Probabilistic Seismic Hazard Model of Inelastic Oscillator Based on Equivalent Linearization Technique

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SUMMARY:
In reliability-based seismic design of a structure, it is important to estimate the maximum response of an inelastic oscillator corresponding to a prescribed exceedance probability during a reference period. This paper first investigates the accuracy of the use of either a uniform hazard spectrum (UHS) or a conditional mean spectrum along with an equivalent linearization technique to estimate the response. Then, a new approximation method is proposed using UHS along with a modified Capacity Spectrum. The accuracy and applicability of the method is discussed using numerical examples as well as a generic model.

Keywords: Performance-based design, Seismic hazard, Uniform hazard spectrum, Inelastic oscillator

1. INTRODUCTION

Predictors of seismic structural demands (such as interstory drift ratios) that are less time consuming than nonlinear dynamic analysis (NDA) are useful for structural performance assessment and design. Several techniques for realizing such predictors have been proposed using the results of a nonlinear static pushover analysis (e.g., Luco 2002; Chopra & Goel 2002; Yamanaka, et al. 2003; Mori, et al. 2006). These techniques often use the maximum response of an inelastic oscillator (computed via NDA) that is "equivalent" to the original frame.

In reliability-based seismic design of a structure, it is necessary to express the maximum response of the inelastic oscillator probabilistically. Such information could be obtained via NDA, which requires thousands of samples; however, the acquisition of such information requires considerable computational effort. In practice, the use of simpler methods such as an equivalent linearization technique (EqLT) using an elastic response spectrum seems more reasonable, and design spectra are being developed on the basis of probabilistic approaches such as a uniform hazard spectrum (UHS) and a conditional mean spectrum (CMS, Baker & Jayaram 2008).

A UHS is obtained by plotting the response with the same (i.e., uniform) exceedance probability for a suite of elastic oscillators with different natural periods, and hence, a UHS does not represent any specific ground motion (Abrahamson 2006). Although there exists some correlation among the spectral responses of elastic oscillators to a ground motion (e.g., Baker & Jayaram 2008), perfect correlation is implicitly assumed in the use of a UHS. In such a scenario, the response could be overestimated via EqLT when a very rare event is considered.

The correlation among the spectral responses could be considered by using a CMS, which is the mean spectrum conditional to the event that the spectral displacement of an elastic oscillator with a certain period, T_c, equals the displacement with, say, 10% exceedance probability in 50 years. However, guidelines for the selection of T_c are not well established.

This paper first investigates the accuracy of the use of either a UHS or a CMS along with an EqLT to estimate the response of an inelastic oscillator. Then, based on the investigation using a generic model expressed in a stationary standard normal stochastic process a new approximation method is proposed using UHS along with a modified Capacity. The accuracy and applicability of the method is discussed using numerical examples.

2 EQUIVALENT LINEARIZATION TECHNIQUE

2.1 Equivalent Linearization Technique

In an EqLT, the maximum displacement of an inelastic oscillator with the elastic natural period, T_1, and the damping factor, h_1, is approximated using the maximum displacement of an elastic oscillator with the equivalent natural period, T_{eq}, and the equivalent damping factor, h_{eq}, as

$$S_D^I(T_1;h_1) \approx S_D^E(T_{eq};h_{eq}) \tag{1}$$

where $S_D(T;h)$ is the spectral displacement of an oscillator with the natural period, T, and the damping factor, h, and the superscripts E and I represent the elastic and inelastic responses, respectively.

T_{eq} and h_{eq} are often expressed as a function of the maximum ductility factor of the inelastic oscillator, μ, which is defined as

$$\mu = S_D^I(T_1;h_1) \big/ \delta_y \tag{2}$$

where δ_y is the yield displacement of an oscillator. Several linearization techniques have been proposed (e.g., Iwan 1980, Shimazaki 1999), and among them, the following formulae based on the idea proposed by Shimazaki are used in for an oscillator with a bilinear backbone curve this paper.

$$T_{eq} = T_1 \cdot \sqrt{\frac{\mu}{1-k_2(1-\mu)}} \tag{3}$$

$$h_{eq} = 0.25 \cdot \left(1 - 1\big/\sqrt{\mu}\right) + h_1 \tag{4}$$

where k_2 is the second stiffness ratio of the backbone curve.

