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COEFFICIENTS OF CROSS-MODAL CONTRIBUTION FOR MDOF SYSTEMS

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SUMMARY

This paper presents equations for computing the coefficients of cross-modal contributions from the power spectral densities of recorded accelerograms. The equations account for the influence of soil condition and for horizontal or vertical direction of ground motion. The findings from this study show that the site-dependent coefficients may be significant not only for systems with closely spaced frequencies but also for systems which are very flexible or very stiff. However, in the majority of cases, the equation proposed by Wilson, Der Kiureghian and Bayo (Ref. 7) for computing the coefficients based on a white noise is accurate and easy to use.

INTRODUCTION

The response spectrum procedure for computing the maximum response of a multi-degree-of-freedom (MDOF) system consists of (1) transforming the governing equations of motion from the geometric coordinates to a set of normal mode coordinates whereby the modal responses are obtained directly from the spectrum, and (2) combining the modal responses to obtain an estimate of the maximum geometric response. Since the modal spectral responses do not occur at the same time, the accuracy of the procedure depends on how the modal and cross-modal contributions to the geometric response are accounted for in the combining method. Among the combining methods, the one which has been used extensively in practice is the SRSS method (Refs. 1, 2 and 3) where the maximum geometric response is obtained as the square root of the sum of the squares of the modal response maxima. Since this combining method disregards the cross-modal contributions, satisfactory results are achieved only for systems where the frequencies are spaced apart from each other. For systems with closely spaced frequencies, however, the cross-modal contributions are significant (Ref. 4) and the use of the SRSS method may lead to incorrect answers.

Der Kiureghian (Refs. 5 and 6) has developed a combining procedure, based on a random vibration approach, which accounts for the cross-modal contributions. His procedure which is referred to as the "Complete Quadratic Combination or CQC" has been recommended (Ref. 7) as a better alternative to the SRSS method. Der Kiureghian presented expressions for the coefficients of the cross-modal contribution using a white noise and a filtered white noise. His results indicate that in addition to damping, the coefficients depend to a large extent on the frequency content of the excitation. It should be noted that simulating earthquakes by an assumed power spectral density does not account for the soil condition and the direction of motion which influence the frequency content of the ground motion (Ref. 8).

This paper uses smooth normalized power spectral densities from ensembles of accelerograms recorded on alluvium and rock to obtain the coefficients of cross modal contributions. The expressions for evaluating the coefficients are presented in terms of the Kanai-Tajimi power spectral density and the Clough-Penzien filter. The influence of the soil condition on the coefficients is examined and discussed. The responses of a 6-DOF structure consisting of two rigid slabs supported on three corner columns to two base excitations are computed using the coefficients from this study, those proposed in (Ref. 7), and the SRSS method and compared with a time history analysis.

COEFFICIENTS OF CROSS-MODAL CONTRIBUTION

The coefficients of cross-modal contributions ρ_{nm} are defined in terms of the correlation between the n-th and m-th modes of vibration (Refs. 5 and 6) as

$$\rho_{nm} = \lambda_{o,nm} / \sqrt{\lambda_{o,nn}\, \lambda_{o,mm}} \tag{1}$$

where $\lambda_{o,nm}$ is the first spectral moment of the response cross-power spectral density. Assuming that the base excitation is a modulated random process $\{ \ddot{x}_g(t) \}$ with a zero mean (Ref. 8), a set of $\lambda_{o,nm}$ can be obtained in terms of the normalized power spectral density of the excitation $G_{\ddot{x}_g}^{<n>}$ as

$$\lambda_{o,nm} = \mathrm{Re}\ [\int_0^\infty H_n(f)\ H_m^*(f)\ G_{\ddot{x}_g}^{<n>}(f)\ df\ ] \tag{2}$$

where $H_n(f)$ is the complex frequency displacement response function of the n-th mode given by

$$H_n(f) = 1 / \{ (2\pi)^2 [(f_n^2 - f^2) + 2i\xi f_n f] \} \qquad i = \sqrt{-1} \tag{3}$$

and the (*) indicates the complex conjugate of H. If the damping ratio ξ is assumed the same for every mode, it can be shown upon substituting from Eq. (3) into Eq. (2) and letting $s = f/f_m$ and $r = f_n/f_m$ that

