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SEISMIC RESPONSE OF STRUCTURE SUBJECTED TO SIX CORRELATED EARTHQUAKE COMPONENTS BY MODE ACCELERATION METHOD

Mohsen GHAFORY-ASHTIANY*

* Ast. Professor, Nuclear Research Center, Atomic Energy Organization of Iran, Tehran, Islamic Republic of Iran.

ABSTRACT

Response Spectrum approach using "mode acceleration" of structural dynamic instead of commonly used "mode displacement" is developed to obtain mean square response of structure subjected to six correlated earthquake components by using relative velocity and relative acceleration spectra of principal components of ground acceleration as input. This approach proves its priority over other methods when using only a first few modes. The formulation is such that the methodology of obtaining the Worst-Case response irrespective of the structural orientation can be used.

INTRODUCTION

For the calculation of seismic design response of classically damped structural system subjected to multicomponent excitations, the method of Square Root of the Sum of the Square (SRSS) of modal response using mode displacement of structural dynamic is commonly used. Often in these approaches, only a first few modes are used in the analysis, as usually the higher modes do not contribute much to the response. However, there are situations involving certain response quantities or certain structures where the contribution of the high frequency modes can not be neglected without affecting the accuracy of the results.

To improve the accuracy of the results with only a first few modes, Singh and Mehta [1,2] have developed an alternative response spectrum approach for the calculation of design response for a single excitation component. This approach is based on the "mode-acceleration" method of structural dynamics. It requires the seismic inputs to be defined in terms of relative acceleration and relavtive velocity spectra. Herein, a SRSS approach based on the method of mode acceleration is developed for the calculation of design response of structures subjected to the six components of earthquake excitation. This formulation also include the effect of the three rotational components of excitation which are commonly neglected "as being of minor consequent" in seismic analysis. Here basically the Newmark approach [3] used by Ghafory-Ashtiany and Singh [4] to obtain the characteristics of the rotational components.

Here, the procedure of obtaining the Worst-Case response irrespective of the direction of the impinging ground motion developed by Singh and Ghafory-Ashtiany [5] is briefly described. The numerical results are also presented to demonstrate the advantage of the mode acceleration formulation over the common mode displacement formulation [6].

DESIGN RESPONSE

General equations of motion for MDOF structral system subjected to six components of earthquake are as follows :

$$[M]\ \{\ddot{u}\} + [C]\ \{\dot{u}\} + [K]\{u\} = -[M]\ [r]\ \{E'(t)\} \qquad (1)$$

where [M], [C], [K] are mass, damping and stiffness matrices, {u} = relative displacement vector; [r] = the influence coefficient matrix (Nx6) with its ℓ^{th} column, $\{r_\ell\}$, being the ground displacement influence vector for the ℓ^{th} components, and $\{E'\}$ is vector of six components (3 translational and 3 rotational) of earthquake defined in terms of 3 principal components of earthquake as [6]:

$$\{E'(t)\} = ([G_1] + \frac{1}{2C}\frac{d}{dt}[G_2])\ [D]\{E(t)\} \quad (2)$$

where [D] is matrix of direction cosine, $\{E(t)\} = \{\ddot{x}_1\ \ddot{x}_2\ \ddot{x}_3\}$ is vector of principal component of earthquake, c is shear wave velocity and $[G_1]$ and $[G_2]$ are constant matrix of transformation defined as :

$$[G_1] = [\ [I] : [0]\ ],\quad [G_2] = [\ [0] : \begin{matrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{matrix}\ ] \quad (3)$$

where [I] is 3x3 identity matrix and [0] is 3x3 null matrix.