2.2 Capacity Spectrum Method

The capacity spectrum (CS) method (Freeman 1978) can be used to estimate graphically the inelastic displacement as the intersection of the capacity spectrum and the demand spectrum. In order to take into account the effect of h_{eq}, the demand spectrum must be adjusted by multiplying it with the damping reduction factor $F_h(h_{eq})$, defined as the ratio of the spectral response of an elastic oscillator with the damping factor h_{eq} to that with the damping factor h_1. Because h_{eq} is a function of the unknown value μ, an iterative procedure is generally required for its determination.

In contrast, the response can be estimated directly by considering the demand and capacity spectra in an ordinal T - S_D coordinate rather than an S_D - S_A coordinate (see Fig.1, Mori & Maruyama 2007). The S_D axis can be transformed linearly into the axis of the maximum ductility factor, μ, by dividing the S_D axis by the yield displacement of the inelastic oscillator. The T axis can also be expressed in terms of μ because T_{eq} is a function of μ, as expressed by Eq.(3). Then, the capacity spectrum can be obtained by connecting the corresponding values in the linear (vertical) and nonlinear (horizontal) μ coordinates.

On the basis of Eq.(3), the capacity spectrum, $C_S(T)$, of an inelastic oscillator with a bilinear backbone curve and whose mass is equal to unity can be expressed as

$$C_S(T) = \delta_y \cdot \mu$$
$$= \frac{9.8 \cdot C_y}{k_1} \cdot \frac{(1-k_2) \cdot T^2}{T_1^2 - k_2 \cdot T^2} \qquad = \frac{9.8 \cdot C_y \cdot T_1^2}{4\pi^2} \cdot \frac{(1-k_2) \cdot T^2}{T_1^2 - k_2 \cdot T^2} \; ; \; T \geq T_1 \tag{5}$$

where k_1 and C_y are the elastic stiffness and the yield base shear coefficient of the oscillator, respectively, and the acceleration due to gravity is 9.8 (m/s^2).

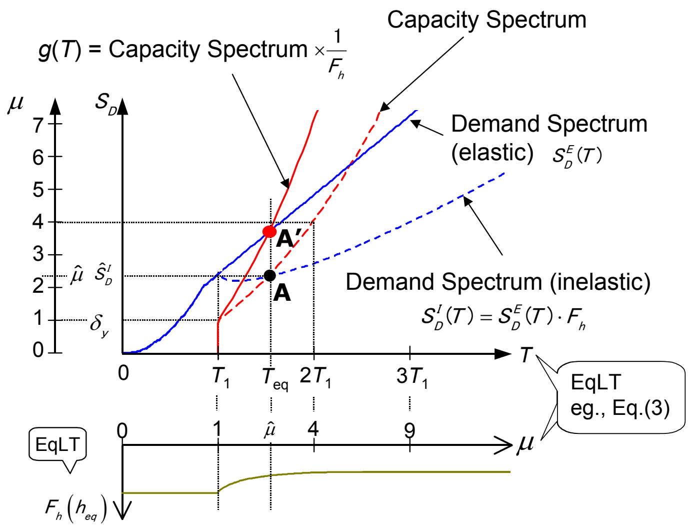


Figure 1. Capacity spectrum method in T - S_D coordinate

3 PROBABILISTIC SEISMIC HAZARD MODEL

3.1 Probability Distribution Function of Elastic Spectral Displacement

It is assumed here that the basic information on the seismic hazard at a construction site is expressed by a suite of probability distribution functions (CDFs) of the maximum spectral displacement of elastic oscillators in n years, $S_{Dn}^E(T;h)$. The CDFs and correlation among spectral displacements can be obtained through a seismic hazard analysis, which generally involves the following steps.

