Structural Control for Asymmetric Buildings Subjected to Earthquake Excitations

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SUMMARY:
A control algorithm suitable for irregular structures subjected to earthquake actions is investigated. The general control strategy consists of monitoring the incoming signal, analyzing it and recognizing its dynamic characteristics, applying the control algorithm in order to calculate the required action, and, finally, applying this action to the structure by means of control devices. The controlled algorithm that is used is a pole placement algorithm. Selection of poles of the controlled structure, which is critical issue for the success of the algorithm, is proposed in this paper by the following procedure: A fictitious symmetric structure is obtained from the irregular structure adding vertical elements, columns, shear walls or braces, at any location is necessary. The poles of symmetric structure are calculated. These poles are forced to be the poles of the irregular controlled structure. Based on locations of these poles, and using the pole placement algorithm, the feedback matrix is obtained. Using this feedback matrix, control forces are calculated at any time instant during the earthquake and are applied to the irregular structure by the control devices, which are installed at specific locations. The proposed control procedure makes the irregular structure to behave like a regular one. From the numerical results it is shown that the above control procedure is efficient in reducing the response of irregular building structures, with small amount of required control action.

Keywords: Structural control, Pole placement, Irregular structures, Structural Dynamics, Earthquake Engineering

1. INTRODUCTION

The last thirty years remarkable progress has been made in the field of control of civil engineering structures subjected to environmental loadings such as winds and earthquakes. Research and implementation in practice have shown that seismic control of structures has a lot of potential but also many limitations, Soong (1990), Housner et al. (1997). Most of studies assumed that the controlled structure is a planar structure. However, it is generally recognized that a real building is actually asymmetric in plan. Because of this asymmetry, it will undergo lateral as well as torsion vibrations simultaneously.

To compensate the tortional effect one approach is to retrofit and strengthen the building creating a new vertical structural elements shear walls in a suitable position in order to cancel the irregularities that exists. This is a traditional approach and is followed when no architectural or functional limitations are exists on the building. A second approach to faced irregularities is to use passive control devices as base isolators as it is shown in the work of Colunga and Soberon (2002 and 2007) and Gavin and Alhan (2002). Other passive control devices as dampers can also be used in suitable locations on the building in order to cancel again the irregularities Lavan and Levy (2006), Goel (2000), Lin and Chopra (2001).
A multiple tuned mass dampers (MTMD) can also be applied on the vibration control of irregular buildings considering soil structure interaction Lin and Wang. (2001). A semi active control system, using MR Dampers, can also be applied in order to reduce the coupled lateral and torsional motions in asymmetric buildings subjected to horizontal seismic excitations Yoshida et al. (2003). Hybrid control system consisting of a passive supplementary damping system and a semi-active tuned liquid column

damper (TLCD) system used to control irregular building under various seismic excitations, Kim and Adeli (2005). When a semi active or hybrid control system is applied to a structure then a control algorithm that used to direct the semi active devices is needed. In research studies and practical applications, various control algorithms have been investigated in designing controllers, such as pole placement control, linear quadratic or Gausian regulator LQR, LQG, $H\infty$ or H_2 control, sliding mode control, control strategies based on fuzzy logic or neural network are employed for the nominal controller design. The most suitable algorithms for structural application and the practical considerations that should be taken into account are described by Soong (1990).

One of the most suitable algorithms for controlling the structure is the pole placement algorithm. Pole placement algorithms have been studied extensively in the general control literature Sage and White (1977), Kwakernaak and Sivan (1972), Brogan (1974), Ogata (1997), Kautsky and Nichols (1985). The application of the algorithm in structural control can be found in the work of Martin and Soong (1976), Wang et al. (1983), Meirovotch (1990), Soong (1990), Utku (1990), and Preumont (2002), In Pnevmatikos and Gantes (2010) pole placement algorithm where used as control algorithm for mitigating the response of structures subjected to earthquake actions. In this work the poles of controlled structure are calculated on-line based on the important frequencies of seismic loading.

In this study the pole placement control algorithm is used in order to control irregular buildings. Firstly, a fictitious symmetrical building is created from the irregular building cancelling all irregularities. The poles of this fictitious symmetrical building are calculated. The poles of the controlled irregular building are forced to be equal with the poles of the symmetrical building. Then, the feedback matrix and the equivalent control force of the semi active devices applied to irregular building during the excitation are estimated. The result of this procedure is that the total controlled structure (irregular building with the devices) behaves like the fictitious symmetrical building.

