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ACCURACY OF MODE-SUPERPOSITION METHODS IN THE EVALUATION OF ASYMMETRIC MULTISTOREY BUILDINGS RELIABILITY

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SUMMARY

The Dynamic Correction method is here extended to the stochastic seismic analysis of a multistorey building. Earthquake ground motion is modelled as a stationary zero mean Gaussian filtered process. The accuracy in the evaluation of the maximum peak of the nodal response is investigated by varying some structural parameters.

INTRODUCTION

It is well known that seismic ground motion can be adequately represented as a stochastic process and the response of structural systems can be evaluated in a probabilistic sense. In the framework of modal analysis, when the degrees of freedom of the systems are numerous, usually only the first few modes are taken into account. The truncation of modes (Mode Displacement method) drastically reduces the computational effort and accurate results are obtained when the frequency content of dynamical loadings is at a low frequency. Unfortunately, modal truncation is usually made a-priori and no information on the contribution of higher modes to the response can be established. For this reason, in deterministic analysis many correction procedures have been proposed (Refs. 1 to 3) in order to improve the nodal solution. The latter consist in adding to the nodal solution, obtained using a reduced number of modes, a pseudo-static response which takes the remaining ones into account. Limited research has been carried out in stochastic analysis (Ref. 4).

The aim of this paper is to investigate computation problems and the accuracy of the seismic response of asymmetric multistorey buildings subjected to earthquake ground motion, using the Dynamic Correction method (Ref. 3) here extended to stochastic analysis. Seismic ground motion is modelled as a zero mean stationary Gaussian process having a Tajimi (Ref. 5) power spectral density function. It is to be emphasized that the correction procedure for stochastic seismic analysis gives greater accuracy with respect to the Mode Displacement method without a noticeable increment in computing time.

EQUATION OF MOTION

The equation of motion of an n-storey building subjected to an earthquake may be cast in a linear form as follows:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = -\mathbf{M}\boldsymbol{\tau}\zeta(t) \tag{1}$$

where $\zeta(t)$ is the ground acceleration; $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ are the inertia, damping and stiffness matrices, respectively, all of order N×N ($N = 3n$); $\mathbf{x}$ is the vector of floor displacements relative to the ground and the dot indicates differentiation with respect to time t. The vector $\boldsymbol{\tau}$ takes into account the building orientation with respect to the input direction.

By means of the usual co-ordinate transformation $\mathbf{x} = \boldsymbol{\Phi}\mathbf{q}$, $\boldsymbol{\Phi}$ being the modal matrix normalized with respect to $\mathbf{M}$ ($\boldsymbol{\phi}_i$ is its i-th column) equation (1) may be transformed into modal co-ordinates in the following form:

$$\ddot{\mathbf{q}} + \boldsymbol{\Lambda}\dot{\mathbf{q}} + \boldsymbol{\Omega}^2\mathbf{q} = \mathbf{u}\zeta(t) \tag{2}$$

where $\mathbf{u} = -\boldsymbol{\Phi}^T\mathbf{M}\boldsymbol{\tau}$ (the superscript T means transpose). In equation (2), $\boldsymbol{\Omega}$ is a diagonal matrix listing the natural frequencies ω_i, $\boldsymbol{\Lambda} = \boldsymbol{\Phi}^T\mathbf{C}\boldsymbol{\Phi}$ is the modal damping matrix here assumed to be a diagonal one.

For the evaluation of the modal response a 2N dimension state vector approach is commonly used. For this purpose equation (2) can be written in reduced form as follows:

$$\mathbf{A}\dot{\mathbf{y}} = -\mathbf{B}\mathbf{y} + \mathbf{V}\zeta(t) \tag{3}$$

where $\mathbf{y}(t) = [\mathbf{q}^T(t) \quad \dot{\mathbf{q}}^T(t)]^T$ is the vector of 2N modal state variables and

$$\mathbf{A} = \begin{bmatrix} \boldsymbol{\Lambda} & \mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} \boldsymbol{\Omega}^2 & \mathbf{0} \\ \mathbf{0} & -\mathbf{I} \end{bmatrix}; \quad \mathbf{V} = \begin{bmatrix} \mathbf{u} \\ \mathbf{0} \end{bmatrix} \tag{4}$$

in which $\mathbf{0}$ is the zero matrix and $\mathbf{I}$ is the identity matrix. The vector of nodal variables can be obtained by using the modal superposition $\mathbf{z} = \mathbf{P}\mathbf{y}$ where the matrix $\mathbf{P}$ and the vector $\mathbf{z}(t)$ are given as:

$$\mathbf{P} = \begin{bmatrix} \boldsymbol{\Phi} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Phi} \end{bmatrix}; \quad \mathbf{z}(t) = \begin{bmatrix} \mathbf{x}(t) \\ \dot{\mathbf{x}}(t) \end{bmatrix} \tag{5}$$