(1) Simulate the occurrence of earthquakes at the faults that could cause a strong ground motion at the site for the next n years.

(2) Estimate the response spectrum at the site for each earthquake considering the variability and the correlation among the residuals from the spectrum estimated by an attenuation formula.

(3) At each natural period, take the maximum response among the ground motions estimated in Step (2) as the sample of the maximum response in n years.

(4) Repeat Steps (1)–(3) many times to obtain the probability distribution functions of the n-year maximum displacement, $S_{Dn}^{E}(T;h)$.

In this research, the seismic hazard at the city hall of Nagoya, which is located in the central part of Japan, is estimated and used in the following analysis.

The seismic hazard is estimated by analyzing 4,000 samples for 50-year seismic activities. An acceleration response spectrum for each ground motion at the site is estimated by the attenuation formula proposed by An-naka, et al. (1997). The corresponding displacement response spectrum is estimated by multiplying the acceleration response spectrum with $(T_1/2\pi)^2$. It is assumed that the residuals of a response spectrum are lognormally distributed with the coefficient of variation (c.o.v.) equal to 0.5 (Ikeura & Noda 2005) and with the correlation coefficient proposed by Baker & Jayaram (2008), which are taken into account in Step (2).

The mean and c.o.v. of the 50-year maximum spectral displacement of an elastic oscillator with $h = 0.05$, $S_{D50}^{E}(T;h=0.05)$, are shown in Fig.2. The correlation coefficients of the 50-year maximum spectral displacement between T_1 = 0.3, 0.5, 1.0, or 1.5 s and other natural periods are shown in Fig.3, where the natural period is expressed in the logarithmic scale. The correlation model proposed by Baker and Jayaram is also presented in the figure (dashed line). It should be noted that the model is a function of only $\xi = \left|\log(T_i) - \log(T_j)\right|$; the shapes of the correlation model do not change, regardless of the value of T_1, and have the mirror image at T_1.

In contrast, the correlation of the 50-year maximum response is not a function of only $\xi = \left|\log(T_i) - \log(T_j)\right|$. The correlation coefficients are relatively close to those of Baker and Jayaram's model when both T_i and T_j are longer than 0.5 s. However, at shorter values of T_i and T_j where several intraplate faults could contribute to the response of an oscillator, they are somewhat smaller than those of the model.