2. THEORETICAL BACKGROUND

The equation of motion of one story, irregular, controlled structure based on orthogonal system with center of axis located at the center of mass of diaphragm is:

$$\mathbf{M}\ddot{\mathbf{U}}(\mathrm{t})+\mathbf{C}\dot{\mathbf{U}}(\mathrm{t})+\mathbf{K}\mathbf{U}(\mathrm{t})=-\mathbf{M}\mathbf{E}a_g(\mathrm{t})+\mathbf{E}_f sat\mathbf{F}(\mathrm{t}\text{-}\mathrm{t}_d)+\mathbf{B}_p\mathbf{P} \tag{2.1}$$

where where **M**, **C, K** denote the mass, damping and stiffness matrices of the structure, respectively. In irregular structure the mass and stiffness matrix is given below:

$$[\mathrm{M}]=\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_{CM} \end{bmatrix}, \qquad I_{CM}=m\cdot\left(\frac{L_X{}^2+L_Y{}^2}{12}\right) \tag{2.2}$$

$$[K]=\sum_i K_i, \quad K_i=\begin{bmatrix} K_{xi} & 0 & -K_{xi}\cdot y_i \\ 0 & K_{yi} & K_{yi}\cdot x_i \\ -K_{xi}\cdot y_i & K_{yi}\cdot x_i & K_{\theta i} \end{bmatrix}, \quad K_{\theta i}=K_{xi}\cdot y_i^2+K_{yi}\cdot x_i^2 \tag{2.3}$$

$$[C]=a_o[M]+a_1[K] \tag{2.4}$$

where m is the diaphragm mass and I_{CM} is the moment of inertia around the vertical axis that pass from center of mass, CM, Lx , Ly are the dimensions of the plate, K_i is the stiffness matrix of the vertical element (column or shear wall), K_{xi}, K_{yi}, $K_{\theta i}$ are the translational and rotational stiffness about x, y and z axis of the vertical elements located at coordinates x_i and y_i from the center of mass.

Alternatively, if the center of rigidity, CR, is calculated then the stiffness matrix **K** of structure has the following form

$$[K]=\begin{bmatrix} K_x & 0 & -K_x \cdot e_y \\ 0 & K_y & K_y \cdot e_x \\ -K_x \cdot e_y & K_y \cdot e_x & \sum (K_{x,i} y_i^2 + K_{y,i} x_i^2) + K_\theta \end{bmatrix} \tag{2.5}$$

$$K_\theta = K_x \cdot e_y^2 + K_y \cdot e_x^2$$

where e_x and e_y are the eccentricities of center of stiffness to the center of mass. K_x, K_y and K_θ are the translational and rotational story stiffness along the x,y and z axis respectively.
E, $\mathbf{E}_f$ are the location matrix for the earthquake and the control forces on the structure,

$$\mathbf{E}=\begin{bmatrix}1\\1\\0\end{bmatrix}, \quad \mathbf{E}_f=\begin{bmatrix}1 & 0\\0 & 1\\ \mathrm{e}_{\mathrm{fy}} & \mathrm{e}_{\mathrm{fx}}\end{bmatrix} \tag{2.6}$$

F is the matrix of equivalent control forces which should be applied to the structure, directly if active control devices are used or indirectly way if semi active control devices are used to control the structure.

$$\mathbf{F}=\begin{bmatrix}\mathrm{F}_{\mathrm{cx}}\\ \mathrm{F}_{\mathrm{cy}}\end{bmatrix} \tag{2.7}$$

Since the control devices have limited capacity and applied to the structure with time delay, saturation delayed function, *sat***F,** should be used and given by:

$$sat\mathbf{F}(\mathrm{t}\text{-}\mathrm{t_d})=\begin{cases}\mathbf{F}(\mathrm{t}\text{-}\mathrm{t_d}), & \mathbf{F}(\mathrm{t}\text{-}\mathrm{t_d})<\mathbf{F}_{\mathrm{allowable}}\\ \mathbf{F}_{\mathrm{allowable}}, & \mathbf{F}(\mathrm{t}\text{-}\mathrm{t_d})\geq\mathbf{F}_{\mathrm{allowable}}\end{cases} \tag{2.8}$$

where t_d is the time delay and $\mathbf{F}_{allowable}$ is the maximum capacity of the control device.
In Eqn. 2.1 **P** and $\mathbf{B}_P$ are the vertical loads and their location, in this study this loads are ignored.
U is the displacement vector with respect to the mass center

$$\mathbf{U}=\begin{Bmatrix}u_x\\u_x\\ \theta\end{Bmatrix} \tag{2.9}$$

The geometric properties, eccentricities, the center of mass and rigidity, the location of control devices are shown in Fig. 2.1.

