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INELASTIC RANDOM RESPONSE OF RC FRAMED STRUCTURES

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SUMMARY

This paper presents a practical method to obtain the inelastic random response of reinforced concrete frames, considering the nonlinear properties of constituent members. By using a simple equivalent linearization method, the differential equation of the covariance response is derived and solved numerically with step by step calculation. The numerical analysis is carried out in the case of three-story one-bay plane frame model, and the results are compared with Monte-Carlo simulation to examine the accuracy of the method.

INTRODUCTION

To evaluate structural safety under severe earthquakes, the statistical characteristics of nonlinear response should be studied. Especially, the response analysis of framed structures subjected to random earthquake motions is quite important for tracing the damage process of each structural members.

In this paper, the reinforced concrete plane frames with weak-girders and strong-columns are studied. In order to get response statistics of frames under random earthquake motions analytically, a simple equivalent linearization method is proposed and applied to the nonlinear properties of constituent members.

METHOD OF ANALYSIS

Linearization of Nonlinear Element   To express the nonlinear properties of structural constituent members, several inelastic beam models have been proposed. 'Generalized Composite Beam Model' (Ref. 1) which was proposed by H.Takizawa is used in this study. As shown in Fig. 1, a simple supported

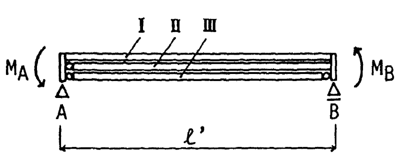


Fig. 1 Inelastic beam model

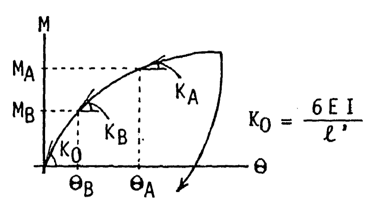


Fig. 2 Moment-rotation relation

member AB is imaginarily divided into three components along its member axis. The relation between end moments M_A, M_B and end rotations τ_A, τ_B is expressed as follows in incremental form.

$$\begin{Bmatrix} \Delta M_A \\ \Delta M_B \end{Bmatrix} = [K] \begin{Bmatrix} \Delta\tau_A \\ \Delta\tau_B \end{Bmatrix}, \quad [K] = \frac{1}{6}\left\{ K_A \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} + (K_B - K_A)\begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix} + (K_0 - K_B)\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\}, \text{ when } K_A < K_B \quad (1)$$

In this study, shear and axial deformations are considered to be negligible. In Eq. 1, the first term is the stiffness matrix of an elastic component I, the second term is that of a component II with end A hinged and the third term is that of a component III with both ends hinged. $K_A(K_B)$ is a instantaneous stiffness of end A(B), and K_0 is a initial stiffness. These stiffnesses are decided from the hysteresis relation between end moment M and end rotation Θ, as shown in Fig. 2. This relationship is usually given under antisymmetric loading condition. Thus, $\Theta_A(\Theta_B)$, which is called 'pseudo antisymmetric rotation' is defined as,

$$\begin{Bmatrix} \Delta\Theta_A \\ \Delta\Theta_B \end{Bmatrix} = \begin{bmatrix} f_A & 0 \\ 0 & f_B \end{bmatrix} \begin{Bmatrix} \Delta M_A \\ \Delta M_B \end{Bmatrix}, \quad f_{A(B)} = 1/K_{A(B)} \quad (2)$$

and from Eq. 1 and 2, the relation between τ_A, τ_B and Θ_A, Θ_B is expressed as,

$$\begin{Bmatrix} \Delta\tau_A \\ \Delta\tau_B \end{Bmatrix} = \begin{bmatrix} 3/2 + f_B/(2f_A) & -1 \\ -f_B/f_A & 2 \end{bmatrix} \begin{Bmatrix} \Delta\Theta_A \\ \Delta\Theta_B \end{Bmatrix}, \quad \text{when } f_A > f_B \quad (3)$$

In the assumption that the moment distribution of the member is nearly antisymmetric, τ-Θ relation turns out to be linear relation by setting $f_A = f_B$,

$$\begin{Bmatrix} \tau_A \\ \tau_B \end{Bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{Bmatrix} \Theta_A \\ \Theta_B \end{Bmatrix} \quad (4)$$