When the degrees of-freedom of the given systems are numerous, only the lower frequencies and the corresponding m lower modes are usually computed. It follows that the matrix $\boldsymbol{\Phi}$ is truncated after the m-th column. Consequently, the matrix $\mathbf{P}$ is of order (2N×2m). Hereafter, all truncated matrices and vectors will be denoted by means of a superimposed symbol "^". The approximated nodal response $\mathbf{z}^a(t)$ can be obtained by various mode superposition methods (Refs. 1 to 3).

The Dynamic Correction (DC) method (Ref. 3) is adopted because all other approximate methods can be obtained as particularizations of the DC method. According to the DC method the approximate response can be written in the form:

$$\mathbf{z}^a(t) = \hat{\mathbf{P}}\hat{\mathbf{y}}(t) + \boldsymbol{\Delta}_1\boldsymbol{\tau}_1\zeta(t) + \boldsymbol{\Delta}_2\boldsymbol{\tau}_1\dot{\zeta}(t) \tag{6}$$

where

$$\boldsymbol{\Delta}_1 = \begin{bmatrix} \mathbf{k}^{-1} - \hat{\boldsymbol{\Phi}}\hat{\boldsymbol{\Omega}}^{-2}\hat{\boldsymbol{\Phi}}^T \\ \mathbf{0} \end{bmatrix}; \quad \boldsymbol{\Delta}_2 = \begin{bmatrix} -(\mathbf{k}^{-1}\mathbf{C}\mathbf{k}^{-1} - \hat{\boldsymbol{\Phi}}\hat{\boldsymbol{\Omega}}^{-2}\hat{\boldsymbol{\Lambda}}\hat{\boldsymbol{\Omega}}^{-2}\hat{\boldsymbol{\Phi}}^T) \\ \mathbf{k}^{-1} - \hat{\boldsymbol{\Phi}}\hat{\boldsymbol{\Omega}}^{-2}\hat{\boldsymbol{\Phi}}^T \end{bmatrix} \tag{7}$$

and $\tau_1 = -M\tau$.

It is worth noting that the response in terms of displacement (first N rows of the vector $z^a(t)$), under the assumption of $\dot{\zeta}(t)$ negligeable, coincides with the analogous rows evaluated by means of the Mode Acceleration (MA) method (Ref. 1).

However, it will be emphasized that the nodal response in terms of velocity (last N rows of the vector $z^a(t)$) in the simpler case of forcing vector constant ($\dot{\zeta}(t) = 0$) also differs from the analogous rows evaluated by the MA method. Indeed, applying equation (6) particularized for $\dot{\zeta}(t) = 0$, the last N rows of the vector $z^a(t)$ remain unchanged, while, using the MA method and then differentiating the vector $x^a(t)$, the derivative $\dot{\zeta}(t)$ appears and this implies serious problems in stochastic analysis, as will be shown further on.

SEISMIC STOCHASTIC ANALYSIS

When earthquake ground motion $\zeta(t)$ is modelled as a zero mean Gaussian process, the vector $z(t)$, which is the solution of the linear system, is fully described in a probabilistic sense by the covariance matrix $\Sigma_{z^a z^a}(t) = E[z^a(t)\ z^{aT}(t)]$, where $E[\cdot]$ means stochastic average. This matrix evaluated for $z^a(t)$ can be obtained by multiplying equation (6) particularized for $\dot{\zeta}(t) = 0$ by its transpose and making expectation we obtain:

$$\Sigma_{z^a z^a}(t) = \hat{P}\,\hat{\Sigma}_{yy}(t)\,\hat{P}^T + E[\zeta^2(t)]\,\Delta_1\tau_1\tau_1^T\Delta_1^T + \hat{P}\,\hat{\Sigma}_{y\zeta}(t)\,\tau_1^T\Delta_1^T + \Delta_1\tau_1\,\hat{\Sigma}_{y\zeta}^T(t)\,\hat{P}^T \qquad (8)$$

In order to evaluate the matrix $\Sigma_{z^a z^a}(t)$ by means of Eq. (8) the modal covariance matrix $\hat{\Sigma}_{yy}(t)$, the variance of input $E[\zeta^2(t)]$ and the input-output cross-covariance matrix $\hat{\Sigma}_{y\zeta}(t)$ must be determined.

