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A DIRECT CONSTRUCTION OF SEISMIC RESPONSE ANALYSIS FOR HYSTERETIC SYSTEM

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SUMMARY

Performed in this paper is the stochastic response analysis of the piece-wise-linear (p.w.l.) hysteretic structural system under physical excitation noises. We develop an improved linearization technique which produces a direct and simple formulation and meanwhile yields a very reliable approximation to the response. Furthermore, in order to apply this linearization technique to the p.w.l. system we present a technique to smooth the p.w.l. hysteretic system. Finally, the 4th order cumulant response equations are derived to study the effect of the non-normality of the excitation upon the stochastic response.

INTRODUCTION

Structural systems under dynamic loading usually exhibit nonlinear hysteretic behavior and for reasons of safety and economy, such behavior has to be taken into account in design and seismic response analysis. Generally, most of this behavior is expressed by a piece-wise-linear(p.w.l.) hysteretic system. To the stochastic response analysis of the p.w.l. structural system the stochastic linearization method seems to have the greatest potential in terms of practical application. However, it has been pointed out[1] that the existing linearization method underestimates the exact statistical moments of the response of the p.w.l. hysteretic system.

On the other hand, the excitation noises imposed in the analysis have been, up till now, assumed to be Gaussian white (GW for short) processes or filtered GW ones which, in fact, are essentially processes dealing with GW excitation. However, physical excitation noises in actual situations are generally Non-Gaussian and Non-White and the effect of the non-normality can not be considered to be negligible in some situations[2][3].

This paper is to perform a simple and direct construction for the stochastic response analysis of the p.w.l. hysteretic system under physical excitation noises. First we develop an improved version of the linearization method on the basis of the concept of the weighted least-square minimization. Next in order to apply this technique to the p.w.l. system we present a technique to smooth the p.w.l. system in an equivalent probabilistic sense. Finally to study the effect of the non-normality of the excitation upon the stochastic response the stochastic response equations characterized by cumulant functions instead of the conventional moment functions are derived.

PHYSICAL EXCITATION NOISES

Normality and Whiteness

The term "physical excitation noises" refers to the excitation processes in practical situations. Generally they are non-Gaussian non-white. Theoretically, any random process $\xi(t)$ can be completely described by its cumulant functions $k_s(t_1,\dots,t_s)_\xi$. In terms of $k_s(t_1,\dots,t_s)_\xi$, we can define the normality and whiteness. By normality we mean that the process $\xi(t)$ satisfies

$$k_s(t_1,\dots,t_s)_\xi = 0 \qquad s=3,4,\dots \tag{1}$$

On the other hand, whiteness describes that $\xi(t)$ is delta-correlated:

$$k_s(t_1,\dots,t_s)_\xi = K_s(t_1)_\xi\,\delta(t_1-t_2)\,\delta(t_1-t_3)\dots\delta(t_1-t_s) \quad s=2,3,\dots \tag{2}$$

In terms of the normality and whiteness, the physical excitation noises can be classified into:

1) Gaussian White noise (GW)
2) Gaussian Non-White noise (GNW)
3) Non-Gaussian White noise (NGW)
4) Non-Gaussian Non-White noise (NGNW)

Replacement of a NW noise by a white process

Even when a linear system is subjected to a NW noise $\xi(t)$ which can not be expressed exactly by a filtered white process, the stochastic response can not be regarded rigorously as a Markov process and therefore the most powerful Markov theory can not be applied directly to the stochastic response analysis of the system. However, for the time intervals which are considerably greater than the correlation time τ_{cor}, the stochastic response becomes a multi-dimensional Markov process asymptotically[4] and then we can show that the NW excitation $\xi(t)$ essentially acts as a white noise $\xi^*(t)$ having the same intensity functions K_s. Here τ_{cor} and K_s are given respectively by

$$\tau_{cor}=\frac{s!}{K_s}\int_0^\infty \tau_1 d\tau_1 \int_0^{\tau_1} d\tau_2 \dots \int_0^{\tau_{s-2}} k_s(t_1,\dots,t_s)_\xi d\tau_{s-1} \tag{3}$$

$$K_s=\int_{-\infty}^{\infty}\dots\int_{-\infty}^{\infty} k_s(t_1,\dots,t_s)_\xi d\tau_1\dots d\tau_{s-1} \tag{4}$$

$$(\tau_1=t_2-t_1,\dots\ \tau_{s-1}=t_s-t_1)$$

As a consequence, we propose herein that the NW excitation noise $\xi(t)$ be replaced by a corresponding white process $\xi^*(t)$ with

$$k_s(t_1,\dots,t_s)_{\xi^*} = K_s\,\delta(\tau_1)\dots\delta(\tau_{s-1}) \tag{5}$$