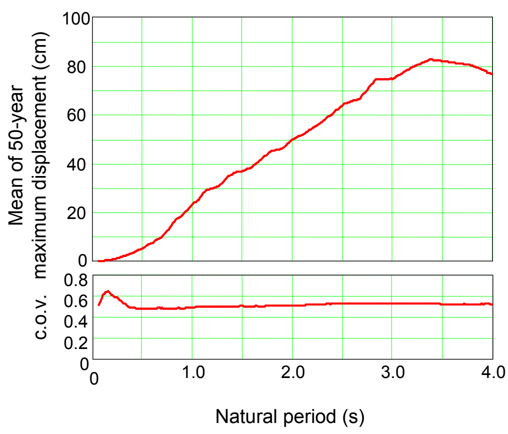


Figure 2. Statistics of 50-year maximum acceleration response spectrum

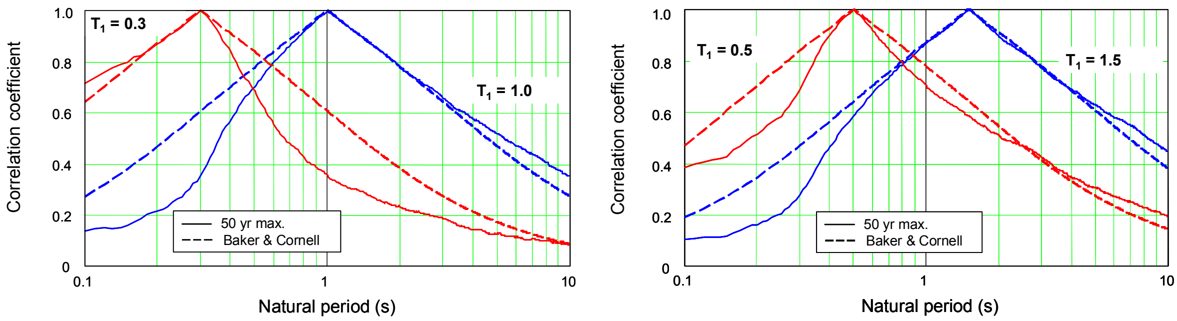


Figure 3. Correlation coefficient of Baker and Jayaram's model and 50-year maximum response

3.2 UHS and CMS

The UHS can be obtained by sorting the samples of the 50-year maximum values at each period obtained in Step (4) and then plotting the values in the same order of the samples.

As mentioned earlier, guidelines for the selection of T_c are not well established for the conditional event considered in the CMS. Here, T_1 is selected as T_c, and a CMS conditional to the event that the spectral displacement of an elastic oscillator at T_1 equals the value with 10% exceedance probability in 50 years is estimated. It is assumed that a spectral displacement is lognormally distributed.

Some argument could arise as to which correlation model should be used: that of the 50-year maximum response or that of the spectral displacements of a single ground motion such as Baker and Jayaram's model. Here, the latter is considered because the former would be site dependent and would require simulation such as the one described in this paper, and because the response of an oscillator to be estimated would eventually be the response to a single ground motion.

4 Hazard model of $S_D^I(T)$

4.1 Approximate Estimate of $S_D^I(T)$ Using UHS/CMS

The exceedance probability of the maximum displacement of an inelastic oscillator in 50 years, S_{D50}^I, estimated by the CS method using either a UHS or a CMS is presented in Figs.4. It is assumed that the damping factor, h_1, is 0.05; the yield base shear coefficient of the inelastic oscillator, C_y, is 0.3 or 0.5; the elastic natural period , T_1, is 0.3, 1.0 (upper figures), 0.5, or 1.5 s (lower figures); and the second stiffness ratio, k_2, is 0.00 or 0.03. For comparison, the "accurate" estimate of the exceedance probability considering the response of each ground motion in each 50-year of 4,000 samples is also shown using solid lines in the figure. The "accurate" probability is estimated by employing the following steps:

(i) Applying the CS method in the T-S_D coordinate (Fig.1) to the response spectrum of each ground motion during each period of 50 years, estimate the maximum response to that ground motion. Both the bias and the error in the EqLT are considered using the function proposed by Kawasaki et al (2011).

(ii) Take the maximum response within each period of 50 years as the sample of the 50-year maximum value.