In cases in which the earthquake is modelled as a stationary process, the statistical characteristics are independent of time. Therefore, in these cases, the quantities that appear in Eq. (8) can be easily evaluated in the frequency domain. In particular, the matrix $\hat{\Sigma}_{yy}$ is given as:

$$\hat{\Sigma}_{yy} = \int_{-\infty}^{\infty} \hat{H}(\omega)\, u\, u^T \hat{H}^{*T}(\omega)\, S_{\zeta\zeta}(\omega)\, d\omega \qquad (9)$$

where, the asterix means complex conjugate, $S_{xx}(\omega)$ is the power spectral density of the input $\zeta(t)$, and $\hat{H}(\omega)$ is the usual transfer function matrix (order 2m×m) ($\hat{H}^T(\omega) = [\hat{h}(\omega)\quad i\omega\hat{h}(\omega)]$, $\hat{h}(\omega)$ being the diagonal transfer function matrix in modal co-ordinates).

The variance of input $E[\zeta^2(t)]$ and the vector listing the input-output cross-covariances can be written respectively as in the form:

$$E[\zeta^2(t)] = \int_{-\infty}^{\infty} S_{\zeta\zeta}(\omega)\, d\omega; \qquad \hat{\Sigma}_{y\zeta} = \int_{-\infty}^{\infty} \hat{H}(\omega)\, u\, S_{\zeta\zeta}(\omega)\, d\omega \qquad (10)$$

Notice that the covariance matrix of the displacement $\Sigma_{x^a x^a}$ is the same using either the MA or the DC method. The displacement-velocity cross-covariance matrix $\Sigma_{x^a \dot{x}^a}$ and the covariance matrix of velocity $\Sigma_{\dot{x}^a \dot{x}^a}$ computed using MA method give serious problems due to the fact that in the corrective terms there appear the cross-covariance between $\zeta(t)$ and $\dot{\zeta}(t)$ and the covariance of $\dot{\zeta}(t)$, which in most cases are divergent quantities. Using the DC method, this drawback is overcome and the derivative $\dot{\zeta}(t)$ does not appear in the velocity vector $\dot{x}^a(t)$.

NUMERICAL EXAMPLE

In this section the method proposed is applied to a shear type ten-storey building, whose layout is shown in Fig. 1, in which the frames have been numbered.

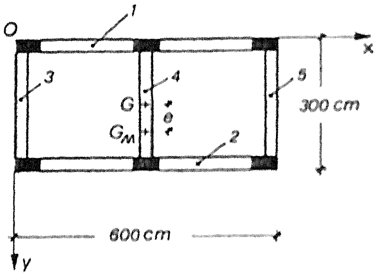

Fig. 1 Layout of the multistorey building

The cross-section of all columns have been assumed to be 30 × 60 cm; the mass of each floor is 18000 kg and the Young's modulus of the material is E = 30000 MPa. The earthquake ground motion is idealized as a unidirectional input in the direction x; its PSD is a Tajimi (Ref. 5) one given as

$$S_{\zeta\zeta}(\omega) = \frac{(\omega_g^4 + 4\zeta_g^2\omega_g^2\omega^2)S_0}{(\omega_g^2 - \omega^2)^2 + 4\zeta_g^2\omega_g^2\omega^2} \tag{11}$$

where S_0 is a constant here assumed to be one. The values ω_g and ζ_g are given as: $\omega_g = 5\pi s^{-1}$ and $\zeta_g = 0.6$. The mean value of the largest peaks of the displacements μ with respect to the ground are evaluated by means of Davenport's formula (Ref.6). The analysis has been carried out varying the position G_M of the centroid of the mass in relation to the geometrical centroid G of the generic storey. In Fig. 2, the displacements μ of frame no. 1 are depicted. These quantities have been evaluated using both the Mode-Displacement (MD) method and the DC method. To compare the results in the same figure the exact response, i.e. evaluated using all the natural modes, has been depicted. Comparing these figures some considerations emerge: using only the first natural mode in all cases the response evaluated by means of the MD method is very different from the exact one. The correction obtained using the DC method does not lead to acceptable results. Using two modes, in the case of e = 0 the DC method does not give a noticeable improvement while for e = 50 cm, two modes without correction give a very different result from the exact solution and the correction has more influence. For e = 100 cm, two mode give almost zero displacement using the MD method and the correction improves the solution though the corrected results underestimate the actual displacements. In order to establish a criterion for selecting the number of modes and to understand a-priori when the correction has an influence, it is necessary to evaluate the ratio between the mean energy corresponding to the approximate response computed with the MD method and the mean energy associated with the response evaluated by means of the DC method, i.e.