The concept of the replacement technique is illustrated in Fig.1. As has been shown in Reference[5], this technique produces a rather satisfactory approximation to the response when τ_{cor} is small (<1). For a physical excitation noise with $\tau_{cor}>1$, it is necessary to adopt a proper linear filter to simulate the noise. Anyway, the replacement technique is of great interest in the sense that it makes the application of the Markov theory possible.

P.W.L. HYSTERETIC SYSTEM

The term "p.w.l. hysteretic system" refers to a nonlinear system in which the hysteretic characteristics consist of piece-wise-linear behavior. This system involves a bilinear model, double bilinear model, poly-linear model, origin-oriented model, peak-oriented model, slip model, Clough's model, and Takeda's model, etc.. The differential formulation for the system has been developed by a lot of researchers[6]. We have developed a somewhat different differential formulation[7]. The nondimensional nonlinear restoring force of the p.w.l. hysteretic system can be described by

$$\ddot{x} + 2h\dot{x} + \alpha x + (1-\alpha)z = f(t) \tag{6}$$

where α is the post-to-preyield stiffness ratio, x is the displacement; and z is the hysteresis and related to $\dot{x}$ through a first order differential equation in the following general form:

$$\dot{z}=A\dot{x}[1-U(\dot{x})U(z-1)-U(-\dot{x})U(-z-1)] \tag{7}$$

Here U is called the U-step function and defined by

$$U(x)=\begin{cases}1 & x\geqq 0\\ 0 & x<0\end{cases} \tag{8}$$

and A is called the hysteresis coefficient which depends on the characteristics of the individual p.w.l. hysteretic models.

IMPROVED STOCHASTIC LINEARIZATION TECHNIQUE

Formulation of the Improved Linearization Technique

Generally, nonlinear systems can be divided into softening systems and hardening systems. Without loss of generality, consider a symmetric nonlinear vibration system:

$$\frac{dX}{dt} = G(X,t) + F(t) \tag{9}$$

where F(t) is the n dimensional Gaussian white excitation vector with zero mean. Linearize the equation of motion (9) into

$$\frac{dX}{dt} = RX + F(t) \tag{10}$$

where the equivalent coefficient matrix R is to be determined so that the linear system (10) will produce the most approximate solution to (9). In estimating R, the difference of the two systems, i.e.

$$e(X) = G(X,t) - RX \tag{11}$$

should be committed. A logical choice to select R is to require that the mean squared value of e(X) multiplied by a weighting function W(X) be a minimum, i.e.

$$E[e^t(X)e(X)W(X)] \rightarrow \text{minimum} \tag{12}$$

where E[x] denotes the expected value of x and the prime t means the transpose. The weighting function W(X) can be considered to be in accordance with the difference between the non-Gaussian distribution of the response x in the nonlinear system(9) and the normal distribution of the response in the corresponding linear system(10). Of course, this difference can not be described precisely but only be assumed qualitatively. Thus, in Reference[8] the weighting functions for a hardening system and a softening system have been supposed to assume the following forms:

$$W(X)=\exp\left\{-\frac{1}{8}X^tS^{-1}X\right\} \text{ (for hardening systems) (13)} \quad W(X)=\exp\left\{\frac{1}{32}X^tS^{-1}X\right\} \text{ (for softening systems) (14)}$$

as shown in Fig.2. Here S is covariance matrix. On imposing (13) and (14) and after some arrangements, the condition (12) leads to the direct form of the coefficient matrix R as follows

$$r_{ij}=E\left[\frac{\partial g_i}{\partial x_j}\bigg|_{X=\frac{2}{\sqrt{5}}X}\right] \text{ (for hardening systems) (15)} \quad r_{ij}=E\left[\frac{\partial g_i}{\partial x_j}\bigg|_{X=\frac{4}{\sqrt{15}}X}\right] \text{ (for softening systems) (16)}$$

It is of interest to compare this version of linearization to Atalik & Utku's technique[9]. It has been proven[8] that the accuracy of the improved technique is over 10% higher than the existing ones.