Modal analysis approach is used for evaluation of design response in terms of ground response spectra. Using normal mode shapes, equation of motion will be decoupled to :

$$\ddot{V}_j + 2\beta_j\omega_j\dot{V}_j + \omega_j^2\ V_j = -\{\gamma_j\}^T\{E'(t)\} \quad (4)$$

where V_j=j^{th} principal coordinate of modal displacement, ω_j=j^{th} natural frequency, β_j=j^{th} modal damping ratio, $\{\gamma_j\}$= j^{th} vector of participation factor, defined as $\gamma_{\ell j} = \{\gamma_j\}^T\ [M]\{r_\ell\}/m_j$, and m_j=j^{th} modal mass. Common mode displacement approach obtains V_j from the solution of the Duhamel integral, but for mode acceleration method, V_j is defined in terms of the modal velocity and acceleration. Thus a response quantity which is linearly related to displacement can be defined as :

$$S(t) = -\sum_{j=1}^{N} \zeta_j[\{\gamma_j\}^T\ \{E'(t)\} + 2\beta_j\omega_j\dot{V}_j + \ddot{V}_j]/\omega_j^2 \quad (5)$$

where ζ_j is response mode shape.

To obtain the design response (S^2_d), mean square response, $Ex[S^2(t)]$, is required, which, in turn, can be obtained from autocorrelation function of S(t).

$$Ex[S(t_1)S(t_2)] = \sum_{j=1}^{N}\sum_{k=1}^{N} \zeta_j\ \zeta_k\ [\ \{\gamma_j\}^T\ Ex[\ \{E'(t_1)\}\ \{E'(t_2)\}^T]\ \{\gamma_k\}$$

$$+ \{\gamma_j\}^T(2\beta_k\omega_k Ex[\{E'(t_1)\}\dot{V}_k(t_2)] + Ex\ [\{E'(t_1)\}\ \ddot{V}_k(t_2)]\ )$$

$$+ \{\gamma_k\}^T\ (2\beta_j\omega_j\ \ Ex[\{E'(t_2)\}\dot{V}_j(t_1)\ ] + Ex[\{E'(t_2)\}\ \ddot{V}_j(t_1)]\ )$$

$$+ 4\beta_j\beta_k\omega_j\omega_k\ Ex[\dot{V}_j(t_1)\dot{V}_k(t_2)] + Ex[\ddot{V}_j(t_1)\ddot{V}_k(t_2)]$$

$$+ 2\beta_j\omega_j\ Ex[\dot{V}_j(t_1)\ddot{V}_k(t_2)] + 2\beta_k\omega_k\ Ex[\dot{V}_k(t_2)\ \ddot{V}_j(t_1)]\ ] \quad (6)$$

Substituting for auto-and cross-correlation terms in above equation [6] and assuming the input and output process to be stationary, the design response can be as :

$$S_d^2 = \sum_{\ell=1}^{3} \{d_\ell\}^T\ [R_\ell]\ \{d_\ell\} \quad (7)$$

where $\{d\ell\}$ is direction cosine and $[R_\ell]$ is response matrix for ℓ^{th} excitation. An element of response matrix is defined as :

$$R_{\ell mn} = \sum_{j=1}^{N}\sum_{k=1}^{N} (\zeta_j\zeta_k/\omega_j^2\omega_k^2)\ [\Gamma_{1mnjk}\ A_{g\ell}^2 + \Gamma_{2mnjk}\ \dot{A}_{g\ell}^2]$$

$$+ \sum_{j=1}^{N} (\zeta_j^2/2\omega_j^4)\{\Gamma_{1mnjk}\ [2\omega_j^2\ (1-2\beta_j^2)R_{v\ell j}^2 - R_{a\ell j}^2] + \Gamma_{2mnjk}\ [\omega_j^4\ R_{v\ell j}^2 - A_{g\ell}^2]\}$$

$$+\sum_{j=1}^{N}\sum_{K=j+1}^{N}(\zeta_j\zeta_k/\omega_j\omega_k)\ \{\Gamma_{1mnjk}[F_1R^2_{v\ell j}+F_2R^2_{a\ell j}+F_3R^2_{v\ell k}+F_4R^2_{a\ell k}+F_5A^2_{g\ell}]$$
$$+\Gamma_{2mnjk}[F_1R^2_{v\ell j}+F_2R^2_{a\ell j}+F_3R^2_{v\ell k}+F_4R^2_{a\ell k}+F_5A^2_{g\ell}]$$
$$+\Gamma_{3mnjk}[F_1R^2_{v\ell j}+F_2R^2_{a\ell j}+F_3R^2_{v\ell k}+F_4R^2_{a\ell k}+F_5A^2_{g\ell}] \quad (8)$$

