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ON THE USE OF LANCZOS CO-ORDINATES IN THE DYNAMIC ANALYSIS OF STRUCTURES

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SUMMARY

In this paper, Lanczos method is applied to different types of structures to show the effectiveness of the method in the dynamic analysis of structures. The structures are analyzed by the following four methods, namely the classical mode superposition (CMS), the Lanczos Mode Superposition (LMS), Wilson-θ step-by-step integration method and Lanczos Wilson-θ step-by-step integration method. The displacements are compared using these four methods. The results show that the Lanczos method is a very powerful and efficient method in determining the frequencies and mode shapes of the structures, but when the displacement response analysis of structures is needed, the method should be applied with special considerations.

INTRODUCTION

Different methods can be used in the analysis of the response of structures subjected to dynamic loads. These methods require the determination of the eigenvalues and eigenvectors of the generalized undamped eigenproblem of the equations of motion. There are different algorithms presently used for determining eigenvalues and eigenvectors such as Householder Method (Ref.1), Determinant Search (Ref.2) and Subspace iteration (Ref.3). Recently the Lanczos algorithm has been used for the solution of structural eigenvalue problems (Ref.4). The use of Lanczos co-ordinates has been suggested by Nour-Omid and Clough (Ref.5 and 6). In this paper, the Lanczos method is applied to dynamic analysis of structures and the results are compared with those of the classical mode superposition and Wilson-θ step-by-step integration methods.

FUNDAMENTAL EQUATIONS

The equations of motion for a structural system can be modelled by a discretized system and can be expressed in terms of joint displacements U and their derivatives $\dot{U}$ and $\ddot{U}$ as

$$\underline{M}\,\underset{\sim}{\ddot{U}} + \underline{C}\,\underset{\sim}{\dot{U}} + \underline{K}\,\underset{\sim}{U} = \underset{\sim}{f} \tag{1}$$

in which $\underset{\sim}{M}$, $\underset{\sim}{C}$, and $\underset{\sim}{K}$ are the mass, damping and stiffness nxn matrices respectively, $\underset{\sim}{f}(t)$ is the externally applied load vector. For simplicity the undamped vibrations are considered.

The Lanczos algorithm used in the analysis is the one given in (Ref.5). Using the following coordinate transformation

$$\underset{\sim}{U} = \underset{\sim}{Q}_m \underset{\sim}{X}(t) \quad (2)$$

and the orthogonality properties of Lanczos matrix $\underset{\sim}{Q}_m$, equation (1) takes the form

$$\underset{\sim}{T}_m \underset{\sim}{X} + \underset{\sim}{I}\ddot{\underset{\sim}{X}} = \underset{\sim}{g}_m \quad (3)$$

in which $\underset{\sim}{T}_m$ is a tridiagonal matrix, $\underset{\sim}{I}$ is the identity matrix and $\underset{\sim}{g}_m = \underset{\sim}{Q}_m^T M K^{-1} \underset{\sim}{f}$, $(m<n)$.

For comparison purposes, the equation (1) is solved using the classical mode superposition and Wilson-θ step-by-step integration method. Equation (3) is also solved by applying the mode superposition and Wilson-θ step-by-step integration methods.

EXAMPLES

Two examples are considered in comparing the results obtained by different methods.

Example 1

The 5-degrees of freedom system shown in Figure 1 with properties given in Table 1 has been analyzed by four different methods. In applying the Lanczos mode superposition and Lanczos Wilson-θ step-by-step integration methods five vectors have been used. Fewer number of vectors than five yielded displacements which are somewhat different than the ones obtained by the classical mode superposition and Wilson-θ step-by-step integration methods. The displacements at the top level are given in Table 2. The time increments are taken as $\Delta t = 2T/19$ in which T is the first fundamental period.