Application to P.W.L. Hysteretic System

Consider the p.w.l. hysteretic system (6) under a GW noise f(t). Clearly, expression (7) involves the U-step functions and therefore has discontinuous differentiations with respect to the state variables. This discontinuity prevents application of the above technique to the system. Consequently, it may be quite reasonable to search for a smooth hysteretic system which is essentially equivalent to the p.w.l. hysteretic system. By "equivalent" we mean here that the smooth system and the original p.w.l. system have similar hysteretic characteristics and that meanwhile almost the same probability information about the concerned variables can be derived after smoothing as well. This aim can be achieved by approximating the U-step functions U(x) and U(z-1) through

$$U(\dot{x}) = \frac{1}{2}(1+\mathrm{sgn}\,\dot{x}) = \begin{cases} 1 & \dot{x}>0 \\ 0.5 & \dot{x}=0 \\ 0 & \dot{x}<0 \end{cases} \tag{17a}$$

$$U(z-1) = \frac{1}{2}|z|^n(1+\mathrm{sgn}\,z) \quad (|z|\leqq 1) \text{ for a positive } n \tag{17b}$$

As a result, we can obtain the smooth differential expression:

$$\dot{z} = \bar{A}\,[\,\dot{x} - 0.5\,|z|^{n}\,\dot{x} - 0.5\,|\dot{x}|\,z\,|z|^{n-1}] \quad \text{for a positive n} \qquad (18)$$

where $\bar{A}$ is the smoothed hysteresis coefficient and can be obtained from A by making use of the same smoothing approximation. It is interesting that the hysteresis expression (18) of the smooth p.w.l. system is similar to Y.K. Wen model[10]. Parameter n in (18) is to control the smoothness of U(z-1) and the transition of the response, as shown in Fig.3. We should notice that parameter n be so chosen that we may obtain the same probabilistic information about what is concerned after smoothing and meanwhile may apply the linearization technique. The propriety of the smoothing technique is verified by applying to the bilinear model and shown in Fig.4. Incorporating (15) to the p.w.l. system yields the equivalent linear system:

$$\frac{dX}{dt} = RX + F \qquad (19)$$

where

$$G = \begin{pmatrix} 0 & 0 & 1 \\ 0 & ac & ab \\ -\alpha & -(1-\alpha) & -2h \end{pmatrix} \quad (20a) \qquad F = \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} \quad (20b) \qquad X = \begin{pmatrix} x \\ z \\ \dot{x} \end{pmatrix} \quad (20c)$$

$$b = 1 - \left(\frac{4}{\sqrt{15}}\right)^{n}\left[\frac{2^{n/2-1}}{\pi}\sum_{k=0}^{n-1}\binom{n}{k}\Gamma\left(\frac{k-1}{2}\right)\Gamma\left(\frac{n-k+1}{2}\right)(1-\rho^{2})^{k/2}\rho^{n-k}\sigma_{z}^{n} + \frac{2^{n/2-1}}{\sqrt{\pi}}\Gamma\left(\frac{n+1}{2}\right)\sigma_{z}^{n}\right] \qquad (21a)$$

$$c = -\left(\frac{4}{\sqrt{15}}\right)^{n} n\,\frac{2^{n/2-1}}{\pi}\sum_{k=0}^{n-1}\binom{n-1}{k}\Gamma\left(\frac{k-1}{2}\right)\Gamma\left(\frac{n-k+1}{2}\right)(1-\rho^{2})^{k/2}\rho^{n-k-1}\sigma_{\dot{x}}\sigma_{z}^{n-1} + n\,\frac{2^{n/2-1}}{\sqrt{\pi}}\Gamma\left(\frac{n+1}{2}\right)\rho\,\sigma_{\dot{x}}\sigma_{z}^{n-1}\Bigg] \qquad (21b)$$

$$a = E[\,\bar{A}\,] \quad (21c) \qquad \rho = \frac{E[\dot{x}z]}{\sigma_{\dot{x}}\sigma_{z}} \quad (21d) \qquad \Gamma(x) = \int_{0}^{\infty} e^{-t}t^{x-1}\,dt \quad (21e) \qquad k = \text{even} \quad (21f)$$

Its application to the bilinear model under GW noises has been illustrated in Reference[8] as shown in Fig.5 where parameter n has been decided to be 1.

