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ROLE OF TENSORIAL FORMULATIONS IN CHARACTERIZING THE POINTWISE MULTI-DIMENSIONAL PERFORMANCE OF DYNAMIC SYSTEMS

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SUMMARY

A family of tensorial formulations is shown to be advantageously used for obtaining better insights into the pointwise multi-D features of earthquake shaking and structural response. This can be rendered sufficiently general and resorts to the tensors of RS values or RMS amplitudes by including also the tensors of their time-domain and/or frequency-domain decomposition. With an interpretation related to the energy fed into simple oscillators of multi-D isotropy, findings in the present study are highlighted by geometric simplicities characterizing the tensor fields of response. Examined in addition is practically important but non-tensorial properties in the pointwise directional dependence of peak amplitudes.

Instructive Equalities For a multi-D time history, $\{a(t)\}$, which exhibits its vectorial nature upon rotating coordinate axes, two matrices of

$$[P(t,\omega)] = \mathrm{Re}(\{F(t,\omega)\}\overline{\{F(t,\omega)\}}^T)$$

$$\text{with} \quad \{F(t,\omega)\} = \int_{-\infty}^{t} \{a(t_1)\} \exp(-j\omega t_1)\, dt_1$$

$$[P'(t,\omega,\zeta)] = \mathrm{Re}(\{F'(t,\omega,\zeta)\}\overline{\{F'(t,\omega,\zeta)\}}^T)$$

$$\text{with} \quad \{F'(t,\omega,\zeta)\} = \int_{-\infty}^{t} (\{a(t_1)\} \exp(-\zeta\omega(t-t_1)))\exp(-j\sqrt{1-\zeta^2}\,\omega t_1)\, dt_1$$

are first introduced. Obviously they prove real, symmetric and non-negative definite tensors. Physical meaning of each tensor is also clear therein; $[P(t,\omega)]^{1/2}$ stands for the ordinary Fourier amplitude of $\{a(t_1)\}$ truncated at $t_1 = t$, while $[P'(t,\omega,\zeta)]^{1/2}$ corresponds to its evolutionary (instantaneous) Fourier amplitude weighted toward t under a suitable choice of the parameter ζ. With relation to the former tensor, a multi-D version of Husid plot:

$$[E(t)] = \int_{-\infty}^{t} \{a(t_1)\}\{a(t_1)\}^T dt_1 \quad \left[= \frac{1}{\pi}\int_0^{\infty} [P(t,\omega)]\, d\omega\right]$$

is additionally used. This includes, as its particular case of $[E(\infty)]$, the squared RS or RMS intensity tensors proposed by Arias or Penzien.

Then let another set of two matrices (real, symmetric, non-negative definite and tensorial) be defined without resorting to the Fourier integral transform but by means of the motion of isotropically multi-D simple oscillators subjected to $\{a(t)\}$:

$$[W(t,\omega,\zeta)] = -\int_{-\infty}^{t} (\{a(t_1)\}\{\dot d(t_1)\}^T + \{\dot d(t_1)\}\{a(t_1)\}^T)\, dt_1$$

$$\left[= \{\dot d(t)\}\{\dot d(t)\}^T + 4\zeta\omega\int_{-\infty}^{t} \{\dot d(t_1)\}\{\dot d(t_1)\}^T dt_1 + \omega^2\{d(t)\}\{d(t)\}^T\right]$$

$$[W'(t,\omega,\zeta)] = \{\dot d(t)\}\{\dot d(t)\}^T + \zeta\omega\frac{d}{dt}(\{d(t)\}\{d(t)\}^T) + \omega^2\{d(t)\}\{d(t)\}^T$$

$$\text{with} \quad \{\ddot d(t)\} + 2\zeta\omega\{\dot d(t)\} + \omega^2\{d(t)\} = -\{a(t)\}$$

(notation of $\{d(t)\}$ used instead of $\{d(t,\omega,\zeta)\}$ for brevity)

Of these, $(1/2)[W(t,\omega,\zeta)]$ provides a necessary and sufficient tool to represent the work transmitted into the oscillator until t under the action of $\{a(t)\}$. Its second term of energy dissipation is replaced in $[W'(t,\omega,\zeta)]$ by a totally different appearence of the damping action having no immediate mechanical meaning. With its insignificant contribution, $(1/2)[W'(t,\omega,\zeta)]$ equals essentially to the tensor of kinetic plus strain energies stored in the oscillator at t.