where F_1 , F_2.....and F_5 are defined in Ref. 7. Ag_ℓ = peak ground acceleration, $\dot{A}g_\ell$ = maximum rate of change of ground acceleration, $R_{v\ell j}$ and $R_{a\ell j}$ are, respectively, relative velocity and relative acceleration response spectra of ℓ^{th} earthquake component for ω_j and β_j. Γ_{1mnjk}, Γ_{2mnjk} and Γ_{3mnjk} are defined as :

$$\Gamma_{1mnjk} = \gamma_{mj}\ \gamma_{nk} + \gamma_{nj}\ \gamma_{mk}$$

$$\Gamma_{2mnjk} = \sum_{p=4}^{6}\sum_{q=4}^{6} G_{2mp}\ G_{2nq}\ (\gamma_{pj}\ \gamma_{qk} + \gamma_{qj}\ \gamma_{pk})/4C^2$$

$$\Gamma_{3mnjk} = [\sum_{p=4}^{6} G_{2mp}(\gamma_{pj}\ \gamma_{nk} - \gamma_{nj}\ \gamma_{pk}) - \sum_{q=4}^{6} G_{2nq}(\gamma_{mj}\ \gamma_{qk} - \gamma_{qj}\ \gamma_{mk})]/2C \quad (9)$$

No special advantage is gained by this approach if Eq.8 is going to be used for exact evaluation of $R_{\ell mn}$ since it also requires complete sets of modes, especially for the evaluation of the first two terms. The following analysis show that the first two terms can be obtained from the following psuedo static problem :

$$[K]\{u_s\} = [M]\ [r]\ \{E'(t)\} \quad (10)$$

where $\{u_s\}$ = a vector of the time dependent displacement obtained as a solution of Eq.10, the psuedo static response quantity, S(t) , in terms of $\{u_s\}$ is :

$$S(t) = \{k_j\}^T\ \{u_s\} \quad (11)$$

Expanding $\{u_s\}$ in terms of $\{\phi_j\}$, the modal vector of Eq.1, and obtaining the coefficient of exponsion from Eq.10, S(t) become :

$$S(t) = \sum_{j=1}^{N} \zeta_j\ (\ \{\gamma_j\}^T\ \{E'(t)\}\ )/\omega_j^2 \quad (12)$$

with a similar approach the design psuedo static response can be written as :

$$S_d^2 = \sum_{\ell=1}^{3} \{d_\ell\}^T\ [R_\ell^{(s)}\ ]\{d_\ell\} \quad (13)$$

where $[R_\ell{}^{(s)}]$ is psuedo static response for ℓ^{th} = excitation which can element of the matrix can be written as :

$$R_{\ell mn}^{(s)} = \sum_{j=1}^{N}\sum_{k=1}^{N} (\zeta_j\zeta_k/\omega_j^2\omega_k^2)\,(\Gamma_{1mnjk}\ A^2_{g\ell}+\Gamma_{2mnjk}\ \dot{A}_{g\ell}) \quad (14)$$

Eq.14 is identical to the first two terms of Eq.8, and can also be obtained by a psuedo-static analysis of Eq. 10. From Eq. 10 $\{u_s\}$ can be written as :

$$\{u_s\} = \sum_{p=1}^{6} \{u_{sp}\}\ E_p'(t) \quad (15)$$

where $\{u_{sp}\}$ is obtained as a solution of the following linear simultaneous equations :

$$[K]\{u_{sp}\} = [M]\{r_p\} \quad (16)$$

the response quantity S(t) can also be obtained from Eq.15 as :

$$S(t) = \sum_{p=1}^{6} S_p'\ E_p(t) = \{S'\}^T\{E'(t)\} \quad (17)$$

where $S_p' = \{k_i\}\{u_{sp}\}$. Again using Eq.12, the design response of S(t) can be written in same form as Eq.13, with $R_{\ell mn}^{(s)}$ of :

$$R^{(s)}_{\ell mn} = S'_m\ S'_n\ A^2_{g\ell} + \tfrac{1}{4}C^2 \sum_{p=4}^{6} \sum_{q=4}^{6} G_{2mp}\ G_{2nq} S'_p S'_q\ \dot{A}_{g\ell} \tag{18}$$