$$r = \frac{E[(\underset{\sim}{\tau}_1 \zeta(t))^T (\hat{\underset{\sim}{\Phi}} \hat{\underset{\sim}{q}}(t))]}{E[(\underset{\sim}{\tau}_1 \zeta(t))^T \underset{\sim}{x}^a(t)]} \tag{12}$$

This ratio lies between zero and one. In particular, the ratio becomes one when the correction term is zero, i.e., when all modes have been computed. The ratio is zero when the response is evaluated in a pseudostatic term only. It follows that the lower r is the more influence the correction term will have. Those observations one confirmed in Table I, in which for the numerical example the coefficients r are appended.

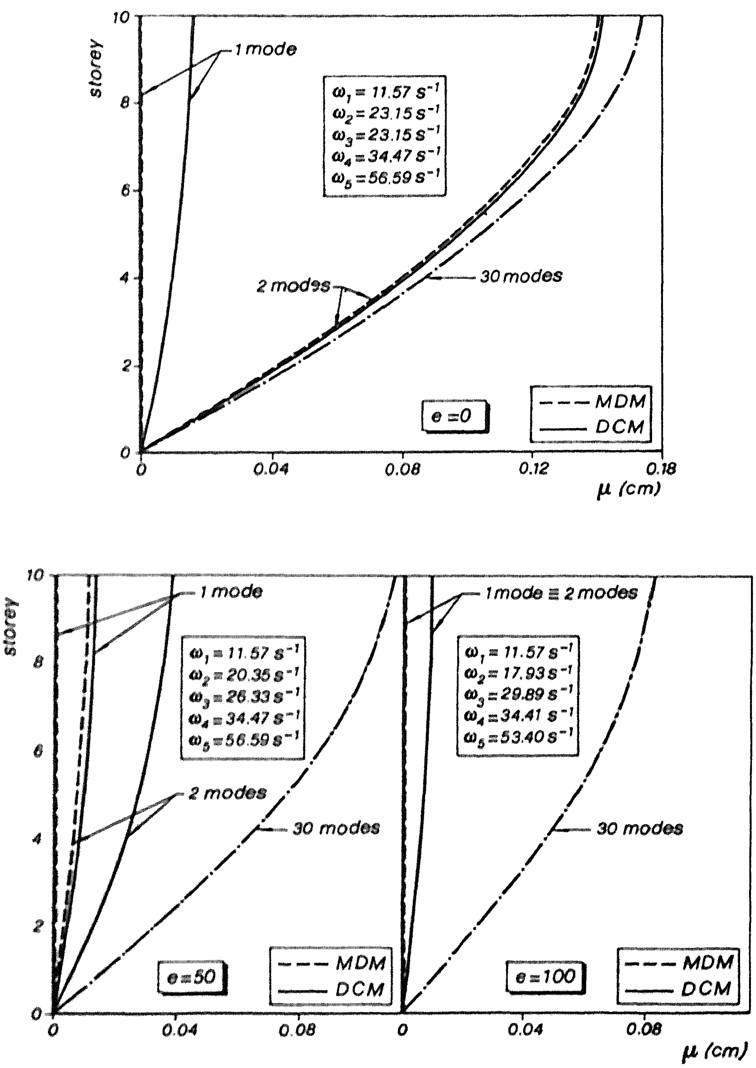

Fig. 2 Mean value of the largest absolute displacements of frame 1.

e (cm)	1° mode	2° mode
0	0	0.97231
50	0	0.58648
100	0	0.70050

TABLE I. Values of the energy ratio r for the structural system examined.

CONCLUSIONS

In this paper an extension of the Dynamic Correction method to the stochastic analysis has been proposed. The procedure affords an accurate evaluation of structural response in terms of covariances of displacements and velocities using a reduced number of mode shapes. The numerical application shows that for all cases considered the method gives a better evaluation of the response. As the seismic event, here schematized as a zero mean Gaussian process filtered by a Tajimi filter (Ref. 5), involves a rather wide frequency range, the improvement due to the correction proved to be minimum in some cases. The procedure adopted makes it possible to estimate a priori the efficiency of the modal correction through the evaluation of a coefficient given by the ratio between the average value of the energy associated with the uncorrected response and that associated with the corrected response.

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