It can be shown that the two groups of tensors are mathematically related one another by the following strict equalities.

$$\int_0^{\infty} w(\omega_1,\omega,\zeta)[P(t,\omega_1)]d\omega_1 = [W(t,\omega,\zeta)]$$

$$\text{with } w(\omega_1,\omega,\zeta) = \frac{(4/\pi)\zeta\,\omega\,\omega_1{}^2}{(\omega_1{}^2-\omega^2)^2+4\zeta^2\omega^2\omega_1{}^2}$$

$$[P'(t,\omega,\zeta)] = [W'(t,\omega,\zeta)]$$

With definite-integral and limiting properties of

$$\int_0^{\infty} w(\omega_1,\omega,\zeta)d\omega_1 \equiv 1 \qquad \lim_{\zeta\to 0} w(\omega_1,\omega,\zeta) = \delta(\omega_1-\omega)$$

the role of $w(\omega_1,\omega,\zeta)$ is understood as a weighting function centered on $\omega_1=\omega$ in the above integral operation. Actually $[W(t,\omega,\zeta)]$ coincides with $[P(t,\omega)]$ averaged around ω, the smoothing in which becomes more notable according to a larger damping factor of ζ. When noting another definite integral of

$$\int_0^{\infty} w(\omega_1,\omega,\zeta)d\omega = \frac{2}{\pi}c(\zeta) \qquad \text{where } c(\zeta) = \begin{cases} \dfrac{\cos^{-1}\zeta}{\sqrt{1-\zeta^2}} & \text{for } 0 \leqq \zeta < 1 \\ 1 & \text{for } \zeta = 1 \\ \dfrac{\cosh^{-1}\zeta}{\sqrt{\zeta^2-1}} & \text{for } \zeta > 1 \end{cases}$$

a remaining link of

$$2\,c(\zeta)[E(t)] = \int_0^{\infty}[W(t,\omega,\zeta)]d\omega$$

can be obtained.

As evidenced in the foregoing formulas, the energy fed into simple oscillators of multi-D isotropy permits to interpret consistently all the tensors of Arias integrated intensity, Husid time-axis growth, Fourier spectral modulus, time-dependent frequency content and the likes of them. Thus the mathematically defined tensors can be assigned their individual roles within the framework of structural dynamics. An illustration in Figure 1 is intended to demonstrate such advantages when applied to the analysis of 2-D earthquake ground motions. This includes $[W'(t,\omega,\zeta)]^{1/2}$ and $[W(\infty,\omega,\zeta)]^{1/2}$ under $\zeta=0.05$ as well as $[E(\infty)]^{1/2}$. A closed curve at sampled points along time and frequency axes stands for the locus obtained by tracing the tip of vectors that specify directional components of tensor. Appealingly the 2-D characteristics of available energy are seen to differ in complicated features over the combined domains of time and frequency. These findings highlight the fact that