The equivalent linearization of the member is defined as follows,

$$\begin{Bmatrix} M_A \\ M_B \end{Bmatrix} = \begin{bmatrix} K_{e,A} & 0 \\ 0 & K_{e,B} \end{bmatrix} \begin{Bmatrix} \Theta_A \\ \Theta_B \end{Bmatrix} + \begin{bmatrix} C_{e,A} & 0 \\ 0 & C_{e,B} \end{bmatrix} \begin{Bmatrix} \dot\Theta_A \\ \dot\Theta_B \end{Bmatrix} \quad (5)$$

where, K_e : equivalent stiffness, C_e : equivalent damping. K_e and C_e are functions of response statistics as described later. By substituting Eq. 4 into Eq. 5, the relation between end moment and end rotation of the linearized member is given as,

$$\begin{Bmatrix} M_A \\ M_B \end{Bmatrix} = [K_e] \begin{Bmatrix} \tau_A \\ \tau_B \end{Bmatrix} + [C_e] \begin{Bmatrix} \dot\tau_A \\ \dot\tau_B \end{Bmatrix} \quad (6)$$

where, $[K_e]$: element stiffness matrix, $[C_e]$: element damping matrix.

When the rigid end zone of the member and the translational movement of the nodes are taken into consideration (Fig. 3), this relation is expressed as,

$$(M_i, M_j, Q_i, Q_j)^T = [K_E](\theta_i, \theta_j, u_i, u_j)^T + [C_E](\dot\theta_i, \dot\theta_j, \dot u_i, \dot u_j)^T \quad (7)$$

$$[K_E] = [A]^T[L]^T[K_e][L][A]$$

$$[C_E] = [A]^T[L]^T[C_e][L][A]$$

$$[L] = \begin{bmatrix} 1+\lambda_A & \lambda_B \\ \lambda_A & 1+\lambda_B \end{bmatrix}, \quad [A] = \begin{bmatrix} 1 & 0 & 1/\ell & -1/\ell \\ 0 & 1 & -1/\ell & 1/\ell \end{bmatrix}$$

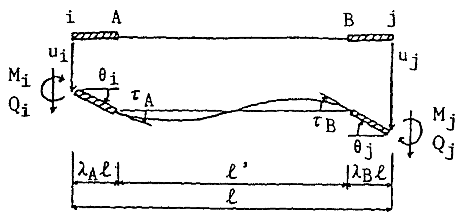


Fig. 3 Member element model

Equation of Motion The stiffness and damping matrices of total frames are easily formulated by assembling all the element stiffness and damping matrices. The total structural equilibrium is, therefore,

$$\begin{Bmatrix} -M(\ddot U + \ddot y_0) \\ 0 \end{Bmatrix} = \begin{bmatrix} K_{UU} & K_{U\theta} \\ K_{\theta U} & K_{\theta\theta} \end{bmatrix} \begin{Bmatrix} U \\ \theta \end{Bmatrix} + \begin{bmatrix} C_{UU} & C_{U\theta} \\ C_{\theta U} & C_{\theta\theta} \end{bmatrix} \begin{Bmatrix} \dot U \\ \dot\theta \end{Bmatrix} \quad (8)$$

where, M : mass matrix, U : horizontal floor displacements, θ : nodal rotations, y_0 : ground motion. When the ground motion is a stationary random white noise with power spectral density S_0, the differential equation of S_X which is a covariance matrix of a vector $X = (U, \theta, \dot{U})$ is derived.

$$\dot{S}_X = S_X G^T + G S_X{}^T + B \tag{9}$$

$$G = \begin{bmatrix} 0 & 0 & 1 \\ -C_{\theta\theta}{}^{-1}K_{\theta U} & -C_{\theta\theta}{}^{-1}K_{\theta\theta} & -C_{\theta\theta}{}^{-1}C_{\theta U} \\ M^{-1}(C_{U\theta}C_{\theta\theta}{}^{-1}K_{\theta U}-K_{UU}) & M^{-1}(C_{U\theta}C_{\theta\theta}{}^{-1}K_{\theta\theta}-K_{U\theta}) & M^{-1}(C_{U\theta}C_{\theta\theta}{}^{-1}C_{\theta U}-C_{UU}) \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & S_0\begin{bmatrix} 1 & \cdots & 1 \\ \vdots & & \vdots \\ 1 & \cdots & 1 \end{bmatrix} \end{bmatrix}$$

Nonstationary statistical responses of frames are obtained by solving Eq. 9 with step by step calculation.