It is seen that in the above equation, the first term is associated with the psuedo-static response for the translatory motion and the second term is due to the rotational effects. Thus Eq.18 would replace the first two terms of Eq.8. The total response can now be written as a sum of the psuedo-static and dynamic responses, that is

$$S^2_d = \sum_{\ell=1}^{3} \{d_\ell\}^T\ ([R^{(s)}_\ell] + [R^{(D)}_\ell])\ \{d_\ell\} \tag{19}$$

which an element of the dynamic response, $R^{(D)}_{\ell mn}$, is same as Eq.8. without the first two terms. The design response for purely translational excitation can be obtained by neglecting the terms associated with the rotational effects. The main advantage of this approach is that only a first few modes are necessary in the calculation of the response, because the terms associated with $R_{V\ell}$ and $R_{a\ell}$ become small for modes with frequencies higher than the input frequency.

WORST-CASE RESPONSE

Eq.19 gives the design response for specific $\{d_\ell\}$. However, the orientation of structural axis relative to the principal component of excitation will never be known in advance. Here, the methodology developed earlier by Singh and M.Gh. Ashtiany is used to obtain the Worst-Case response irrespective of the structural orientation. The procedure is as follows:

$$[R_\ell]\{d_\ell\} - \lambda_\ell\{d_\ell\} = 0 \qquad \ell=1,2,3 \tag{20}$$

Solution of each of the above equation provides three eigenvalues, $\lambda_{\ell 1}$, $\lambda_{\ell 2}$ and $\lambda_{\ell 3}$, respectively, defines the response due to ℓ^{th} excitation when it is applied along the $\{d^{(1)}\}$,$\{d^{(2)}\}$and$\{d^{(3)}\}$directions, where $\{d^{(i)}\}$ are eigenvectors of Eq. 20.

To obtain the Worst-Case response a search through 18 possible combination (6 per each set of eigenvector) have to be made. These possibilities are numerated in Ref 7. For example one of the combination is :

$$S^2_d = \lambda_{11} + \{d^{(2)}_1\}^T\ [R_2]\{d^{(2)}_1\} + \{d^{(3)}_1\}^T\ [R_3]\{d^{(3)}_1\} \tag{21}$$

where$\lambda_{11}$is the response due to excitation-1 applied along director $\{d^{(1)}_1\}$, the second term represents the response due to excitation-2 applied along direction $\{d^{(2)}_1\}$ and similarly for third term.

NUMERICAL RESULTS

The main purpose of this proposed formulation is to solve the problems associated with inclusion of high frequency modes in evaluation of design response. Here some numerical result is shown to support the claim that "using the mode acceleration approach does not affect the accurracy of response when higher mode get exculed from the summation process" the input motion, relative velocity and acceleration spectra, in these analysis are defined in terms of three independent Kanai-Tajimi spectral density functions representing the major, intermediate and minor principal excitation components.

To show the advantage of the new mode combination, the three story torsional structure shown in Fig.1 have been considered here. Each floor has 3 degrees-of-freedom : two horizontal translation and a rotation about the vertical axis. The results have been obtained for various structural frequency parameter, $\omega=\sqrt{k/m}$=10,33.4 and 50 cps., to represent soft to stiff structures. Table 1 shows the modal frequencies, damping ratios and participation factors for five earthquake component (2 translational and 3 rotational) for the system with ω=10cps and eccentricity parameter, e/r=0.01.