STOCHASTIC RESPONSE EQUATIONS

When the p.w.l. hysteretic system is subjected to a physical excitation noise $\xi(t)$, the response exhibits the non-normality. Therefore, it is desirable to characterize the stochastic response in terms of cumulant functions. From the viewpoint of engineering, the 2nd and 4th order cumulant responses are most concerned to us. Since the 2nd order cumulant response functions are the elements of the conventional covariance matrix response, only the 4th order cumulant response equations are developed here. For this purpose it is necessary to determine $k_4(x^{n}z^{m}\dot{x}^{h}\xi)$, $(n+m+h=3,\ n,m,h=1\sim3)$. Generally, their proper values are not obvious immediately. In Reference[5], we have presented an approach to decide $k_4(x^{n}\dot{x}^{m}\xi^{*})$ $(n+m=3,\ n,m=1\sim3)$ for a linear system on the basis of the Markov theory and have obtained the results as follows.

$$k_4(x^{n}\dot{x}^{m}\xi^{*})=0 \quad (n+m=3,\ n\geqq1) \qquad (22a) \qquad\qquad k_4(\dot{x}^{3}\xi^{*}) = \frac{1}{4}B(t_1)Q_0 \qquad (22b)$$

where $\xi^{*}(t)$ is a NG white noise with

$$k_5(t,t_1,t_2,t_3)_{\xi^{*}} = B(t)\,\delta(t-t_1)\,\delta(t-t_2)\,\delta(t-t_3) \qquad (23)$$

For a physical noise $\xi(t)$ with small τ_{cor}, the above expressions still hold after we apply the replacement technique to $\xi(t)$.

At this stage we can incorporate the improved linearization technique approximately after smoothing the p.w.l. hysteretic system. As a result, we have the equivalent linear system (19) except that f(t) is replaced by $\xi(t)$. Multiplying (19) by $x^{n}z^{m}\dot{x}^{h}$ $(n+m+h=3,\ n,m,h=1\sim3)$ and taking cumulant operation after making use of (12) and (13) leads to the 4th order cumulant response equations. For a special case $\alpha=1$, i.e., for the corresponding linear system, we have the following 4th order cumulant equations.

$$\dot{k}_4(x^{4})=4k_4(x^{3}\dot{x}) \qquad (24a) \qquad\qquad \dot{k}_4(x\dot{x}^{3})=k_4(\dot{x}^{4})-3k_4(x^{2}\dot{x}^{2})-6hk_4(x\dot{x}^{3}) \qquad (24d)$$