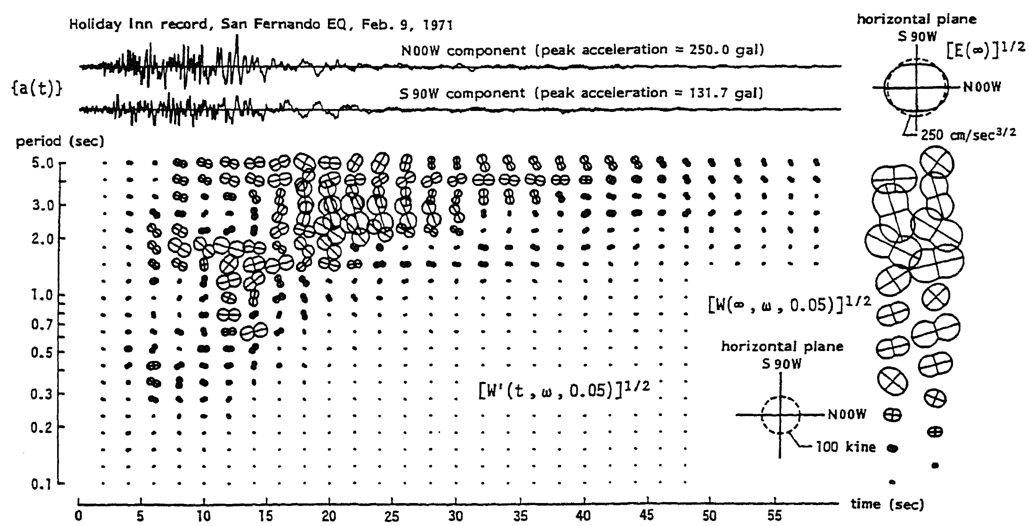

Figure 1

the overall tensorial measure of $[E(\infty)]$ becomes of little use for describing specific situations of multi-D dynamics.

Complex-valued Tensors The previous definitions of $[P(t,\omega)]$ and $[P'(t,\omega,\zeta)]$ were based on deletion of imaginary part in complex-valued matrices. Then a question may arise concerning what the deleted part implies. Such complex matrices can be conditioned, in general, to be Hermitian and non-negative definite tensors. Furthermore they were composed of tensor product of vectors in the deterministic instances. The latter condition of tensor-product decomposition is however not necessarily assumed in the following, which allows to include stochastic problems.

For a tensorial Hermitian matrix, [R], it goes without saying that its symmetric real part is tensorial. On the other hand, its skew-symmetric imaginary part is under different circumstances during rotation of coordinate axes. According to its component representation of

$$[R]=\begin{bmatrix} p_{11} & p_{12}+jq_{12} \\ p_{12}-jq_{12} & p_{22} \end{bmatrix}, \quad \begin{bmatrix} p_{11} & p_{12}+jq_{12} & p_{31}-jq_{31} \\ p_{12}-jq_{12} & p_{22} & p_{23}+jq_{23} \\ p_{31}+jq_{31} & p_{23}-jq_{23} & p_{33} \end{bmatrix}$$

in 2-D and 3-D instances, respectively, the single component of q_{12} in 2-D instances turns out to be invariant, while the three-component set of (q_{23}, q_{31}, q_{12}) in 3-D instances, when picked up and arrayed in the current manner, is subjected to a vectorial transformation. Discussions are henceforth focused upon simpler 2-D instances due to space limitation.

Among the 4-dof parameters of [R], invariants consist of $p_{11}+p_{22}$, $p_{11}p_{22}-p_{12}^2$ and q_{12}. Its positive definite condition is given by $p_{11}+p_{22}>0$ and $p_{11}p_{22}-p_{12}^2>q_{12}^2$. Also this is not positive definite but non-negative definite if and only if $p_{11}+p_{22}>0$, $p_{11}p_{22}-p_{12}^2=q_{12}^2$ and $q_{12}\neq 0$, by disregarding trivial cases that reduce to 1-D problems. Under the two situations combined, normalization by use of the principal axes identified for the real-part tensor leads to

$$[R]=p_{major}\left(\begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & r^2 \end{bmatrix}\begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix}+\begin{bmatrix} 0 & jr\sigma \\ -jr\sigma & 0 \end{bmatrix}\right)$$
$$=p_{major}\begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix}\begin{bmatrix} 1 & jr\sigma \\ -jr\sigma & r^2 \end{bmatrix}\begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix}$$

where $p_{major}(>0)$, $p_{minor}(>0)$ and ψ stand for major and minor principal values and orientation angle of major axis, respectively, and

$$r=\sqrt{\frac{p_{minor}}{p_{major}}} \quad (0<r\leqq 1) \qquad \sigma=\frac{q_{12}}{\sqrt{p_{11}p_{22}-p_{12}^2}}=\frac{q_{12}}{\sqrt{p_{major}\,p_{minor}}} \quad (|\sigma|\leqq 1)$$

Without loss of generality, a particular case of $\psi=0$ is chosen below.