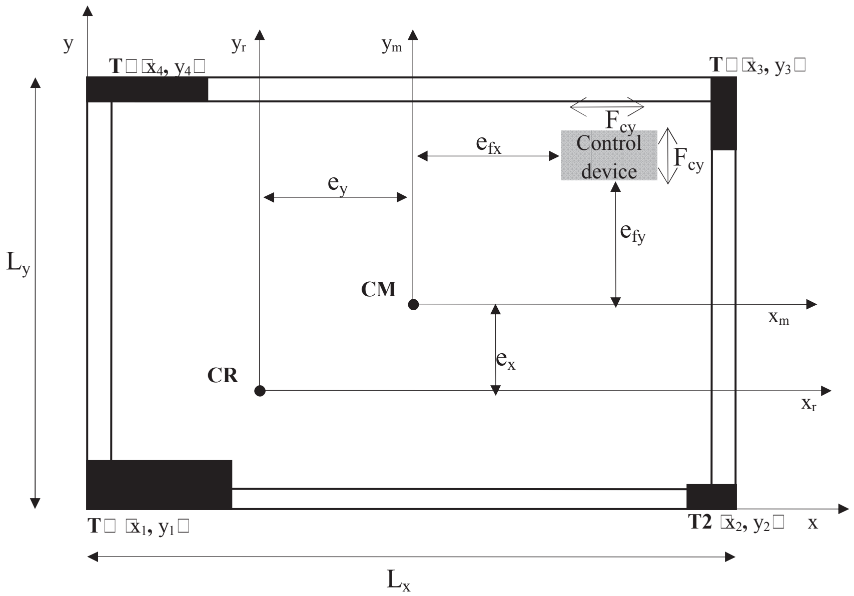
In the state space approach the above Eqn. 2.1 can be written as follows:

$$\begin{aligned}\dot{\square}(\mathrm{t}) &= \mathbf{A}\square(\mathrm{t})+\mathbf{B}_g a_g(\mathrm{t})+\mathbf{B}_f sat\mathbf{F}(\mathrm{t}\text{-}\mathrm{t_d})\\ \square(\mathrm{t}) &= \mathbf{C}\square(\mathrm{t})+\mathbf{D}_{\mathrm{f}} sat\mathbf{F}(\mathrm{t}\text{-}\mathrm{t_d})+\mathbf{D}_{\mathrm{g}} a_{\mathrm{g}}(\mathrm{t})+\square\end{aligned} \tag{2.10}$$

The matrixes □, **A, B**$_g$, **B**$_f$ are given by:

$$\square = \begin{bmatrix} \mathbf{U} \\ \dot{\mathbf{U}} \end{bmatrix}_{2nx1}, \ \mathbf{A} = \begin{bmatrix} \square & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}_{2nx2n}, \ \mathbf{B}_g = \begin{bmatrix} \square \\ -\mathbf{E} \end{bmatrix}_{2nx1}, \ \mathbf{B}_f = \begin{bmatrix} \square \\ \mathbf{M}^{-1}\mathbf{E}_f \end{bmatrix}_{2nx1} \quad (2.11)$$

The matrixes **Y, C, D□, D□,** and □are the output states, the output matrix, the feed forward control force matrix, the excitation matrix and the noise matrix, respectively. Consider the case where the output variables are the same as the states of the system and there is no application of the control forces to the output variables, so the matrixes **C, D** are the identity and zero matrix, respectively. The noise matrix depends on the sensor that is used to measure the response of the system. The above equation can be solved numerically in MATLAB software using the simulink toolbox.



F□□□□2.□Geometrical properties, eccentricities e_x, e_y, center of mass CM, center of rigidity CR, and location of control devices e_{fx}, e_{fy}.

□ CONTROL ALGORITHM FOR IRREGULAR □TRUCTURE

The control force **F** is determined by linear state feedback:

$$\mathbf{F} = -\mathbf{G}_1\mathbf{U} - \mathbf{G}_2\dot{\mathbf{U}} = -\begin{bmatrix} \mathbf{G}_1 & \mathbf{G}_2 \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \dot{\mathbf{U}} \end{bmatrix} = -\mathbf{G}\square \quad (3.1)$$

G is the gain matrix, which is calculated with pole placement algorithm. This algorithm requires desired poles location of the controlled system.
The eigenvalues or poles of a structural system are given by:

$$\lambda_i = -\xi_i \omega_i \pm j\omega_i \sqrt{1-\xi_i^2} \quad (3.2)$$

where ω_i and ξ_i are the cyclic eigenfrequencies and the damping ratio, respectively. If a state space formulation is adopted, these poles are obtained directly from the eigenvalues of matrix **A**:

$$\det[\lambda \mathbf{I} - \mathbf{A}] = 0 \rightarrow \lambda_i = \alpha_i \pm j\beta_i \quad (3.3)$$