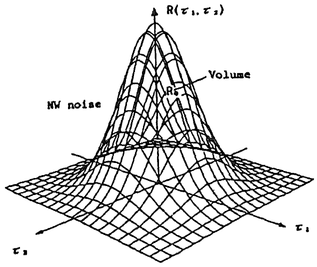

Fig.1a cumulant function of a nonwhite noise

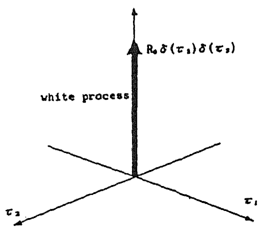

Fig.1b cumulant function of the corresponding white noise

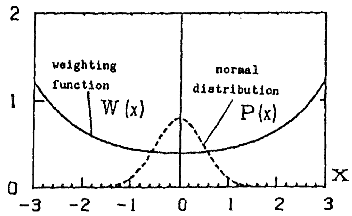

Fig.2a weighting function for a softening system

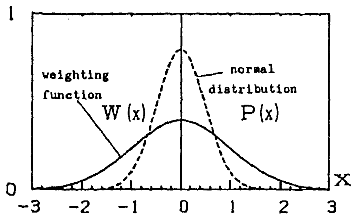

Fig.2b weighting function for a hardening system

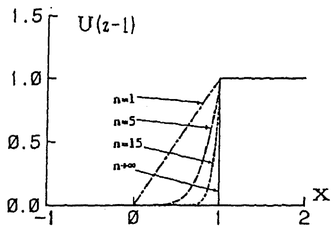

Fig.3a control of smoothness of U(z-1) by parameter n

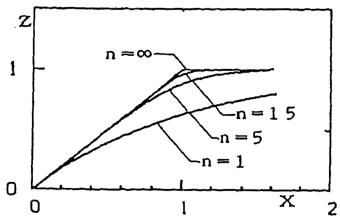

Fig.3b control of transition of response by parameter n

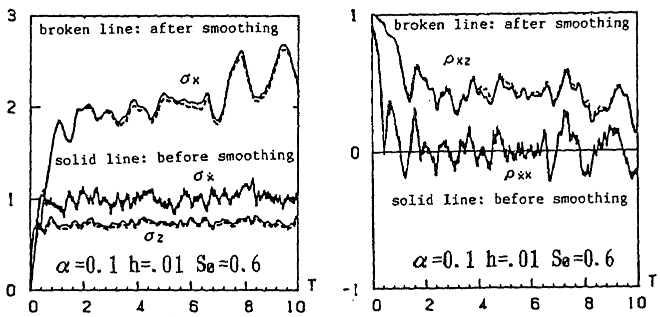

Fig.4 application of the smoothing technique to the bilinear model

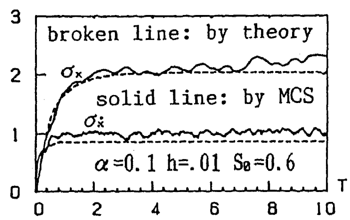

Fig.5a rms history for the bilinear model (case 1)

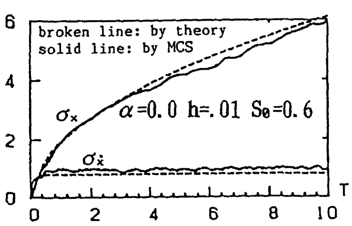

Fig.5b rms history for the bilinear model (case 2)

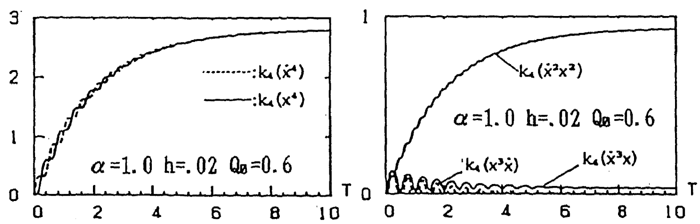

Fig.6 4th order cumulant response history for the corresponding linear system

Explanation of the Parameters in the Figures

σ: standard deviation ρ: correlation function
T: normalized time by the natural period(2π) of the system
S_0: 2nd order intensity of the noise Q_0: 4th order intensity of the noise

$$\dot{k}_4(x^3\dot{x})=3k_4(x^2\dot{x}^2)-2hk_4(x^3\dot{x})-k_4(x^4) \quad (24b)$$

$$\dot{k}_4(x^2\dot{x}^2)=2k_4(x\dot{x}^3)-4hk_4(x^2\dot{x}^2)-2k_4(x^3\dot{x}) \quad (24c)$$

$$\dot{k}_4(\dot{x}^4)=B(t)Q_\theta-8hk_4(\dot{x}^4)-4k_4(x\dot{x}^3) \quad (24e)$$

Some examples of the 4th order response for the corresponding linear system are given in Fig.6. We should mention that it is necessary to adopt a proper linear filter in the case of a physical excitation noise with large correlation time before applying the present theory.

CONCLUSIONS

In order to perform the direct construction of seismic response analysis for the piece-wise-linear hysteretic system under physical excitation noises, the following useful and reliable approaches are presented in detail.

1) An approach to replace a non-white physical noise by a white process is described. This approach yields a good approximation to the response when the correlation time τ_{cor} of the excitation is small, although for a physical excitation noise with large τ_{cor} it is necessary to adopt a proper linear filter.

2) An improved version of linearization method is developed on the basis of the weighted least-square minimization. This version produces a direct and simple formulation and meanwhile yields very dependable approximations to the response even for the case of large nonlinearities.

3) In order to apply the above linearization technique to the piece-wise-linear system a technique to smooth the piece-wise-linear hysteretic system is presented.

4) Finally the 4th order cumulant response equations are derived to study the effect of the non-normality of the excitation upon the stochastic response.

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