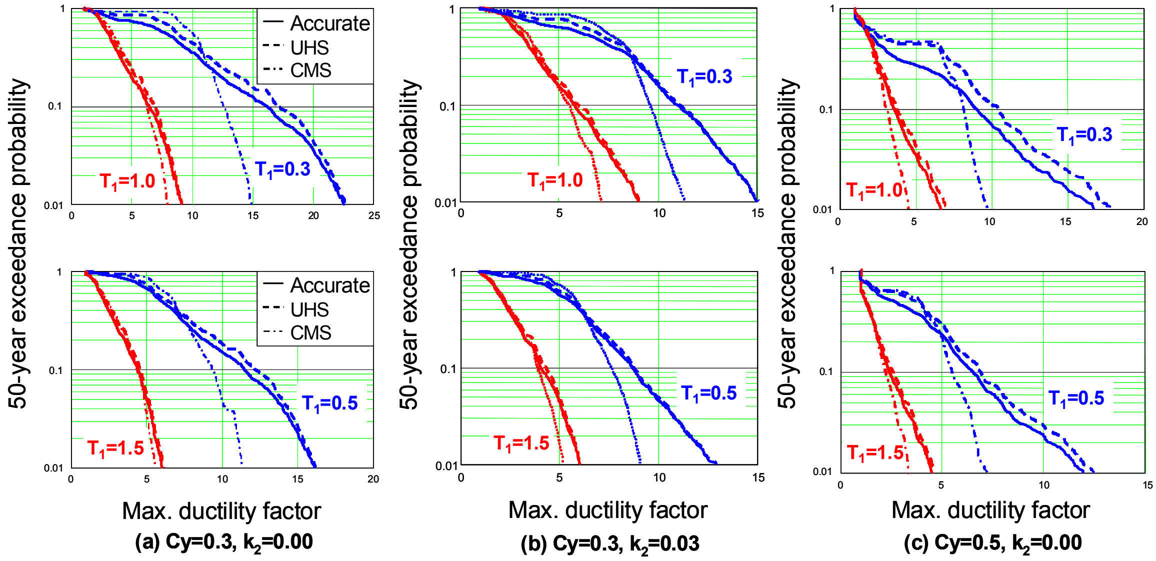


Figure 4. The exceedance probability of S^{I}_{D50} estimated by the CS method using a UHS or a CMS

As seen in Figs.4, there are many cases in which the use of the EqLT with a UHS (dashed line) provides fairly accurate estimates, especially when the elastic natural period is longer than or equal to 1.0 s. The error becomes smaller when $k_2 = 0.03$ (Fig.4(b)). However, when $T_1 = 0.3$ or 0.5 s, the error in the estimation obtained using a UHS becomes large. As T_1 is smaller, the correlation coefficient is relatively small, which is far from the perfect correlation implicitly assumed in the UHS. The error is more noticeable when $C_y = 0.5$ (Fig.4(c)).

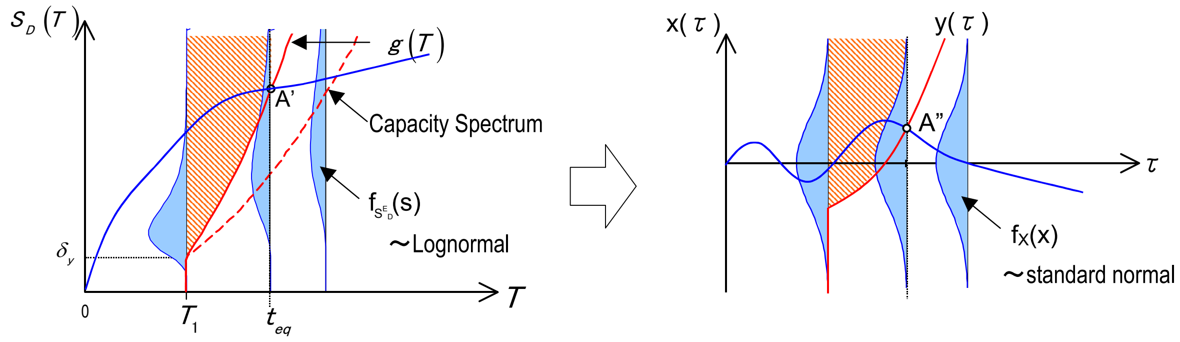
On the contrary, CMS always provides optimistic estimates when exceedance probability is 20% or less in 50 years. The error is especially noticeable when C_y= 0.3 and k_2 = 0.00. In a CMS, the condition considered is the elastic spectral displacement at T_1 is equal to the value with same exceedance probability of S^{I}_{D50}. However, there could be many other possible events of e.g.10% in 50 years. Such ignorance of possible events could lead to underestimation of the response with a large error.

