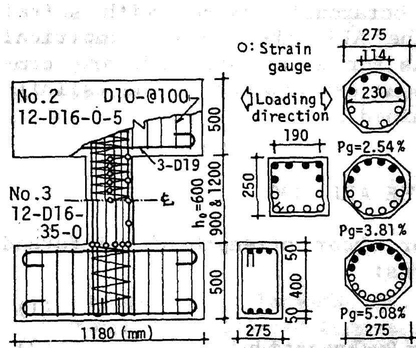

Fig. 2 Details of Specimen

Table 1 Main Variables and No. of Specimen

Longi-tudinal Bar	Clear height of h_0 (cm)		60					90			120
	6φ hoop @ mm		No	100	75	50	35	No	75	35	75
12-D16	σ_0 = 0	Fc=300	–	1	–	2	–	–	15	16	–
	σ_0=35	200	–	–	–	–	–	–	22	–	–
		300	3	4 [5]	17	6 [7]	8	18	19	20	21
		400	–	–	–	–	–	–	23	–	–
	σ_0=70	200	–	–	–	–	–	–	27	–	–
		300	11	12	24	13	14	–	25	–	26
		400	–	–	–	–	–	–	28	–	–
16-D16	σ_0=35	300	–	–	–	9	–	–	–	–	–
8-D16	σ_0=35	300	–	–	–	10	–	–	–	–	–

1) No. 5 & 7 : square columns with square hoops (6φ=0.272 cm², σ_{wy}=3720kg/cm²). Spiral hoop (6φ=0.271 cm², σ_{wy}=3750kg/cm²)
2) Series I (1986) : No.1–14, Series II (1987) : No.15–28.
3) D16 : Series I σ_y=3730kg/cm², Series II σ_y=3700 kg/cm².

Test Apparatus and Instrumentation As shown in Fig. 3, axial and lateral loads were applied to an L-shaped loading frame by actuators ② and ③, respectively. Actuator ① was used for keeping the upper and lower beams horizontal. The influence of vertical actuators' angular deviation from vertical on shear forces was considered.

All specimens were loaded using displacement control. The basic increment of displacement in each cycle was 2 mm. It was changed to 3 mm after the maximum load for a column height of 90 cm and 4 mm for that of 120 cm. Lateral and axial loads were measured by load cells which were mounted on the loading and restraining actuators ① to ③. In order to obtain the lateral displacement of column, two pairs of linear variable differential transducers were positioned on both sides at the mid-depth of the upper and lower beams.

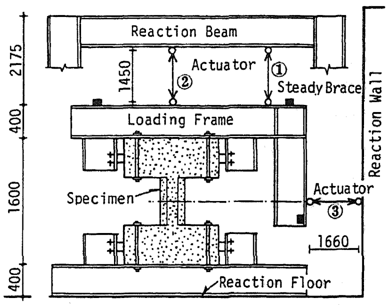

Fig. 3 Test Setup

Strain gauges (2 mm) were attached to both longitudinal reinforcing bars and spiral hoops, as shown in Fig.2 by ○ marks. Data acquisition processing and strage were done by minicomputer and disk memory for all of these test points.

DISCUSSION OF TEST RESULTS

Fracture Processes Soon after loading, flexural cracks occured at either end of column. And then one or two flexural cracks appeared in a region (D/2 or D from end) of large bending moments, which sometimes resulted in inclined cracks. At the same time, several shear cracks were observed in this region. With more loading cycles, many cracks occured along the diagonal of the column. The maximum loads were reached, while these existing cracks started to spread and widen (shear failure). As shown in Fig. 4(a), the load carrying capacity of columns in shear failure dropped very sharply as many cracks were generated along the main bars. However, it was different from that of the columns in flexural failure [see Fig.4(b)] which had sufficient ductility to absorb and dissipate energy by postelastic deformations when subjected to several cycles of loading after maximum load.

Load-Deflection Envelope Curves Fig. 5 shows the comparisons of load-deflection envelope curves which indicate the average value of positive and negative cycle of peak loads. Fig. 5(a) gives the difference between square columns and octagonal columns. The maximum loads of octagonal columns were slightly larger than those of square columns. Figs.5(b) and 5(e) indicate that the shear strength both increases with increasing axial compressive stress and decreases with increasing column height. The load carrying capacities dropped more after reaching the maximum