Evaluation of K_e and C_e The equivalent stiffness K_e and the equivalent damping C_e are determined from the equivalent linearization of a stationary hysteresis loop as shown in Fig. 4. The amplitude of the loop is $\alpha\sigma_\Theta$, where σ_Θ is a standard deviation of rotation Θ in random responses and α is a parameter which determine the level of the equivalent stationary amplitude. By using the "Geometrical Method" (Ref. 2) which was proposed for sinusoidal excitations, K_e and C_e are expressed as,

$$\begin{cases} K_e = C_1(\alpha\sigma_\Theta)K_0 \\ C_e = -S_1(\alpha\sigma_\Theta)K_0/\omega_e + C_0 \end{cases} \tag{10}$$

where, K_0 : initial stiffness, C_0 : initial damping, ω_e : the first natural frequency of the structure calculated assuming the first mode shape. The functions C_1, S_1 are determined from the shape of the hysteresis loop. In case of Clough model (Fig. 5) which is considered to represent the hysteresis restoring force characteristics of RC members (Ref. 3),

$$\begin{cases} C_1 = (1-r)/\mu \\ S_1 = -2(1-r)(1/\mu-1/\mu^2)\{1+r(\mu-1)\}/\pi \end{cases} \tag{11}$$

$$\mu = \alpha\sigma_\Theta/\Theta_Y, \quad \Theta_Y = M_Y/K_0$$

where, r : stiffness ratio of the second to the first branch, M_Y : yield moment of the member. The standard deviation σ_Θ is obtained from the nodal response statistics using the following transformation,

$$(\Theta_A, \Theta_B)^T = [T](\theta_i, \theta_j, u_i, u_j)^T, \quad [T]=[N][L][A], \quad [N]=\frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \tag{12}$$

therefore

$$S_\Theta = [T]S_{\theta u}[T]^T \tag{13}$$

where, S_Θ : covariance matrix of Θ, $S_{\theta u}$: covariance matrix of vector (θ, u). Clearly, $S_{\theta u}$ is a element matrix of S_X in Eq. 9. Thus, at each step calculation in Eq. 9, the values of K_e and C_e are varied according to Eqs. 10 and 13.

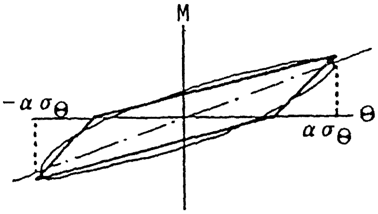


Fig. 4 Stationary hysteresis loop

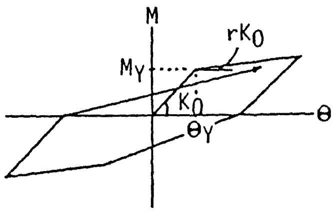


Fig. 5 Clough model

NUMERICAL STUDIES

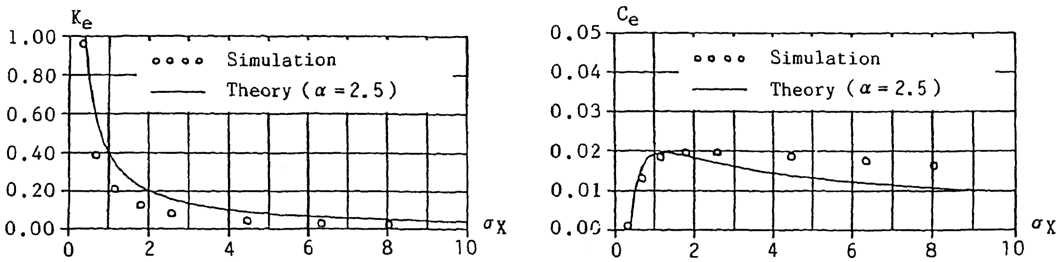
Evaluation of Parameter α Since the equivalent linearization coefficients K_e and C_e in Eq. 10 are both functions of parameter α, the appropriate value of α must be determined at first. For this purpose, the response of a single degree of freedom hysteresis system subjected to stationary random white excitation is examined.