The representation of the poles in the complex plane is shown in Fig. 3.1

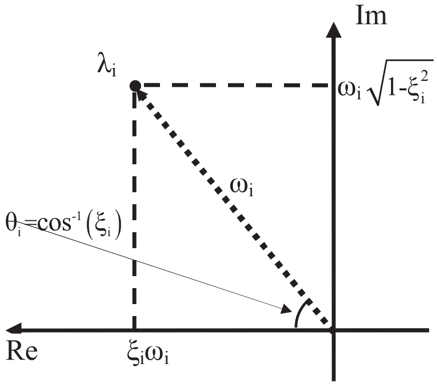


Figure 3.1. Representation of the poles in the complex plane

Replacing the force **F** into Eqn. 2.1 the controlled system can be described by:

$$\mathbf{M}\ddot{\mathbf{U}}(t)+\left(\mathbf{E}_f\mathbf{G}_2+\mathbf{C}\right)\dot{\mathbf{U}}(t)+\left(\mathbf{E}_f\mathbf{G}_1+\mathbf{K}\right)\mathbf{U}(t)=-\mathbf{M}\mathbf{E}a_g(t) \tag{3.4}$$

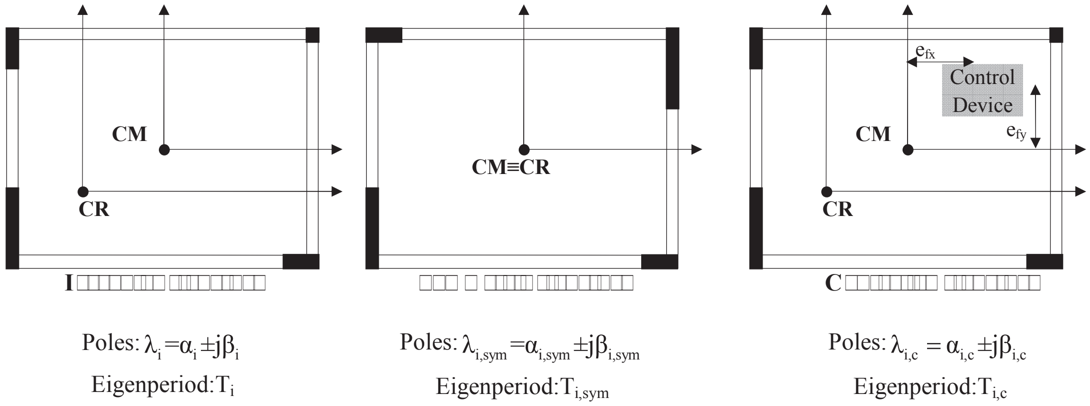
From the above equation it is clear that control of structures causes the change of their stiffness or damping and, consequently, their dynamic characteristics.

In this study the control force or the feedback matrix **G** is estimated in such way that the irregular controlled structure has similar dynamic characteristics to a fictitious symmetric structure. The procedure of calculating the feedback matrix is as follows: Firstly, a symmetric structure is obtained by the irregular one, adding at a specific locations new structural elements (shear walls or columns) cancelling all the irregularities. In this stage no limitations for the location of the elements exist because this symmetric structure is a fictitious structure which is not going to be constructed. The poles of irregular structure λ_i, and those from the symmetric structure $\lambda_{i,\,sym}$ are calculated. Then poles of the controlled irregular structure, $\lambda_{i,\,c}$, are forced to be equal to the poles of the fictitious symmetric structure $\lambda_{i,\,sym}$. Then additional equivalent damping is added and the final location of the poles of the controlled irregular structure, $\lambda_{i,\,c}$, is obtained. Based on these poles the feedback matrix, **G**, is calculated with the pole placement algorithm. The three structures and the relative poles locations in complex plane are shown in Fig. 3.2 and Fig. 3.3 respectively. Based to the feedback matrix, **G**, a control analysis takes place according to the scheme shown in Fig. 3.4. and the response of the controlled and the uncontrolled structure subjected to earthquake excitation is calculated.