Decomposition of the above tensor into a tensor product of mutually complex conjugate vectors:

$$[R]=\{V\}\{\bar{V}\}^T \qquad \{V\}=\sqrt{p_{major}}\begin{Bmatrix} 1 \\ -jr\,\mathrm{sgn}\,\sigma \end{Bmatrix}\exp(j\phi)$$

is strictly contingent upon $|\sigma|=1$, and a single phase factor of ϕ remains indeterminate there. On the contrary, its Choleski decomposition:

$$[R]=[L][\bar{L}]^T \qquad [L]=\sqrt{p_{major}}\begin{bmatrix} 1 & 0 \\ -jr\sigma & r\sqrt{1-\sigma^2} \end{bmatrix} \text{ (: lower triangular)}$$

becomes always possible under the non-negative definite requirement. Associating this with a stochastic phase vector of $\lfloor \exp(j\phi_1) \quad \exp(j\phi_2) \rfloor$ where ϕ_1 and ϕ_2 are random variables satisfying $\mathbf{E}[\exp(j(\phi_1-\phi_2))]=0$, an alternative expression of the latter decomposition is given by

$$[R]=\mathbf{E}[\{V'\}\{\bar{V'}\}^T] \qquad \{V'\}=\sqrt{p_{major}}\begin{Bmatrix} \exp(j\phi_1) \\ r(-j\sigma\exp(j\phi_1)+\sqrt{1-\sigma^2}\exp(j\phi_2)) \end{Bmatrix}$$

This indicates that even the tensor-product decomposition can be free from any requisite when considering the problem in a stochastic sense. The dimensionless factor of σ is to describe therein the degree of statistical dependence or independence observed between major and minor components. Restricted to completely dependent situations of $\sigma=\pm 1$, only a single phase factor of ϕ_1 is retained which corresponds directly to the deterministic cases. Even though the role of σ is totally inconceivable in deterministic problems save for the ambiguity of its sign (no more than the relation of $q_{12}=\pm\sqrt{p_{11}p_{22}-p_{12}^2}$), such a stochastic extension will certainly merit some attentions.

Tensor Fields Consider a multi-D time history, $\{b(t)\}$, which is specified by a linearly combined set of multi-D time histories;

$$\{b(t)\} = \sum_i [{}_i(\beta u)]\{{}_id(t)\}$$

These time histories can be either deterministic or stationarily stochastic with constant rectangular matrices of $[{}_i(\beta u)]$. Then form a matrix of squared RS or RMS intensity, $[B]$, for $\{b(t)\}$.

$$[B] = \begin{cases} \int_{-\infty}^{+\infty} \{b(t)\}\{b(t)\}^T\,dt & \text{when } \{b(t)\} \text{ is deterministic} \\ E[\{b(t)\}\{b(t)\}^T] & \text{when } \{b(t)\} \text{ is stationarily stochastic} \end{cases}$$

$$= \sum_i \sum_j [{}_i(\beta u)][{}_{ij}R][{}_j(\beta u)]^T$$

$$\text{where} \quad [{}_{ij}R] = \int_{-\infty}^{+\infty} \{{}_id(t)\}\{{}_jd(t)\}^T\,dt \quad \text{or} \quad E[\{{}_id(t)\}\{{}_jd(t)\}^T] \quad ([{}_{ji}R] = [{}_{ij}R]^T)$$

Such relations of a practical importance are found in multi-D performance of systems for which the classical modal formulation becomes applicable under a multi-D and vectorial ground acceleration of $\{a(t)\}$. In these instances, $\{b(t)\}$ stands for a subset in the multi-D response, $\{{}_id(t)\}$ and $[{}_i(\beta u)]$ coinciding with the motion of i-th order modal oscillator of multi-D isotropy and the associated matrix of modal participation factors, respectively. Following the equation of motion of

$$\{{}_i\ddot{d}(t)\} + 2\,{}_i\zeta\,{}_i\omega\{{}_i\dot{d}(t)\} + {}_i\omega^2\{{}_id(t)\} = -\{a(t)\}$$

the matrices of $[{}_{ij}R]$ (unsymmetric for $i \neq j$) are characterized by their tensorial nature. Still the resulting symmetric matrix of $[B]$ may or may not be a tensor.