A simple way of choosing K_e and C_e of equivalent linear system is to match the standard deviations of displacement and velocity to the corresponding results of Monte-Carlo simulation of a single degree of hysteresis system under stationary condition. It can be written as,

$$\begin{cases} K_e/m = (\sigma_{\dot{X}}/\sigma_X)^2 \\ C_e/m = S_0/(2\sigma_{\dot{X}}^2) \end{cases} \tag{14}$$

where, m : the mass, $\sigma_X(\sigma_{\dot{X}})$: simulation result of standard deviation of displacement(velocity). The hysteresis model of the system is Clough model. And the system parameters are selected as $m = 1$, $K_0 = 1$, $M_Y = 1$, $r = 0.0$ and $h_0 = 0.03$, where h_0 is a initial damping coefficient. The sample of stationary random white excitation is generated using the method of Ref. 4.

From the simulation calculations for several values of power spectral density S_0, the results of K_e and C_e are plotted against σ_X in Fig. 6. The solid line in the figure shows the relations obtained from Eq. 10 for $\alpha = 2.5$. It seems that theoretical results of $\alpha = 2.5$ agree well with simulation results. So, assume that this value of α is still valid under nonstationary condition, $\alpha = 2.5$ is adopted in this study for Clough model.



(a) Equivarent stiffness K_e (b) Equivarent damping C_e

Fig. 6 Relation between K_e, C_e and σ_X

Analysis of RC Frame The proposed method is applied to a reinforced concrete plane frame model described in Fig. 7. Sizes and yield moment values of the members are also indicated in the figure. The rigid end zone and the slab effect of the member are neglected. Natural periods and participation functions of this frame are listed in Table 1. For the hysteresis characteristics of the member, Clough model is used. As shown in Fig. 8, its initial stiffness is $\alpha_Y K_0$, where α_Y is the stiffness degradation ratio assuming 1/3, and post yielding stiffness is $0.01K_0$. The excitation is stationary random white noise with power spectral density $S_0 = 1600\ cm^2/sec^3$ and 10 sec duration.

In order to examine the accuracy of the method, Monte-Carlo simulation was carried out using 100 sample waves. Fig. 9 shows the locations of yield hinges and the values of average maximum ductility factors obtained from the simulation calculation. It is seen that yield hinges are located at the beam ends of all floors and the column end of the first floor.

The theoretical results are shown in Figs. 11, 12 and 13 for the standard deviations of horizontal floor displacement U, its velocity $\dot{U}$ and nodal rotation θ respectively. For σ_U and $\sigma_{\dot{U}}$ in Figs. 11 and 12, the theoretical results are in good agreement with the simulation results. For σ_θ in Fig. 13, however, there is some discrepancy between these results.

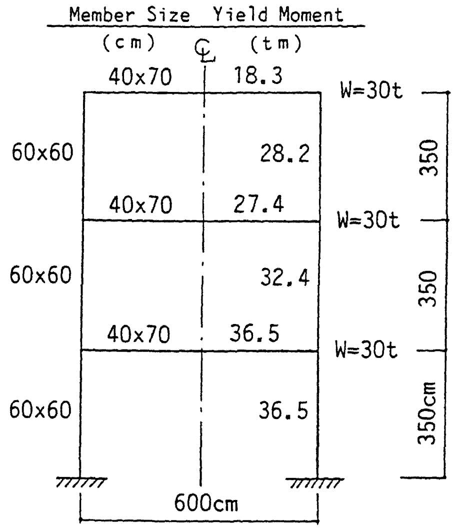


Fig. 7 Frame model

Table 1 Property of frame

Mode	Period (sec)	Participation Function 1F	2F	3F
1	0.57	0.40	0.91	1.25
2	0.18	0.40	0.28	-0.33
3	0.10	0.20	-0.19	0.08