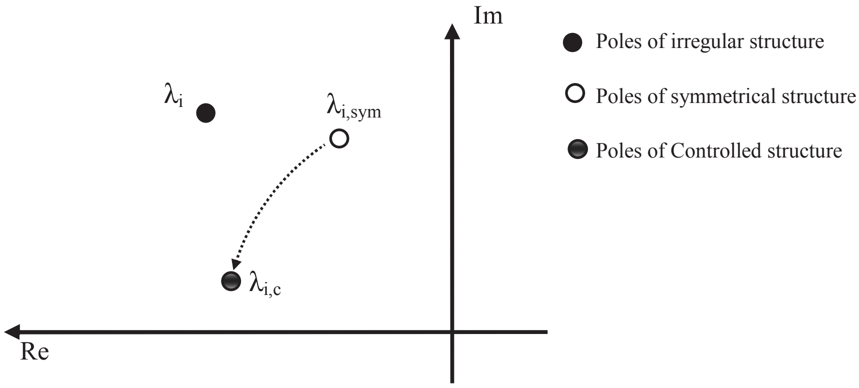
This controlled procedure has as a result that the irregular controlled building behaves like the symmetrical one. This procedure can be followed when there is a difficulty of retrofitting or adding new structural elements, shear walls, due to architectural or functional limitations. Instant of adding new structural element to the initial irregular structure active or semi active devices are installed to the irregular structure at a specific locations and as a result is that the controlled structure behaves like the symmetrical one.

4. EXAMPLES AND NUMERICAL EXPERIMENTS

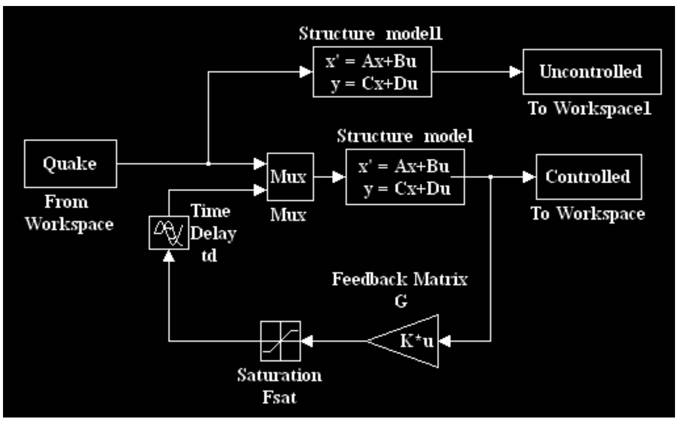
A single story irregular structure with its geometrical characteristics shows in Fig. 4.1(a) is investigated. The mass of the structure is 25 tones. Based to the irregular structure a fictitious symmetrical structure where the center of rigidity, CR, is coinciding with the center of mass, CM is shown in Fig. 4.1(b). The irregular structure with the control device is shown in Fig. 4.2. The poles of each structure are shown in figures Fig. 4.1(a, b) and Fig. 4.2 The control analysis procedure is performed according to the Fig. 3.4 and the response is estimate. The time delay was taken t_d=1msec, and saturation control force capacity Fsat=500kN. The Ducze earthquake was use as the excitation signal. The location of the control device was 2m far from the center of mass as shown in Fig. 4.2.



F□□□□□.2 Poles of irregular, symmetrical and controlled structure.