With a view to demonstrating advantages in applying the above simple formula, taken up is a single-story rigid-floor system of two-way eccentricity subjected to a 2-D ground acceleration. Response of 2-D vectorial drift at an arbitrary point (x, y) on the horizontal plane is assigned to $\{b(t)\}$. The x and y axes are, for specificity, oriented according to principal axes of its overall translational stiffness with the origin located on its gravity center. Then

$$[{}_i(\beta u)] = \frac{1}{1 + {}_i\bar{e}_x{}^2 + {}_i\bar{e}_y{}^2} \begin{Bmatrix} {}_i\bar{e}_y - \bar{y} \\ -({}_i\bar{e}_x - \bar{x}) \end{Bmatrix} \lfloor {}_i\bar{e}_y \quad -{}_i\bar{e}_x \rfloor \qquad (i = 1 \text{ to } 3)$$

in which ${}_i\bar{e}_x = {}_ie_x/\boldsymbol{i}_m$, ${}_i\bar{e}_y = {}_ie_y/\boldsymbol{i}_m$, $\bar{x} = x/\boldsymbol{i}_m$ and $\bar{y} = y/\boldsymbol{i}_m$ with $({}_ie_x, {}_ie_y)$ and $\boldsymbol{i}_m$ representing the position of i-th order modal center of twist and the radius of gyration, respectively. Substitution of this expression into the preceding formula yields

$$[B] \left(= \begin{bmatrix} B_{xx} & B_{xy} \\ B_{xy} & B_{yy} \end{bmatrix}\right) = \sum_{i=1}^{3}\sum_{j=1}^{3} {}_{ij}A \begin{Bmatrix} {}_i\bar{e}_y - \bar{y} \\ -({}_i\bar{e}_x - \bar{x}) \end{Bmatrix} \lfloor {}_j\bar{e}_y - \bar{y} \quad -({}_j\bar{e}_x - \bar{x}) \rfloor$$

$$\text{where} \quad {}_{ij}A = \frac{1}{(1 + {}_i\bar{e}_x{}^2 + {}_i\bar{e}_y{}^2)(1 + {}_j\bar{e}_x{}^2 + {}_j\bar{e}_y{}^2)} \lfloor {}_i\bar{e}_y \quad -{}_i\bar{e}_x \rfloor [{}_{ij}R] \begin{Bmatrix} {}_j\bar{e}_y \\ -{}_j\bar{e}_x \end{Bmatrix}$$

Differing from $[{}_{ij}R]$, symmetry upon interchanging indices, ${}_{ji}A = {}_{ij}A$, features the current modal factors of ${}_{ij}A$. A more compact formulation for three components of the symmetric tensor of $[B]$ is

$$B_{xx} = \alpha(\bar{y} - \beta_y/\alpha)^2 + (\gamma_{xx} - \beta_y{}^2/\alpha)$$

$$B_{xy} = -\alpha(\bar{x} - \beta_x/\alpha)(\bar{y} - \beta_y/\alpha) - (\gamma_{xy} - \beta_x\beta_y/\alpha)$$

$$B_{yy} = \alpha(\bar{x} - \beta_x/\alpha)^2 + (\gamma_{yy} - \beta_x{}^2/\alpha)$$