Table 1 Properties of Example 1

Member Number	Length (cm)	Moment of Inertia (cm^4)	Mass ($t\ sec^2/cm$)
1	300	48020	0.11841
2	300	39220	0.11841
3	300	39220	0.11841
4	300	31400	0.11841
5	300	31400	0.07894

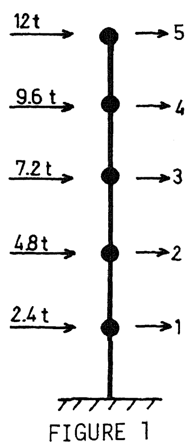

FIGURE 1

Table 2 Displacement $U_5(t)$, cm

t(sec)	CMS and LMS with 5 vectors	Wilson-θ	Lanczos Wilson-θ
0.122	0.943	0.912	
0.244	2.848	2.813	
0.366	5.046	4.945	
0.488	6.698	6.613	
0.610	7.213	7.141	
0.732	6.024	6.233	
0.854	3.796	4.155	
0.976	1.757	1.930	
1.098	0.345	0.526	
1.220	0.305	0.325	
1.342	1.716	1.407	
1.464	4.140	3.484	
1.586	6.053	5.669	
1.708	7.073	6.958	
1.830	6.755	6.857	
1.952	5.039	5.508	
2.074	2.614	3.446	
2.196	0.789	1.408	

Example 2

The building with flexible beams having 30 degrees of freedom shown in Figure 2 is considered as a second example. The properties are given in Table 3.

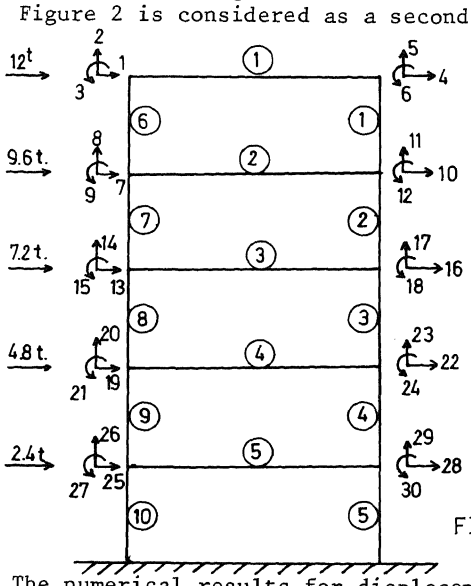

FIGURE 2

Table 3 Properties of Example 2

Beam lengths = 6 m
Column heights = 3 m
Moment of Inertias

$I_1=I_2= 125120\ cm^4$

$I_3=I_4=I_6=I_7=I_{11}=I_{12}= 15700 cm^4$

$I_8=I_9=I_{13}=I_{14}=19610\ cm^4$

$I_{10}=I_{15}= 24010 cm^4$

Mass at each joint = 0.059205 t sec^2/cm each
Mass at top story joints= 0.039470 t sec^2/cm each

The numerical results for displacement U_1, U_{13} and U_{25} are shown in Figures 3, 4 and 5. In calculating these displacements seven Lanczos vectors are used which give very good results as compared to CMS. Higher number of vectors yield displacements greater than the values obtained by CMS. The numerical values are given in Tables 4,5 and 6. The classical mode superposition method and Lanczos mode superposition method with 7 vectors have a very good match while Wilson-θ step-by-step procedures applied to equations (1) and (3) yield results which are also close to each other.