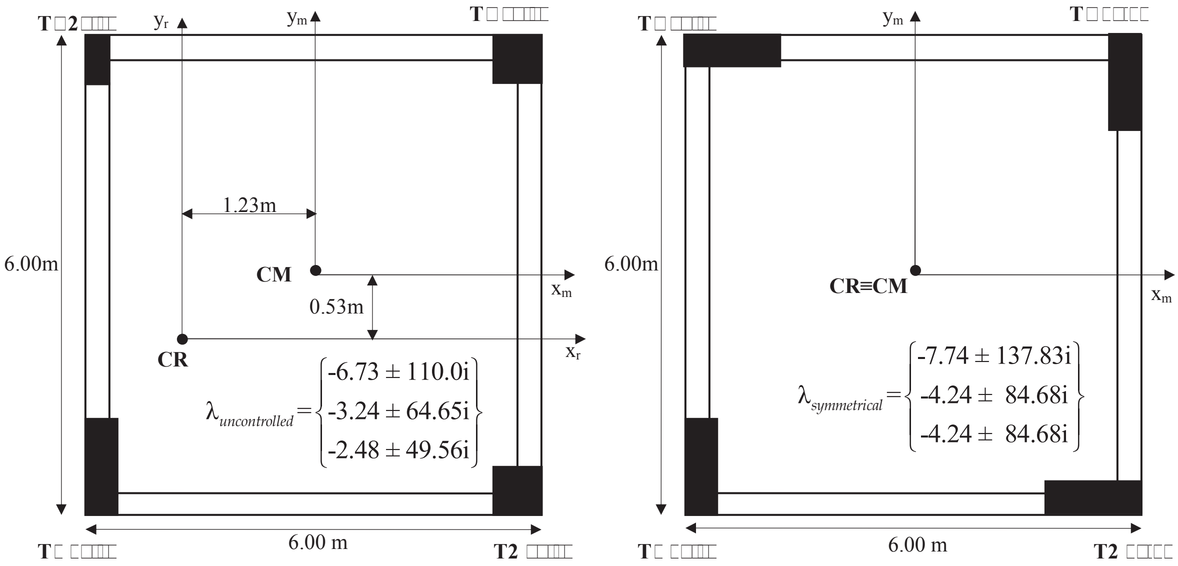


F□□□□□.□ Location of poles ion complex plane of irregular, symmetrical and controlled structure.

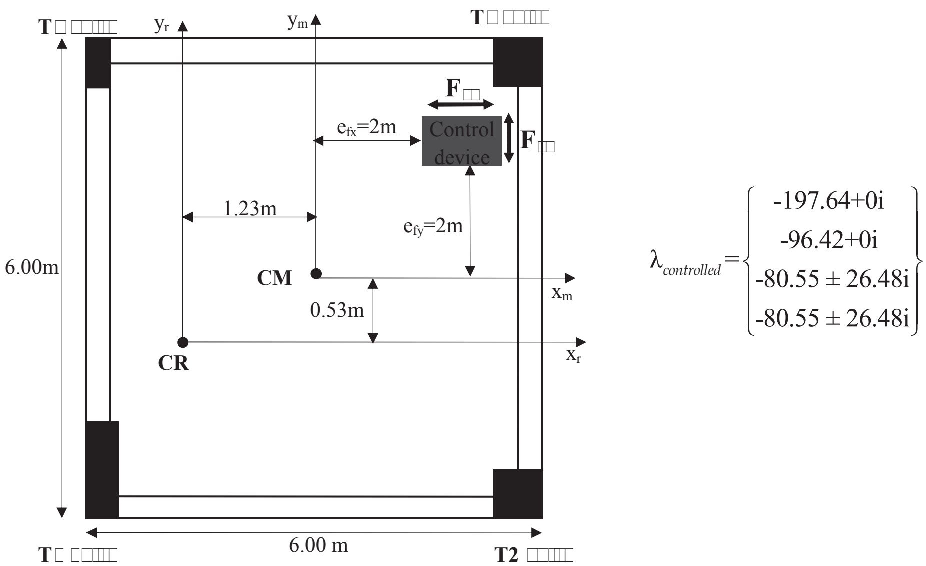


F□□□□□.□ Control procedures for estimating the response of the controlled and the uncontrolled subjected to earthquake excitation.

The Displacement, rotation and acceleration response of the controlled and the uncontrolled structure are shown in Fig. 4.3. The equivalent control forces in both directions are shown in Fig. 4.4. Generally, the response, displacement, rotation, and acceleration are reduced. Specifically, the displacement is reduced one order of magnitude. The rotation of the controlled structure is the half of the rotation of the uncontrolled structure, while the acceleration is reduced in 20%. From the other hand, in order to have the above reductions, control force should be applied to the structure. The maximum control force is 60kN and 40kN for x and y direction respectively.



F[illegible]. Irregular (a) and the fictitious symmetrical (b) structure irregular, symmetrical and controlled structure.



F[illegible].2 Irregular structure with the control device.