where

$$\alpha = {}_{11}A + {}_{22}A + {}_{33}A + 2({}_{12}A + {}_{13}A + {}_{23}A)$$

$$\beta_x = {}_1\bar{e}_x\,{}_{11}A + {}_2\bar{e}_x\,{}_{22}A + {}_3\bar{e}_x\,{}_{33}A + ({}_1\bar{e}_x + {}_2\bar{e}_x){}_{12}A + ({}_1\bar{e}_x + {}_3\bar{e}_x){}_{13}A + ({}_2\bar{e}_x + {}_3\bar{e}_x){}_{23}A$$

$$\beta_y = {}_1\bar{e}_y\,{}_{11}A + {}_2\bar{e}_y\,{}_{22}A + {}_3\bar{e}_y\,{}_{33}A + ({}_1\bar{e}_y + {}_2\bar{e}_y){}_{12}A + ({}_1\bar{e}_y + {}_3\bar{e}_y){}_{13}A + ({}_2\bar{e}_y + {}_3\bar{e}_y){}_{23}A$$

$$\gamma_{xx} = {}_1\bar{e}_y{}^2\,{}_{11}A + {}_2\bar{e}_y{}^2\,{}_{22}A + {}_3\bar{e}_y{}^2\,{}_{33}A + 2({}_1\bar{e}_y\,{}_2\bar{e}_y\,{}_{12}A + {}_1\bar{e}_y\,{}_3\bar{e}_y\,{}_{13}A + {}_2\bar{e}_y\,{}_3\bar{e}_y\,{}_{23}A)$$

$$\gamma_{xy} = {}_1\bar{e}_x\,{}_1\bar{e}_y\,{}_{11}A + {}_2\bar{e}_x\,{}_2\bar{e}_y\,{}_{22}A + {}_3\bar{e}_x\,{}_3\bar{e}_y\,{}_{33}A + ({}_1\bar{e}_x\,{}_2\bar{e}_y + {}_1\bar{e}_y\,{}_2\bar{e}_x){}_{12}A + ({}_1\bar{e}_x\,{}_3\bar{e}_y + {}_1\bar{e}_y\,{}_3\bar{e}_x){}_{13}A + ({}_2\bar{e}_x\,{}_3\bar{e}_y + {}_2\bar{e}_y\,{}_3\bar{e}_x){}_{23}A$$

$$\gamma_{yy} = {}_1\bar{e}_x{}^2\,{}_{11}A + {}_2\bar{e}_x{}^2\,{}_{22}A + {}_3\bar{e}_x{}^2\,{}_{33}A + 2({}_1\bar{e}_x\,{}_2\bar{e}_x\,{}_{12}A + {}_1\bar{e}_x\,{}_3\bar{e}_x\,{}_{13}A + {}_2\bar{e}_x\,{}_3\bar{e}_x\,{}_{23}A)$$

Therefore the distribution of $[B]$ on the horizontal plane is seen to be quadratic indicating the existence of a center of response at $(\beta_x/\alpha, \beta_y/\alpha)$. Upon shifting the origin of $\bar{x}$ and $\bar{y}$ axes to this characteristic point, three components of B_{xx}, $-B_{xy}$ and B_{yy} consist, in addition to separately different constants, of only the single 2nd-order term of $\bar{y}^2$, $\bar{x}\bar{y}$ and $\bar{x}^2$, respectively, under a common coefficient of α. Moreover the parameters of α, β_x, β_y, γ_{xx}, γ_{xy} and γ_{yy} are to be related simply to the response of modal oscillators reflected upon $[_{ij}R]$.

A closer mathematical examination concerning the tensor field of [B] developed on the horizontal plane leads to the following finding of interesting rules. When noting the contour lines drawn by the principal values of pointwise tensors, they form a family of confocal quadratic curves and, at the same time, a family of orthogonal trajectories. More specifically the curves for major-axis and minor-axis components are elliptic and hyperbolic, respectively. Another markedly simple feature becomes also apparent in the flow lines describing the orientation of the principal axes of pointwise tensors. Actually the latter can be shown to coincide strictly with the above family of quadratic curves.