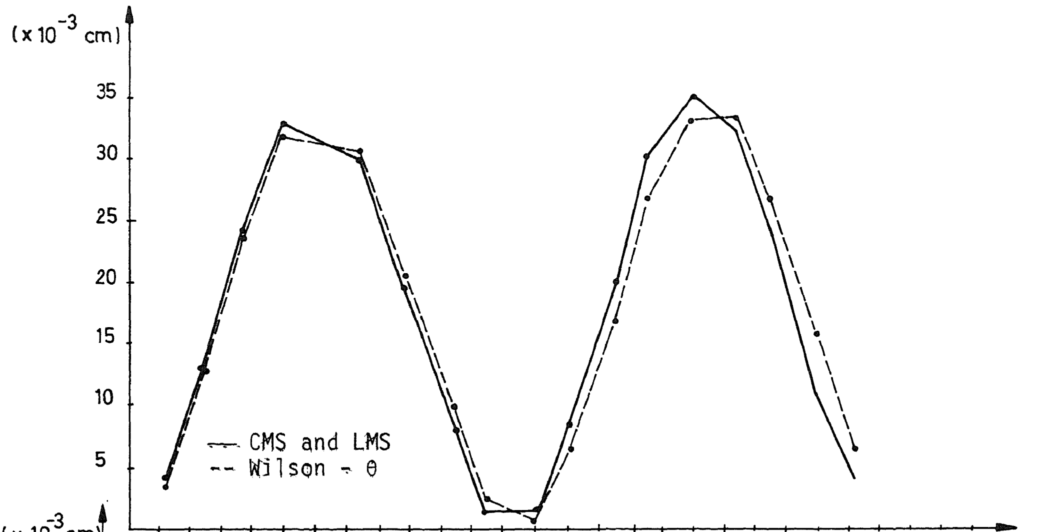

FIGURE 3

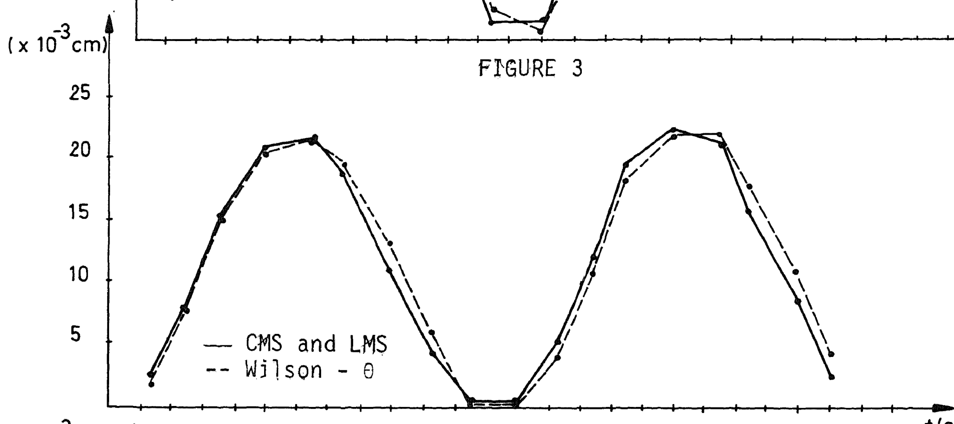

FIGURE 4

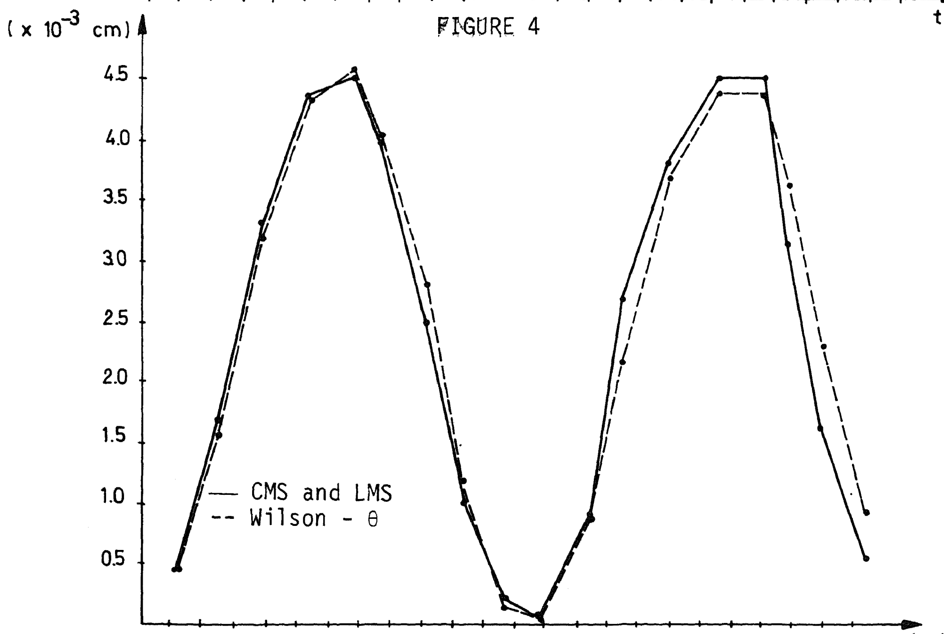

FIGURE 5

Table 4 Displacement $U_1(t)$, cm

t(sec)	CMS	LMS with 7 vectors	Wilson-θ	Lanczos Wilson-θ
0.258	4.167	4.162	3.839	4.027
0.516	13.592	13.578	13.164	13.248
0.774	24.409	24.391	24.003	24.032
1.032	33.193	33.177	32.421	32.425
1.290	34.945	34.936	35.058	35.052
1.548	39.205	29.203	30.213	30.205
1.806	19.277	19.275	20.331	20.321
2.064	8.276	8.276	9.750	9.741
2.322	1.081	1.062	2.266	2.257
2.580	1.406	1.388	0.897	0.887
2.838	8.354	8.343	6.587	6.578
3.096	19.014	19.011	16.863	16.854
3.354	29.557	29.558	27.207	27.197
3.612	35.029	35.023	33.713	33.703
3.870	32.807	32.798	33.874	33.865
4.128	24.666	24.647	27.351	27.342
4.386	13.619	13.609	16.834	16.825
4.644	3.738	3.739	6.866	6.856

Table 5 Displacement $U_{13}(t)$, cm

t(sec)	CMS	LMS with 7 vectors	Wilson-θ	Lanczos Wilson-θ
0.258	2.067	2.059	1.948	2.062
0.516	8.241	8.226	7.989	8.030
0.774	15.838	15.825	15.368	15.378
1.032	20.914	20.904	20.822	20.818
1.290	22.173	22.165	22.127	22.120
1.548	18.903	18.899	19.224	19.216
1.806	11.914	11.907	13.010	13.001
2.064	4.863	4.845	5.827	5.817
2.322	0.554	0.539	0.927	0.918
2.580	0.361	0.355	0.303	0.294
2.838	4.879	4.874	3.925	3.916
3.096	12.186	12.177	10.396	10.387
3.354	18.715	18.746	17.292	17.283
3.612	22.157	22.144	21.614	21.605
3.870	21.156	21.143	21.534	21.525