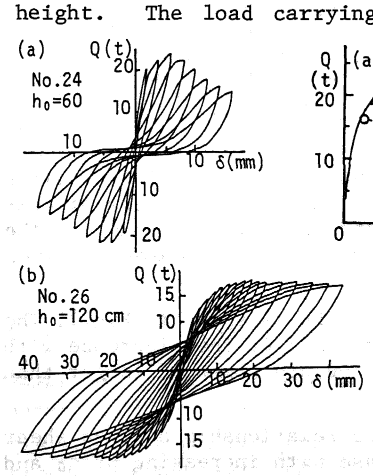

Fig. 4 Load-Deflection Curves

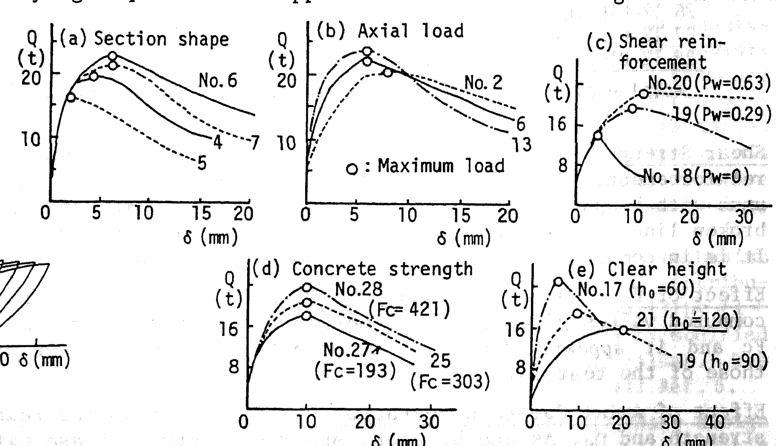

Fig. 5 Load-Deflection Envelope Curves

lateral loads. Fig. 5(d) shows that shear strength increases with the increasing of concrete compressive strength but deflections at the maximum loads and the decreasing rate of loading were nearly the same. And Fig.5(c) shows that with increasing of spiral reinforcement ratio, the shear strength can be in excess of the maximum flexural strength.

Effect of Section Shape The test results are listed in Table 2. There were two pairs of octagonal (No.4 & 6) and square (No.5 & 7) specimens compared at the same conditions. On average the shear strength of specimen with octagonal section was about 10 % higher than that with square section, which can be considered because of the difference of octagonal depth from square depth (or M/QD) as a reason. The shear strength of octagonal columns can be calculated conservatively using the current equations for shear strength by replacing the octagonal sections with squares having the same gross area and the same reinforced as shown in Fig. 2.

Table 2 Test Results

Column No.	Number of bar - Bar size - σ_0 (kg/cm²) - Hoop @ - Height h_0	Concrete strength Fc (Ft) (kg/cm²)	Axial comp. factor σ_0/Fc	Ratio of web reinf. Pw (%)	At diagonal crack load tQsc (ton)	$t\delta sc$ (mm)	At maximum load tQsu (ton)	$t\delta su$ (mm)	Failure mode
1	12-D16-00-100- 60	294 (28.0)	0	0.216	9.05	1.21	16.69	6.08	S
2	12-D16-00- 50- 60	299 (30.5)	0	0.433	10.13	1.31	20.25	8.04	S
3	12-D16-35- No- 60	292 (30.4)	0.120	0	15.60	1.73	15.78	2.06	S
4	12-D16-35-100- 60	304 (29.1)	0.115	0.216	13.87	1.26	19.84	4.00	S
5	=4, Square column	324 (27.3)	0.108	0.217	14.55	1.28	17.20	2.35	S
6	12-D16-35- 50- 60	292 (27.2)	0.120	0.433	14.91	1.43	22.34	6.07	S
7	=6, Square column	308 (29.5)	0.114	0.435	14.97	1.45	21.40	6.02	S
8	12-D16-35- 35- 60	320 (29.5)	0.109	0.619	16.26	1.74	25.41	8.01	S
9	16-D16-35- 50- 60	311 (29.3)	0.113	0.433	14.76	1.32	23.33	5.88	S
10	8-D16-35- 50- 60	308 (30.2)	0.114	0.433	14.15	1.46	21.86	6.82	S
11	12-D16-70- No- 60	293 (29.7)	0.239	0	16.25	1.22	16.26	2.01	S
12	12-D16-70-100- 60	283 (28.2)	0.247	0.216	17.85	1.68	18.97	4.01	S
13	12-D16-70- 50- 60	311 (29.6)	0.225	0.433	19.90	1.78	24.07	5.99	S
14	12-D16-70- 35- 60	319 (31.1)	0.219	0.619	19.09	1.51	27.74	6.05	S
15	12-D16-00- 75- 90	326 (32.4)	0	0.293	9.01	2.82	17.82	14.14	S
16	12-D16-00- 35- 90	319 (33.3)	0	0.619	9.56	3.22	17.97*	14.02*	F
17	12-D16-35- 75- 60	319 (28.9)	0.110	0.293	14.58	1.20	24.55	6.05	S
18	12-D16-35- No- 90	317 (31.6)	0.110	0	12.05	2.80	13.48	4.02	S
19	12-D16-35- 75- 90	318 (32.9)	0.110	0.293	12.00	2.84	18.69	9.64	S
20	12-D16-35- 35- 90	299 (31.3)	0.117	0.619	11.95	2.62	21.45*	12.04*	F
21	12-D16-35- 75-120	311 (30.0)	0.113	0.293	10.89	5.63	15.73*	20.04*	F
22	12-D16-35- 75-90L	209 (23.8)	0.167	0.293	11.05	2.82	17.10	10.08	S
23	12-D16-35- 75-90H	430 (37.5)	0.081	0.293	13.34	2.94	21.84	12.07	S
24	12-D16-70- 75- 60	317 (25.8)	0.221	0.293	18.66	1.60	22.83	5.93	S
25	12-D16-70- 75- 90	303 (31.1)	0.231	0.293	15.50	3.63	20.25	10.11	S
26	12-D16-70- 75-120	315 (29.0)	0.222	0.293	12.50	5.79	17.64*	20.17*	F
27	12-D16-70- 75-90L	193 (23.2)	0.363	0.293	13.96	3.44	17.83	10.07	S
28	12-D16-70- 75-90H	421 (38.0)	0.166	0.293	17.41	3.66	23.61	10.06	S