By use of the same ground motion as in Figure 1, results of an example study are given in Figure 2. The three systems examined therein have an identical relative stiffness for their coupled lateral and torsional motion, only the absolute stiffness being designed to provide different fundamental periods of 0.3, 1.0 and 2.5 seconds. Part (a) is intended to illustrate the 2-D nature of response drifts at sampled points. Individual closed figures represent directional properties of the RS tensor of $[B]^{1/2}$ in a similar way as in Figure 1. This includes $[_{11}R]^{1/2}$ shown on the same scale for comparisons. More complete data are presented in part (b) which involves full information on the tensor field by means of the above-noted contour and flow lines. The geometric simplicities observed there are striking enough to highlight an advantageous role of the tensor formulation. Numerals discriminating each contour line stand for the increasing or decreasing factors compared to the major-axis

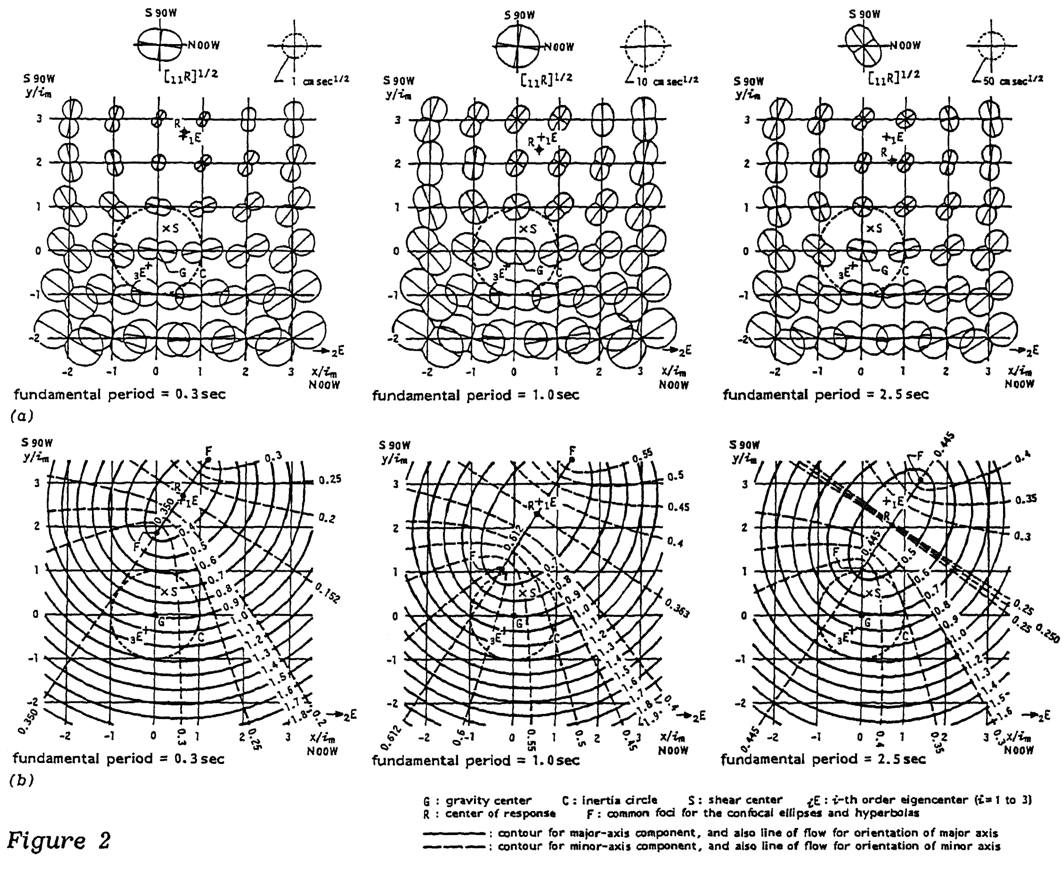

Figure 2

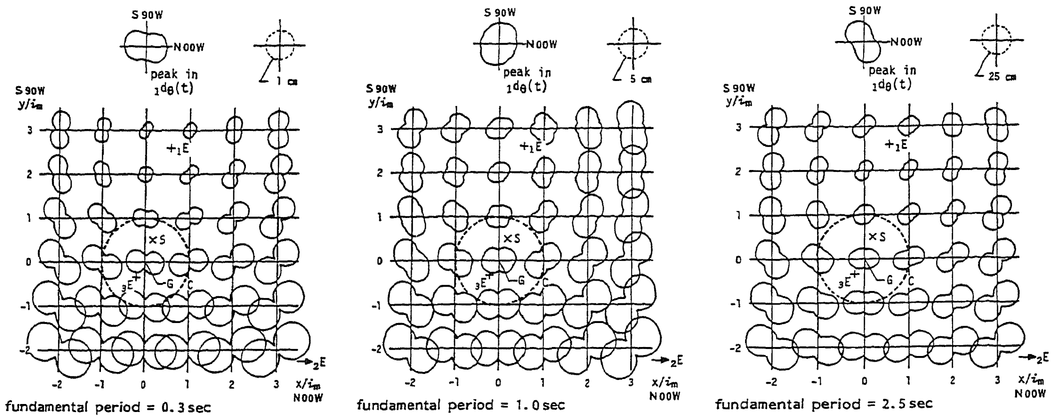

Figure 3

component in $[{}_{1}{}_{1}R]^{1/2}$, thus clarifying the effects of torsion.

Peak Amplitudes The current tensorial approach is indeed quite useful for understanding important trends behind a messy appearence of the pointwise directional characteristics in multi-D response and their spatial distribution. However this does not necessarily comply with the conventional notions in the practical design of earthquake-resistant structures. From the latter standpoints, envelope of Lissajous' locus including the associated rotational properties in orbital motion, for example, may be more preferable instead of the RS or RMS tensor. With such lines of extension in mind and by keeping still a straightforward relation to the tensorial understanding, studied hereafter is the pointwise directional dependence of peak amplitudes.