$$\rho_{nm} = \frac{[\int_0^\infty q_1(r,s)\, G_{\ddot{x}_g}^{<n>}(s)\, ds]}{\{ [\int_0^\infty q_2(r,s)\, G_{\ddot{x}_g}^{<n>}(s)\, ds]\ [\int_0^\infty q_3(r,s)\, G_{\ddot{x}_g}^{<n>}(s)\, ds] \}^{1/2}} \tag{4}$$

where

$$q_1(r,s) = \frac{(r^2 - s^2)(1 - s^2) + (2\xi s)^2 r}{\{ [(r^2 - s^2)(1 - s^2) + (2\xi s)^2 r]^2 + [(r^2 - s^2) - r(1 - s^2)]^2 (2\xi s)^2 \}}$$

$$q_2(r,s) = 1 / [(r^2 - s^2)^2 + (2\xi s r)^2] \quad ; \quad q_3(r,s) = 1 / (1 - s^2)^2 + (2\xi s)^2]$$

Der Kiureghian used the residue theorem to integrate Eq. (2) for both a white noise and a filtered white noise. While the resulting expression for the white noise is simple, it is complicated for the filtered white noise which is more representative of earthquakes. Wilson, Der Kiureghian, and Bayo (Ref. 7) proposed a simplified expression for the coefficients of cross-modal contributions for lightly damped systems by assuming that the earthquake spectrum is smooth over a wide range of frequencies. For a constant modal damping ratio, their expression reduces to

$$\rho_{nm} = 8\xi^2 (1 + r)\, r^{3/2} / [(1 - r^2)^2 + 4\xi^2 r (1 + r^2)] \tag{5}$$

The above expression does not reflect the influence of soil condition nor does it account for the direction of the motion which influence the frequency content of the excitation. These influences can be included using the power spectral densities of recorded accelerograms on different soil conditions. The Kanai-Tajimi expression for power spectral density (Refs. 9 and 10) together with the Clough-Penzien filter (Ref. 11) which eliminates the singularities in the Kanai-Tajimi expression at low frequencies are used to express the normalized power spectral density as

$$G_{\ddot{x}_g}^{<n>}(f) = |H_1(f)|^2 |H_2(f)|^2 G_o \tag{6}$$

where $H_1(f)$ and $H_2(f)$ are the Kanai-Tajimi and the Clough-Penzien filters, respectively, given by

$$|H_1(f)|^2 = [1 + 4\xi_g^2 (f/f_g)^2] / \{ [1 - (f/f_g)^2]^2 + (2\xi_g f/f_g)^2 \} \tag{7}$$

$$|H_2(f)|^2 = (f/\bar{f})^4 / \{ [1 - (f/\bar{f})^2]^2 + [2\bar{\xi}(f/\bar{f})]^2 \} \tag{8}$$

and G_o is the intensity of the normalized power spectral density. The parameters of the filters f_g, ξ_g, $\bar{f}$, $\bar{\xi}$ and G_o computed using the power spectral densities of the accelerograms on various soils (Ref. 12) are given in Table 1. To avoid lengthy mathematical expressions, numerical integration was used to evaluate the coefficients in Eq. (4). The Nyguist frequency of 25 cps used to compute the power spectral densities, was selected as the upper limit of the integration.

Table 1 Summary of Kanai-Tajimi and Clough-Penzien Parameters Computed from Normalized Power Spectral Density of the Ensemble of Accelerograms

Soil	No. of records	G_o	f_g	ξ_g	$\bar{f}$	$\bar{\xi}$
Horizontal						
Alluvium	161	0.102	2.92	0.34	0.388	0.29
Rock	26	0.070	4.30	0.34	0.486	0.26
Vertical						
Alluvium	78	0.080	4.17	0.46	0.272	0.27
Rock	13	0.053	6.18	0.46	0.502	0.24

RESULTS

Equations 1 and 2 indicate that the coefficients of cross-modal contributions are symmetric. Taking advantage of symmetry, the coefficients can be presented in terms of the larger frequency and a frequency ratio between 0 and 1.0. Figure 1 shows typical plots of the cross-modal contributions as a function of the frequency ratio r and the larger frequency f_m for horizontal motion on alluvium. The plots indicate that in addition to damping and frequency ratio, the coefficients depend on the larger frequency, and at times they may be negative. The plots also show that as f_m and r increase (a stiffer system) or as f_m and r decrease (a more flexible system) ρ_{nm} tend to increase. Figure 2 shows the comparison of the coefficients from this study with those from Eq. (5) and indicates that soil condition and the direction of motion influence the coefficients. The figure also illustrates that for stiff systems, the coefficients of cross-modal contributions may be significant even for cases where the frequencies are spaced apart.