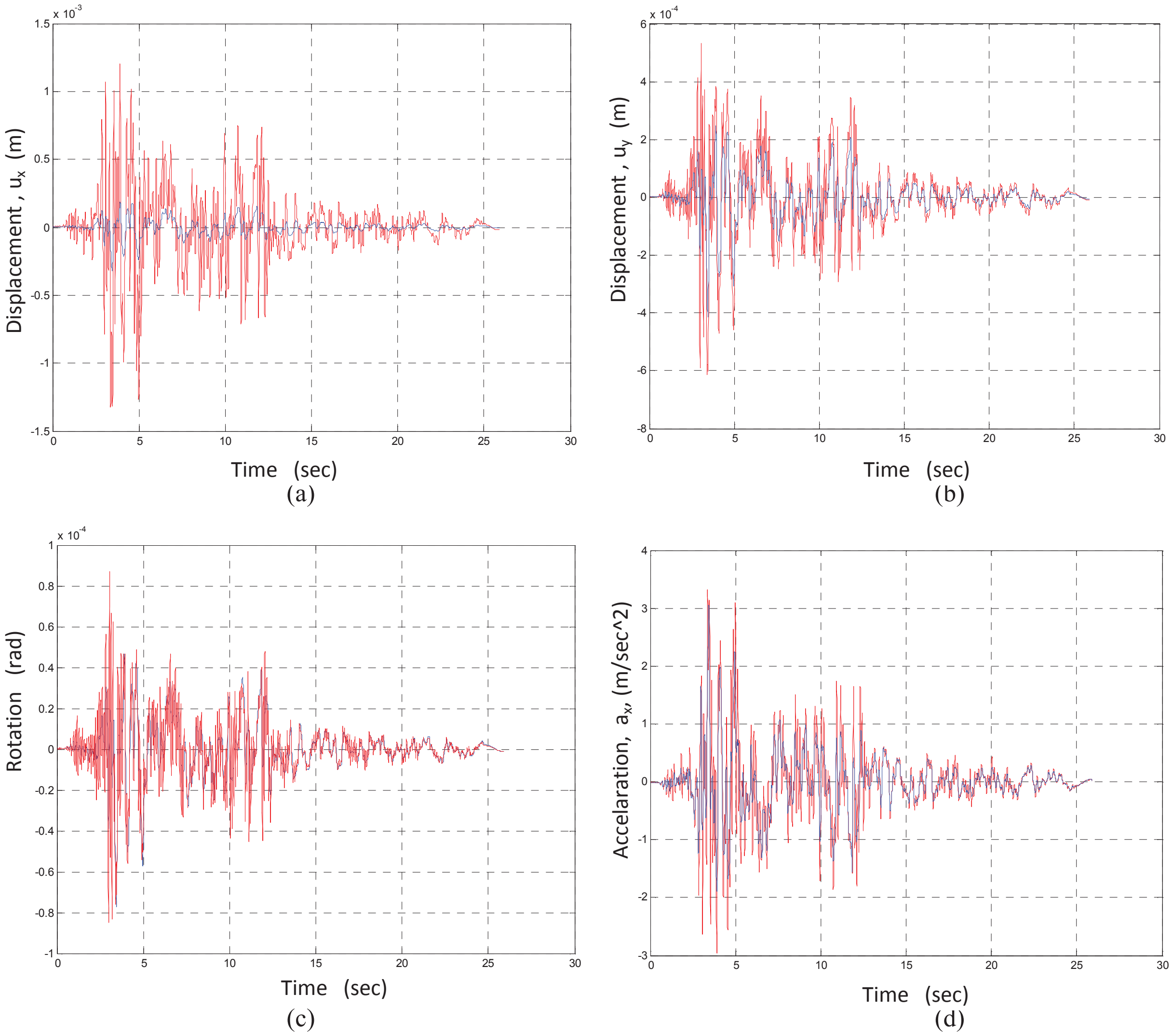


Figure 4.3 Displacement along x and y direction (a), (b), rotation (c) and acceleration in x direction (d) of the controlled and the uncontrolled structure.

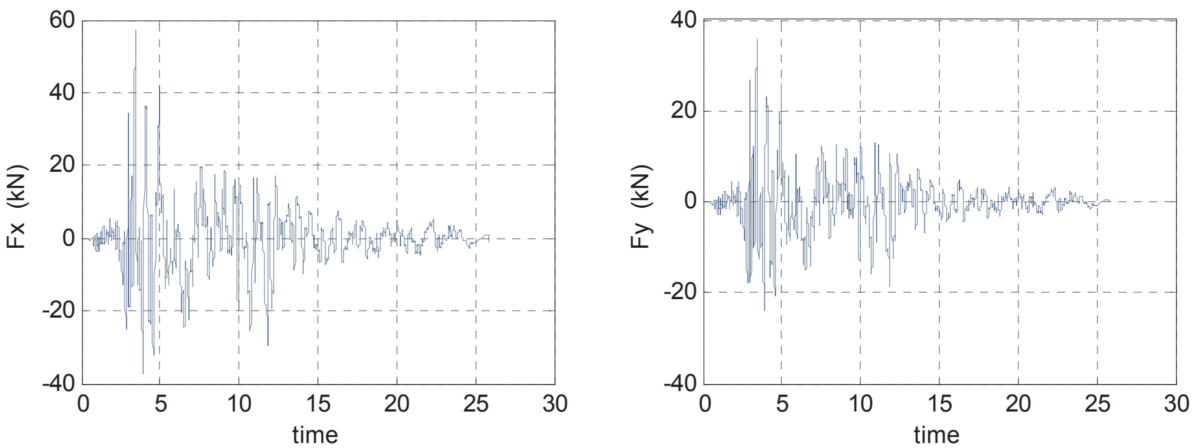


Figure 4.4 The equivalent control force in both directions.