The examination follows the setup in Figure 2, and its immediate concern is directed toward peak amplitude in a 1-D time history, $b_\theta(t)$, extracted from $\{b(t)\}$ along a direction of θ on the horizontal plane;

$$b_\theta(t) = \lfloor \cos\theta \quad \sin\theta \rfloor \{b(t)\}$$

$$\left[\text{also, } {}_1d_\theta(t) = \lfloor \cos\theta \quad \sin\theta \rfloor \{{}_1d(t)\} \quad \text{supplementarily} \right]$$

Under complete lack of operational ease, clumsy repetition of ad hoc evaluations must be continued along each direction as well as at each point.

Figure 3 summarizes results of the examination concerning the variation of peak amplitude depending upon θ and the associated field developed on the horizontal plane. Employing a presentation form corresponding to Figure 2, direct comparisons are intended therein between RS values and peak amplitudes. For both pointwise directional and field characteristics, discrepancies are seen to be minor enough from practical points of view. Hence it is concluded that essential features in the non-tensorial properties of peak amplitude may be replaced by the tensorial formulation. Note that the directional distribution of peak amplitudes is, in general, inconsistent with the envelope of Lissajous' orbit.

REFERENCES

1. Takizawa, H., "Work done by earthquake ground shakings upon various types of dynamic systems," Proc. 6th Jap. Earthq. Eng. Symp., Dec. 1982, pp. 1065-1072.
2. Takizawa, H., "Coupled lateral and torsional failure of buildings under two-dimensional earthquake shakings," Proc. 8th Wld Conf. Earthq. Eng., July 1984, Vol. IV, pp. 195-202.
3. Takizawa, H., "Toward a better understanding of the torsionally-coupled dynamic performance of buildings," Proc. 7th Jap. Earthq. Eng. Symp., Dec. 1986, pp. 1789-1794.
4. Takizawa, H., "Nature of the effects of torsional coupling examined on the tensor field of deformation response and compared to the current regulations of seismic design," Jour. Struct. Eng., Arch. Inst. Jap., Vol. 33B, March 1987, pp. 111-124.