4.3 Investigation through Stationary Standard Normal Stochastic Process

Here, some of the results presented in the previous section are further investigated using a generic model expressed in a stationary standard normal stochastic process. In the CS method, the event that the equivalent natural period, T_{eq}, is longer than t_{eq} corresponds to the event that $S^{E}_{D50}(T;h)$ is always above $g(T)$ in the range of $\left(T_1, t_{eq}\right)$ (the hatched area in Fig.6 (a)). By transforming $S^{E}_{D50}(T;h)$, which is assumed to be lognormally distributed on the basis of the hazard analysis, into a standard normal stochastic process, $y(\tau)$ (Eq.(9)), the function $g(T)$ (see Fig.1) and the hatched area in Fig.5(a) are transformed into the function $y(\tau)$ and the hatched area in the $\tau - y$ coordinate in Fig.5(b), respectively.

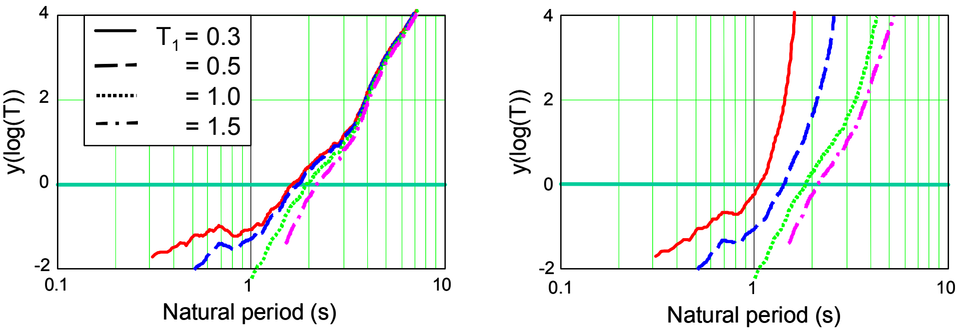
$$y(\tau) = \frac{\ln(S^{E}_{D50}(T;h)) - \mu_{\ln\left(S^{E}_{D50}(T)\right)}}{\sigma_{\ln\left(S^{E}_{D50}(T)\right)}} \tag{6}$$

where $\mu_{\ln\left(S^{E}_{D50}(T)\right)}$ and $\sigma_{\ln\left(S^{E}_{D50}(T)\right)}$ are the mean and standard deviation of $\ln(S^{E}_{D50}(T;h))$, respectively. Note in Eq.(6) and Fig.5(b) that the horizontal axis is transformed to $\tau = \log(T)$.



(a) T - S_D coordinate (b) τ - X coordinate

Figure 5. Schematic illustration of capacity spectrum method for standard normal stochastic process



(a) C_y = 0.3, k_2 = 0.00 (b) C_y = 0.3, k_2 = 0.03

Figure 6. Capacity spectrum and demand spectrum in standard normal stochastic process

Considering the fact that the correlation coefficient of two spectral displacements is a function of only $\left|\log(T_i)-\log(T_j)\right|$ (see Fig.3), the problem of estimating the probability that the stochastic process $S_{D50}^E(T;h)$ stays above the threshold $g(T)$ in the range of (T_1, t_{eq}) can be interpreted as the first passage problem of a stationary standard normal stochastic process crossing the threshold $y(\tau)$ downward.

The correlation coefficient between $y(\tau_i)$ and $y(\tau_j)$, ρ_{ij}, is expressed as (Der Kiureghian and Liu 1985)

$$\rho_{ij} = \frac{\ln\left(1+\rho_{Sij}\cdot V_{SD50(T_i)}\cdot V_{SD50(T_j)}\right)}{\sqrt{\ln\left(1+V_{SD50(T_i)}^2\right)}\sqrt{\ln\left(1+V_{SD50(T_j)}^2\right)}} \tag{7}$$

where $V_{SD50(T_i)}$ is the c.o.v. of $S_{D50}^E(T_i;h)$.