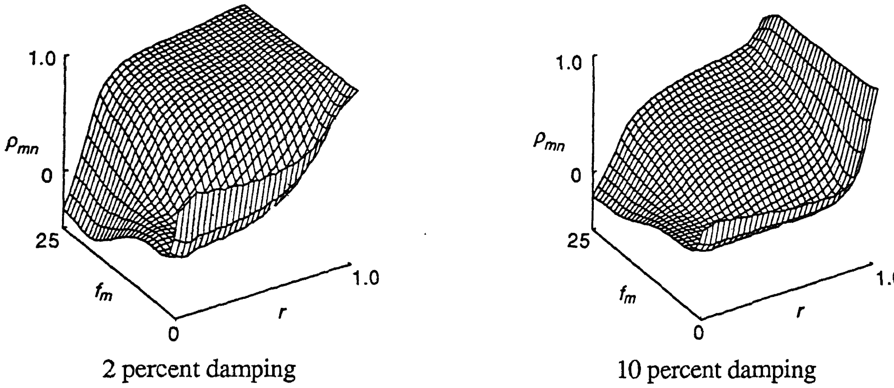

Figure 1 Variation of coefficient ρ_{mn} with frequency ratio r and larger frequency f_m for horizontal motion on alluvium.

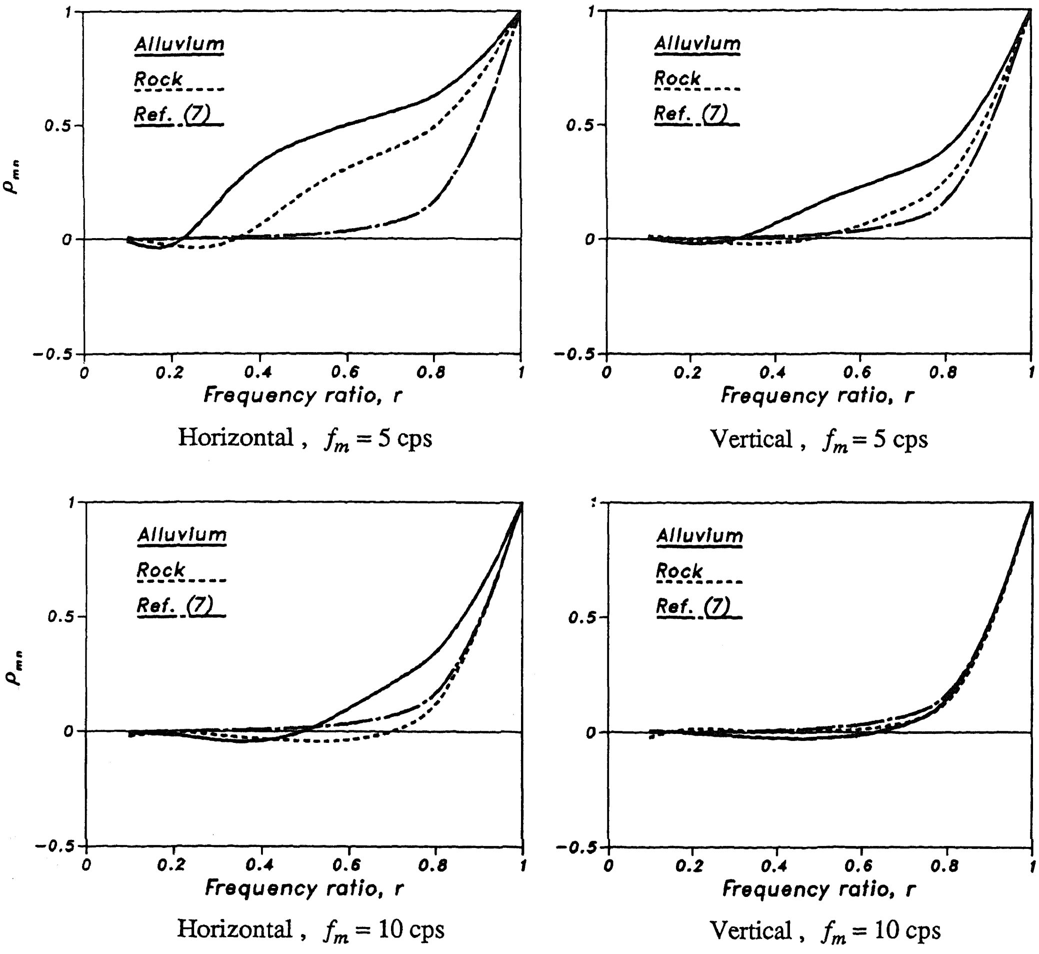

Figure 2 Influence of soil condition on coefficient ρ_{mn} -- 5 percent damping.

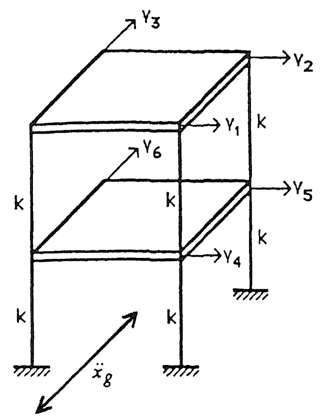

Fig. 3 Rigid slab structure used in the example problem.

The coefficients were used to compute the response of a two story frame consisting of two rigid square slabs supported on columns at three of the four corners (Fig. 3). This example is similar to the one story structure considered in Ref. (11). The structure was subjected to the N21E component of Taft, 1952 (alluvium) and the N21E component of Castaic (rock). The column properties were selected to represent a stiff structure with frequencies of 4.95, 5.39, 9.57, 12.97, 14.10, and 25.00 cps. The response spectrum of each record was used to compute the six modal responses. The square root of the sum of the squares (SRSS) and the complete quadratic combination (CQC) with both the coefficients in Eq. (5) and this study were used to combine the modal responses. The results are presented in Table 2 together with those from the time-history analysis. The table shows that for both alluvium and rock, the results from this study are in close agreement with those using the coefficients in Ref. (7) and compare far better with the response from a time history analysis than do the results from the SRSS combination. In addition, the responses computed from the CQC methods do not show the peculiar behavior (equal displacements at v_2 and v_3, and v_5 and v_6) which is obtained using the SRSS combination.