4.128	15.611	15.608	17.294	17.285
4.386	8.153	8.154	10.620	10.611
4.644	2.231	2.313	4.065	4.056

CONCLUSIONS

Eigen problem of equation (3) is usually solved not for all eigenvalues n, but is truncated to a value m where $m \ll n$. It is found that choosing m as twice the eigenvalues desired yields good results. For determining the displacements a special care has to be exercised as to the number of Lanczos vectors used. From the examples above, it can be concluded that in using modal superposition on equation (3), one may take the number of modes as half of the Lanczos vector m. The method has to be tested on a system with number of degrees of freedom much higher than the ones considered in this paper.

Table 6 Displacement $U_{25}(t)$, cm

t(sec)	CMS	LMS with 7 vectors	Wilson-	Lanczos Wilson-
0.258	0.449	0.441	0.422	0.459
0.516	1.675	1.666	1.537	1.547
0.774	3.291	3.286	3.241	3.243
1.032	4.359	4.356	4.300	4.297
1.290	4.501	4.493	4.526	4.521
1.548	3.963	3.962	3.961	3.951
1.806	2.477	2.477	2.722	2.716
2.064	0.887	0.881	1.181	1.176
2.322	0.212	0.207	0.151	0.146
2.580	0.039	0.042	0.088	0.083
2.838	0.893	0.892	0.811	0.806
3.096	2.636	2.632	2.113	2.108
3.354	3.792	3.792	3.568	3.563
3.612	4.500	4.496	4.448	4.480
3.870	4.477	4.469	4.427	4.422
4.128	3.121	3.117	3.552	3.547
4.386	1.653	1.652	2.206	2.201
4.644	0.547	0.537	0.836	0.831

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