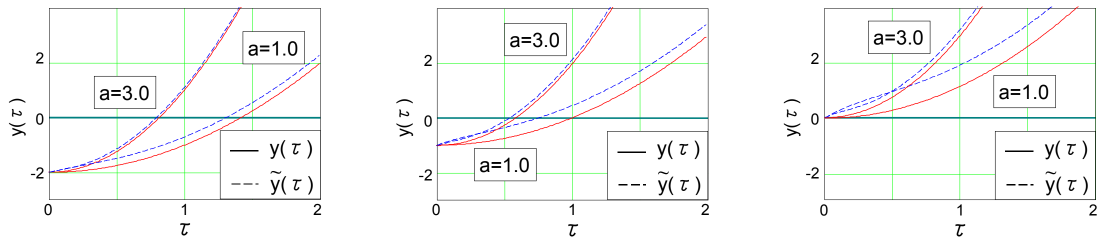
Fig.6 illustrates the capacity spectra of an inelastic oscillator with a bilinear backbone curve in a standard normal stochastic process when C_y = 0.3 and k_2 = 0 or 0.03. It should be noted that the horizontal axis is expressed in a logarithmic scale. When k_2 = 0, as T_1 becomes longer, the slope of $y(\tau)$ becomes steeper and closer to being linear. As the slope becomes steeper, the possible range in which the capacity spectrum crosses $y(\tau)$ downward becomes narrower. Thus, the response becomes less dependent on the correlation coefficient, and the EqLT that uses a UHS, which is now expressed by a horizontal line (see Fig.6), could provide estimates that are more accurate. As the slope becomes steeper when k_2 is larger, estimates of higher accuracy are obtained.

When C_y = 0.5, the capacity spectrum shifts upward. Then, the slope of the part of the capacity spectrum for T_1 = 0.3 or 0.5 s becomes gentler as it approaches the horizontal axis; consequently, the EqLT that uses a UHS becomes more sensitive to the correlation among the elastic spectral displacements.

The accuracy of EqLT using UHS is further investigated using generic CS's shown in Figs.7(a), (b), and (c) by solid lines. The CS's are modeled by parabolic functions expressed by the following equation with a equal to 1.0 or 3.0, and y_0 equal to (a) –2, (b) –1, or (c) 0. Note that τ =0 can be interpreted as the natural period of an inelastic oscillator.

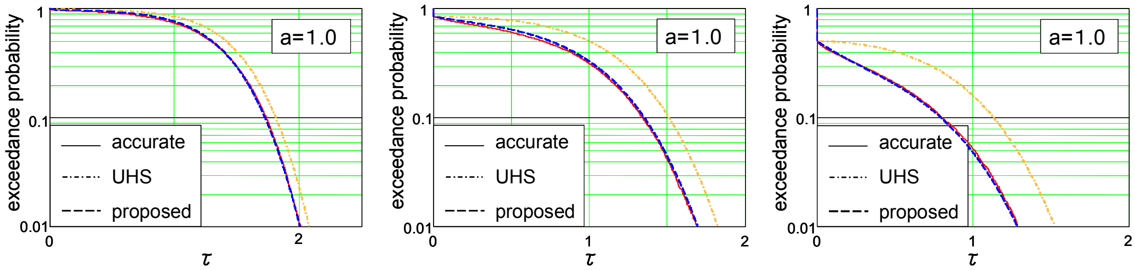
$$y(\tau) = a \cdot \tau^2 + y_0 \tag{8}$$

The exceedance probability associated with the CS's in Figs.7(a), (b), and (c) are shown in Figs.8(a), (b), and (c) for a equal to 1, and in Figs.9(a), (b), and (c) for a equal to 3, respectively. The accurate estimates are also presented by solid lines in each figure. It can be seen in these figures that the part of CS with gentle slope is closer to the horizontal axis (i.e. as y_0 increases), exceedance probabilities estimated using UHS become less accurate. Also, as the slope of CS is steeper, exceedance probabilities estimated using UHS become more accurate.