Table 2 Comparison of Displacement Response for 5 Percent Damping

Method	Displacement (in)					
	v_1	v_2	v_3	v_4	v_5	v_6
Taft N21E (alluvium)						
Time history	.057	.092	.174	.035	.056	.108
SRSS	.088	.132	.132	.054	.082	.082
CQC (Ref. 7)	.064	.088	.165	.039	.055	.102
CQC (This study)	.055	.074	.168	.034	.046	.106
Castaic N21E (rock)						
Time history	.067	.142	.304	.042	.089	.196
SRSS	.144	.258	.258	.089	.159	.159
CQC (Ref. 7)	.097	.188	.312	.060	.116	.193
CQC (This study)	.093	.184	.309	.058	.114	.194

An examination of the coefficients of cross-modal contributions for the structure considered in this study and several others indicates that the coefficients computed from the site-dependent power spectral densities are generally larger than those from Eq. (5). However, since the contributions from the higher modes are not as significant, the results from the two CQC methods are in close agreement. The coefficients computed from Eq. (5) give, in the majority of cases, satisfactory results. For structures which are either very flexible or very stiff, the site-dependent coefficients may be more appropriate to use in combining the modal displacements and modal forces.

CONCLUSIONS

The coefficients of cross-modal contributions are influenced by soil condition and may be significant not only for systems with closely spaced frequencies but also for systems which are very flexible or very stiff. In the majority of cases, however, Eq. (5) proposed in Ref. (7) for computing the coefficients is adequate, gives satisfactory results and is easy to use. The equations presented herein may also be used to compute the coefficients of cross-modal contributions for vertical motion.

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