*:Value at flexural failure. S : Shear failure, F : Flexural failure.

Shear Strength Carried by Truss Mechanism Fig. 6 shows the effect of transverse reinforcement. Here τ_w is based on the difference between the test values of columns with hoops and those without hoops at the same conditions [Ref. 4 & 5]. The broken line is given by the $\tau_w = P_w \cdot \sigma_{wy}$ (truss action in the B-method where $\cot\phi=1$). It is in comparatively good agreement with test data.

Effect of Fc The relationship between test results and Fc is shown in Fig.7. The computed values given by the B-method are also plotted. All of them increase with Fc and it appears that the increments of the B-method are somewhat larger than those of the test data.

Effect of Axial Compressive Factor η_0 Fig. 8 shows the relationship between shear strength and η_0. As can be seen, shear strength increase with increasing of η_0 and there are some differences between test results and calculated from the B-method,

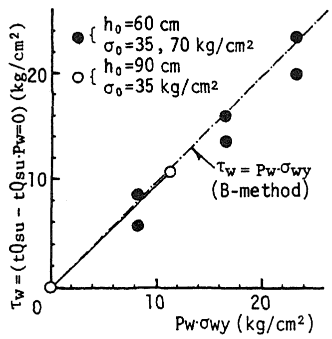

Fig. 6 Shear Strength Carried by Truss

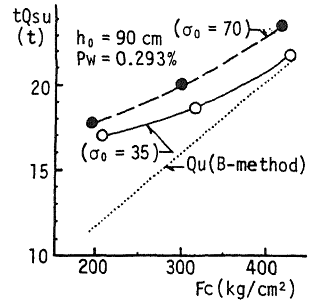

Fig. 7 Shear Strength and Fc

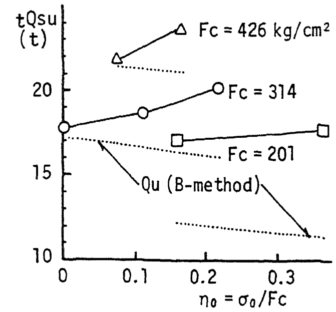

Fig. 8 Shear Strength and η_0

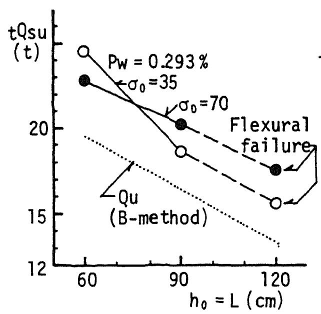

Fig. 9 Shear Strength and h_0

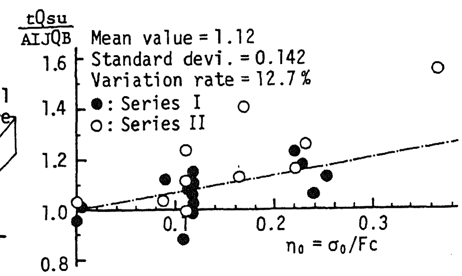

Fig. 10 Relationship Between Shear Strength Ratio and η_0

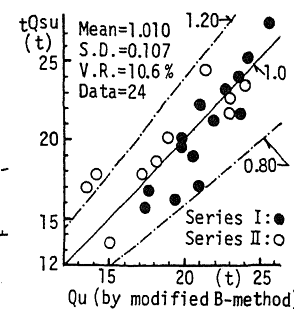

Fig. 11 tQsu and cal.Q by Modified B-method

especially with lower Fc. The reason for that is that the influence of axial compression on shear strength is not considered in the B-method.