To show the effectiveness of the mode acceleration formulation, the story shear, torsional moments and column bending moments design response have been obtained with all nine modes (Exact Response) as well as with only first three modes (Approximate Response). The results are shown in Table 2 and are in the mg-ft. units. the mode displacements and the mode acceleration method provide exactly the same value when all nine modes are used in the analysis and thus the verification of proposed method. Two types of response values are obtained for each case:

1. The Worst-Case (Maximum) Response "MR". These are shown in columns(1),(3) and (5). The values in columns (3) and (5) are shown in the ratio of column(1).
2. The Response"SRSS",with inputs applied along the geometric axes of the structure, without making any search for Worst-Case response. Ratio of the these values to the values of column (1) are shown in column (2), (4) and (6). These results show that if no search is made for maximum value, the calculated response is approximately 10 to 35% underestimated.

Comparison of results in columns (3) and (5) with values in columns (4) and (6) clearly show that mode acceleration approach provides more accurate response values than mode displacement approach for the same number of modes used in the analysis.

CONCLUSION

In this paper a response spectrum approach based on the mode acceleration method of structural dynamics is developed. The use of such an approach is especially desirable if the high frequency structural modes contribute to the response significantly. In the commonly used mode displacement approach, such high modes must be calculated explicitly and included in the modal analysis. In the mode acceleration formulation, these modes, however, need not be calculated explicity, their effect can be included through a simple static analysis of the structure. The input in this approach must be prescribed in terms of the relative acceleration and relative velocity spectra of the ground motion. Again, the formulation considers all six correlated earthquake. The numerical results, demonstrating the benefits of this alternative formulation, and also the importance of the rotational components in the calculation of the design response, are presented.

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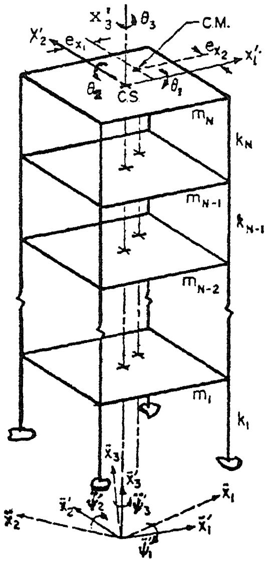

Fig. 1 - A MULTISTORY TORSIONAL BUILDING WITH ECCENTRIC MASS AND STIFENESS CENTERS

TABLE 1 : DYNAMIC CHARACTERISTICS OF STRUCTURE IN FIG.1 FOR e/r=0.01, ω = 10cps

Mode No.	Freq. cps	Damping Ratio	Participation Factors in X_ℓ direction				
			X_1	X_2	X_4	X_5	X_6
1	6.236	.0097	1.44	-1.44	62.3	62.3	0.03
2	6.237	.0097	1.44	1.44	-62.3	62.3	0.00
3	8.294	.0050	-.03	0.03	-1.14	-1.14	1.57
4	12.79	.0293	-.07	0.07	-7.29	-7.29	0.03
5	12.80	.0293	-.07	-.07	7.30	7.30	-.08
6	16.03	.0792	-.04	0.04	-.45	-.45	0.11
7	17.72	.0206	0.67	0.67	-8.03	8.03	0.00
8	17.72	.0207	0.66	0.66	-8.03	-8.03	-.04
9	36.85	.0144	-.00	0.00	-.04	-.04	1.52

TABLE 2: BASE SHEAR, TORSIONAL MOMENT AND COLUMN BENDING MOMENT OF STRUCTURE IN FIG.1, e/r=.01, ω=10 cps.

	Exact Response Value 9 Modes		Approximate Response Using 3 Modes By			
			Mode Displc.		Mode Accl.	
Type of Response	Maximum Resp. (1)	SRSS / Max. (2)	R-Max. / A-Max. (3)	SRSS / A-Max. (4)	R-Max. / A-Max. (5)	SRSS / A-Max. (6)
Base Shear in X_1- Direction	4.329	.921	.800	.718	.991	.913
Base Shear in X_2- Direction	4.329	.830	.800	.656	.991	.822
Torsional Moment	0.068	.982	.430	.423	1.000	.984
Mom. of Col.1 in X_1- Direc.	10.539	.917	.802	.715	.991	.909
Mom. of Col.2 in X_2- Direc.	11.117	.834	.798	.662	.991	.825