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A control algorithm for irregular structures subjected to earthquake excitation was proposed. According to this algorithm the poles of the controlled irregular structure are forced to be equal with the poles of a fictitious symmetrical structure. Then the feedback matrix is calculated with pole placement algorithm and the control forces which should be applied to the structure are obtained. The proposed control procedure makes the irregular structure together with control devices behave like a symmetrical one. The proposed procedure was applied to one story concrete structure in three directions, two translational and one rotation. From the numerical results it is shown that the above control procedure is efficient in reducing the response of irregular building structures, with small amount of required control action.

[illegible]

Soong T.T. (1990). Active structural control: Theory and practice. London/New York: Longman Scientific &Technical/Wiley.

Housner, G. W., Bergman, L. A., Caughey, T. K., Chassiakos, A. G., Claus, R. O., Masri, S. F., Skelton, R. E., Soong, T. T., Spencer, Jr., B. F., and Yao, J. T. P. (1997). Structural control: Past, present and future. *Journal of Engineering Mechanics.* [illegible]**3**[illegible]897–971.

A. Tena-Colunga, L. Gómez-Soberón. (2002). Torsional response of base-isolated structures due to asymmetries in the superstructure. *Engineering Structures.* [illegible]**4**[illegible], 1587-1599.

Arturo Tena-Colunga, José Luis Escamilla-Cruz. (2007). Torsional amplifications in asymmetric base-isolated structures. *Engineering Structures.* [illegible], 237-247.

Gavin H, Alhan C. (2002). Control of torsionally asymmetric structures. *Proceedings of the American Control Conference.* Anchorage, AK, 357.

Lavan, O., and Levy, R. (2006). Optimal peripheral drift control of 3D irregular framed structures using supplemental viscous dampers. *Journal of Earthquake Engineering.* [illegible], 903-923.

Lavan, O., and Levy, R. (2005). Optimal design of supplemental viscous dampers for irregular shear-frames in the presence of yielding. *Earthquake Engineering and Structural Dynamics.* **34**, 889-907.

Goel, R. K. (2000). Seismic behaviour of asymmetric buildings with supplemental damping. *Earthquake Engineering and Structural Dynamic.* [illegible], 461–480.

Lin, W.-H., Chopra, A. K. (2001). Understanding and predicting effects of supplemental damping on seismic response of asymmetric one-storey system. *Earthquake Engineering and Structural Dynamics.* **3**[illegible], 1475–1494.

Chi-Chang Lin, Jer-Fu Wang. (2001). Vibration Control of Irregular Building-MTMD Systems Considering Soil-Structure Interaction Effect. *SMiRT 16*, paper # 1864.

Osamu Yoshida, Shirley J. Dyke1, Luca M. Giacosa, Kevin Z. Truman. (2003). Experimental verification of torsional response control of asymmetric buildings using MR dampers. *Earthquake Engineering and Structural Dynamics.* **3**[illegible]2085–2105.

Hongjin Kim, Hojjat Adeli. (2005). Hybrid control of irregular steel highrise building structures under seismic excitations. *International journal for numerical methods in engineering*. [illegible]**3**[illegible]1757–1774.

Sage A. P., White C. C. (1977). Optimum systems control. 2nd edition Prentice Hall, Englewood Cliffs NJ.

Kwakernaak H. and Sivan R. (1972). Linear optimal control systems. Wiley, New York NY.

Brogan W. L. (1974). Application of determinant identity to pole-Assignment and observer problems. *IEEE Transactions on automatic control AC-19*, 689-692.

Ogata K. (1997). Modern control engineering. 3rd edition Prentice Hall International Inc.

Kautsky, J. and Nichols N.K. (1985). Robust Pole Assignment in Linear State Feedback. *International Journal Control.* **4**[illegible], 1129-1155.

Martin R. C. and Soong T. T. (1976). Modal control of multistory structures. *ASCE Journal of engineering mechanics.* [illegible], 613- 623.

Wang P. C., Kozin F. and Amini F. (1983). Vibration control of tall buildings. *Engineering Structures,* [illegible], 282-289.

Meirovotch L. (1990). Dynamics and control of structures. John Willey & Sons.

Utku S. (1998). Theory of adaptive structures: Incorporative intelligent into engineering products. CRC press LLC.

Preumont A. (2002). Vibration control of active structures. An introduction. 2nd edition Kluwer academic publishers.

Nikos G. Pnevmatikos, Charis J. Gantes, (2010). Control strategy for mitigating the response of structures subjected to earthquake actions. *Engineering Structures*. **3**□□3616–3628.