Effect of Clear Height h_0 The relationship between shear strength and h_0 is illustrated in Fig. 9. As can be seen, the test results decrease with increasing h_0 just as those calculated from the B-method. The results of 4 columns in flexural failure are also illustrated in Fig. 9. The values calculated from flexural equation are closed to the test results. Their ductility factor at maximum loads ranged from 1.8 to 2.6 and the load carrying capacity dropped little even at a ductility factor of 4.5 and presented a very good ductility.

Influence of Axial Compression on Shear Strength The comparisons of test results of 24 with A and B Methods were made. In the A-method, The ratios of test values to computed values were between 0.999 and 1.664 with an average of 1.310, standard deviation of 0.166 and variation rate of 12.7%. In the B-method, the ratios were 0.896 to 1.554 with an average of 1.120, standard deviation of 0.142 and variation rate of 12.7%. Fig. 10 shows the comparison of test values with computed values in the B-method. As can be seen, those ratios of tests to calculations increase with the axial compressive factor η_0. It is recommended that the influence of η_0 can be considered in the B-method.

Now, it is assumed here that the increment of shear strength in arch mechanism is directly proportional to the axial compressive factor η_0. Then, the contribution of arch mechanism to shear strength can be given by multiplying Eq.(3)by $(1+\eta_0)$. Fig. 11 shows the comparison between tests and values calculated by modified B-method. These calculated values are in a good agreement with test values. The upper limit of η_0 can be considered to be 0.4 based on this research and Ref. 6.

Current Building Codes and Empirical Equations for Shear Strength Many design provisions and empirical equations for shear strength have been proposed. In this section, the adequacy of these equations is examined based on 118 of the most recenly available data [Ref. 5] which were chosen according to the following:

a) The columns in shear failure with sectional area of above 400 cm^2
b) With normal concrete of $F_c = 180 \sim 470$ kg/cm^2 and $\eta_0 = 0 \sim 0.70$
c) With transverse reinforcement of $\sigma_{wy} = 2130 \sim 16300$ kg/cm^2 but less than $25F_c$, without special transverse reinforcement
d) With longitudinal reinforcement of $\sigma_y = 3400 \sim 9800$ kg/cm^2 and $P_t = 0.69 \sim 2.70$ %
e) $M/QD = 0.5 \sim 3.0$

The comparisons of test results of 118 with the values calculated using these equations are given in Table 3. The modified B-method and Chinese design provision are in better agreement with test data as shown in Table 3.

Table 3 Comparison of Test Results of 118 Data

Proposed Equations	Arithmetical Mean	S.D.	V.R.%
Ohno-Shibata-Hattori's Eq. : ${}_{OSH}Q_{SU}$	1.281 (0.775～2.119)	0.241	18.8
Ohno-Arakawa's modified Eq. extended by Hirosawa : ${}_{H}Q_{SU}$	1.208 (0.698～1.907)	0.232	19.2
Muguruma-Watanabe's Eq. : ${}_{MW}Q_{SU}$	1.130 (0.578～1.918)	0.310	27.4
Wakabayashi-Minami's Eq. : ${}_{WM}Q_{SU}$	0.986 (0.507～1.665)	0.219	22.2
New Zealand Design Code : ${}_{NZ}Q$	1.173 (0.587～1.990)	0.274	23.3
ACI Design Code : ${}_{ACI}Q$	1.160 (0.430～2.361)	0.305	26.3
AIJ Design Code (A-method) : ${}_{AIJ}Q_A$	1.278 (0.565～2.326)	0.297	23.3
AIJ Design Code (B-method) : ${}_{AIJ}Q_B$	1.232 (0.670～1.990)	0.282	22.9
Chinese Design Code : ${}_{CH}Q$	1.005 (0.491～1.567)	0.229	22.7
Modified A-method : ${}_{AIJ}Q_A$(mod.)	1.156 (0.527～1.828)	0.242	20.9
Modified B-method : ${}_{AIJ}Q_B$(mod.)	1.077 (0.632～1.589)	0.233	21.6

S.D. : Standard deviation, V.R. : Variation rate.

CONCLUSION

From the investigation reported in this paper, the following conclusion can be drawn:

(1) Effect of transverse reinforcement can be evaluated by using Eq. (2) in the B-method proposed by the AIJ.

(2) Shear strength calculations of the B-method are closer to test results than those of the A-method.

(3) The modified B-method in which the influence of axial compression on shear strength is considered in Eq. (3) multiplied by $(1+\eta_0)$ is in a very good agreement with both tests reported here and 118 available tests. The upper limit of η_0 can be considered to be about 0.4. Further work is needed in this area.

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