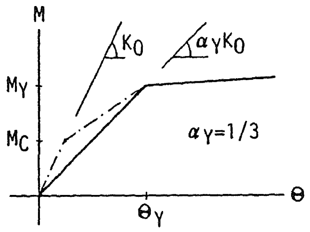


Fig. 8 Moment-rotation relation

CONCLUSIONS

The practical method is presented for evaluating the statistical characteristics of nonlinear responses of reinforced concrete frames. The nonlinear properties of structural constituent members are linearized by using a simple equivalent linearization method. In this method, the stationary hysteresis loop of the restoring force characteristics of the member is used. And setting its amplitude $\alpha\sigma_\Theta$, the equivalent linearization method for sinusoidal excitations can be applied for random excitations. From the simulation calculation, it appears that $\alpha = 2.5$ is adequate for Clough model. Using this method, three-story one-bay RC frame model is analyzed, and the results are compared with simulation results. The accuracy of this method is satisfactory for the evaluation of horizontal floor responses.

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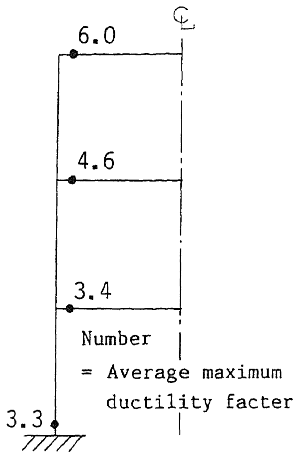

Fig. 9 Location of hinges (from simulation)

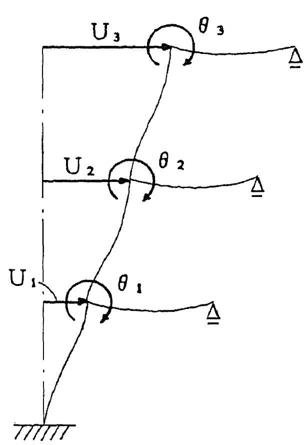

Fig. 10 Displacements of frame

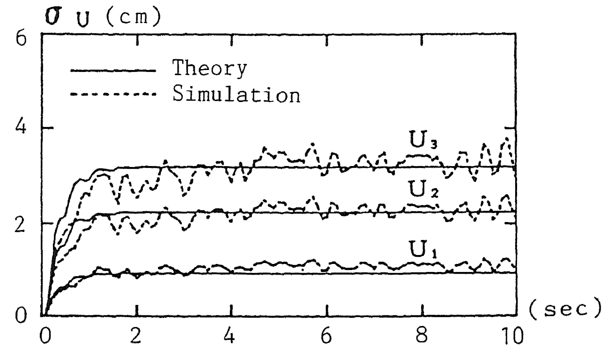

Fig. 11 Standard deviation of horizontal displacement U

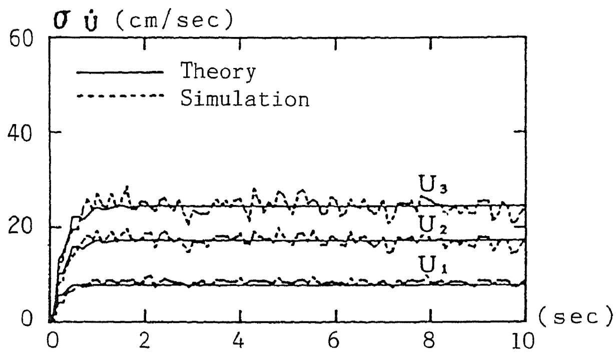

Fig. 12 Standard deviation of horizontal velocity $\dot{U}$

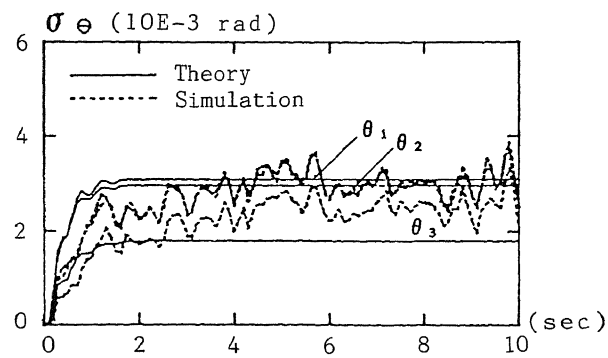

Fig. 13 Standard deviation of rotational displacement θ