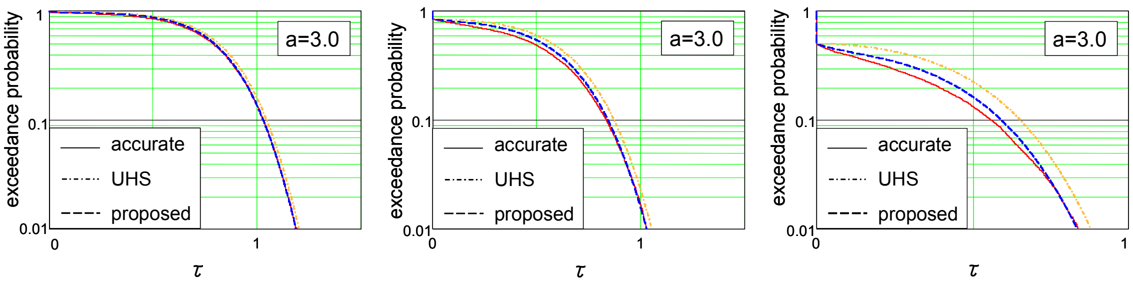
(a) y_0 =-2.0 (b) y_0 =-1.0 (c) y_0 =0.0

Figure 7. Capacity spectrum and modified capacity spectrum



(a) y_0=-2.0 (b) y_0 =-1.0 (c) y_0 =0.0

Figure 8. The exceedance probability estimated in generic model (a =1.0)



(a) y_0 =-2.0 (b) y_0 =-1.0 (c) y_0 =0.0

Figure 9. The exceedance probability estimated in generic model (a =3.0)

4.4 Improvement of EqLT using UHS

When the CS is flat, there is no chance that a UHS intersects with the CS; however, there is some possibility that the stochastic process could cross the CS downwards. The similar observations can stand when the slope of the CS is fairly gentle; the stochastic process would crosses the CS much earlier that that UHS intersects the CS. Based on such observations, one might consider increasing the slope depending on the degree in order to make a UHS crossing the CS a little earlier. Here it is proposed to modify the CS by multiplying the following function, which is a function of the slope and the value y_0 in Eq.(8).

$$h(\tau) = 1 + h^{*}\left(\frac{dy(\tau)}{d\tau},\ y_0\right) \tag{9}$$

For the CS expressed by Eq.(8), the following modified CS is obtained empirically.

$$\tilde{y}(\tau) = a \cdot \tau^2 + \frac{1-\exp\left(-4 \cdot a \cdot \tau\right)}{4 \cdot a \cdot \exp\left(-0.4 \cdot b - 1\right)} + y_0 \tag{10}$$

The modified CS is presented in Fig.7 by dashed lines, and the exceedance probability estimated by the proposed method is also presented in Figs.8 and 9. In any cases, the probabilities agree fairly well with the accurate estimates. Note in Figs.7 that the larger the error of the EqLT using original CS and UHS, the larger the modification of CS is. The applicability of Eqs.(9) and (10) for more general type of CS would be investigated further in the future.

5 CONCLUSIONS

In this study, we investigated the accuracy and applicability of the use of either a UHS or a CMS along with an EqLT to estimate Exceedance probability of $S^{I}_{D50}(T_i;h)$. The results can be summarized as follows:

- Exceedance probability of $S^{I}_{D50}(T_i;h)$ can be estimated fairly accurately by an EqLT that uses a UHS when T_1 is longer than or equal to 1.0 s. The accuracy increases with increase in k_2 and with decrease in C_y.
- The EqLT that uses CMS could provide fairly optimistic estimates when exceedance probability is 20% or less in 50 years. This could be due to the ignorance of many other possible events of e.g.10% in 50 years.

The accuracy of EqLT with UHS is investigated in stationary standard normal stochastic process. It is shown that the part of CS with gentle slope is closer to the horizontal axis, exceedance probabilities estimated using UHS or CMS become less accurate, and that as the slope of CS is steeper, exceedance probabilities estimated using UHS become more accurate. Based on such observations, it is proposed to modify the CS by multiplying a modification function. Through numerical examples it is shown that the proposed method provide fairly accurate estimates for most of the cases when CS is a parabolic function in stationary standard normal stochastic space. The applicability is investigated further in the future.

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