Fuzzy-Random Fragility Model to Estimate Non-Structural Damage to Reinforced-Concrete Frames

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SUMMARY
Damage has to be quantified to assess the seismic performance of buildings but, in fact, its definition is vague by nature. The limit states associated with non-structural damage especially are verbal and qualitative rather than analytical and quantitative. Therefore, it is proper to consider the fuzziness together with the randomness. A probabilistic model to include fuzziness in the fragility computation is formulated herein, consistent with the time-invariant first-order second-moment reliability method. Such a fuzzy-random model is characterized in terms of moments, distribution, and percentiles, in comparison with the classical reliability model. The fuzziness causes the fragility mean value to increase at lower seismic intensity, but to decrease at higher intensity. The fuzziness also causes the fragility dispersion to decrease in most cases. As application, the model is implemented to compute the fuzzy-random fragility curves of non-structural damage to a masonry-infilled reinforced-concrete frame. All results by the proposed model are reasonable.

Keywords: Seismic fragility, fuzziness, non-structural damage, inter-storey drift

1. INTRODUCTION

Seismic design to current codes is based on checking several limit states differing from each other in the degree of damage. Unfortunately, a lot of uncertainty is inherent in any quantification of damage, the seismic one especially, insomuch that the probabilistic approach is necessary. In addition to the stochastic uncertainty, or randomness, the cognitive source of uncertainty has been stressed recently. This is the so-called fuzziness, or vagueness, meaning imprecision which basically arises with the use of natural language, subjective judgement, perception rather than measurement (Zimmermann, 1991). For instance, according to Eurocode 8 (CEN, 2004b) "ultimate limit states are those associated with collapse or with other forms of structural failure which might endanger the safety of people". Apart from randomness of the seismic action, mechanical properties, and so on, any engineering verification against such limit state will be affected by the fuzziness inherent in its definition, which in fact is linguistic, qualitative, and open to individual judgement.

As the damage states are imprecise, assertion about having attained them may be questionable. The fuzzy set theory suggests abandoning the dichotomy between opposite states (Klir and Yuan, 1995), e.g. either collapsed or not. Indeed, it is rational to express a degree of membership in each damage state, considered as a fuzzy set. This degree of membership may range from 0 to 1 with continuity, unlike dichotomous 0 for the safe state and 1 for the failed one. Any intermediate membership in the damage state is not excluded. This also implies the gradual transition from each state to the opposite one, which is fully rational when, for instance, the structural behaviour is ductile. Moreover, any condition may have a non-zero degree of membership in both the opposite states, which may be the case when, for example, both repairing and rebuilding seems to be feasible.

The author recently proposed a simple fuzzy-random model to incorporate fuzziness into the seismic fragility computation (Colangelo, 2012). This is a single-parameter analytical model focused on the membership function, consistent with the time-invariant first-order second-moment reliability method. The demand and capacity are considered as lognormal random variables independent of each other. The study is extended herein in that fragility is treated as a random variable, its mean value being the membership expectation. The second moments, distribution, and percentiles are also formulated, and compared with those in the case of randomness alone. Application to the fragility curve of seismic non-structural damage to a masonry-infilled reinforced-concrete frame is implemented.

2. FUZZY FRAGILITY VERSUS CLASSICAL FRAGILITY

The fuzzy-random fragility is treated within the probabilistic reliability theory (Klir and Yuan, 1995). Attaining or exceeding a damage state is considered as a fuzzily described event whose occurrence is uncertain. The probability of such an occurrence is defined according to Zadeh (1968), which is convenient for engineering applications because: (i) extending the fragility computation to fuzziness is clear and straightforward; and (ii) close-form formulas of the probabilistic quantities can be derived to draw general conclusions, as opposed to the empirical results from particular case studies.

Recalling that seismic fragility is the probability of at least attaining a damage state conditional on the intensity of the ground motion, the probability measure writes (Zadeh, 1968)

$$\Pr = \iint_D m(r,s)\, f_{RS}(r,s)\, \mathrm{d}r\, \mathrm{d}s \tag{2.1}$$

where f_{RS} is the joint probability density function (PDF) of capacity R against that damage state and demand S at the seismic intensity that the probability is conditional on; D is the domain over which the demand and capacity are defined; m is the membership function giving the degree of membership in the (fuzzily described) damage state, considered as a fuzzy set. Therefore, the probability measure according to Zadeh is the expectation of the membership function. Moreover, since any function of random variables itself is a random variable, one may introduce the random variable $M = m(R,S)$, termed membership indicator, and see the fuzzy-random fragility as its mean value μ_M.

Eqn. 2.1 can be readily seen as extension of the classical time-invariant reliability formulation. In fact, referring to the event of at least attaining the damage state, there exists a lower threshold below which it holds $m \equiv 0$, meaning no membership in the damage state, and an upper threshold above which it holds $m \equiv 1$, meaning full membership. For instance, these thresholds may be expressed in terms of the inter-storey drift ratio, or the strength decrease. The membership function increases monotonically and gradually in between because, as discussed above, the membership function measures every possible, partial degree of membership in the damage state. As the membership function becomes steeper and steeper, any gradual transition into the damage state vanishes. The membership function degenerates into the dichotomous characteristic function used in the classical reliability approach

$$c(r,s) = \begin{cases} 0 & s < r \quad \text{(safe state)} \\ 1 & s > r \quad \text{(failed state)} \end{cases} \tag{2.2}$$

This is illustrated in Fig. 2.1a. In such a case, Eqn. 2.1 becomes

$$\Pr = \iint_D c(r,s)\, f_{RS}(r,s)\, \mathrm{d}r\, \mathrm{d}s = 0 + \iint_{s>r} 1 \cdot f_{RS}(r,s)\, \mathrm{d}r\, \mathrm{d}s = \iint_{s>r} f_{RS}(r,s)\, \mathrm{d}r\, \mathrm{d}s \tag{2.3}$$

that is classical fragility. Therefore, also the classical fragility may be intended as the expectation of a function: the characteristic function rather than the membership one. Similar to the variable M, one may introduce the random variable $C = c(R,S)$, termed state indicator, and see the classical fragility as its mean value μ_C, as opposed to μ_M relevant to fuzzy fragility.

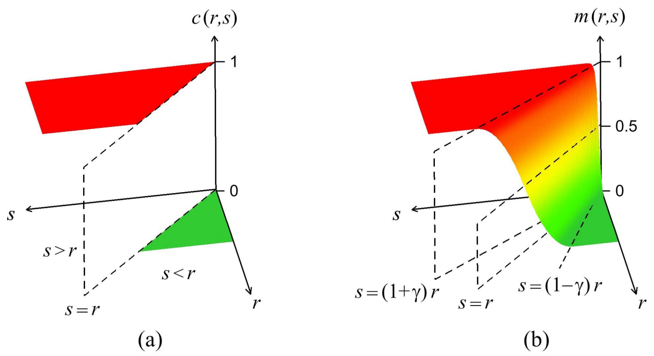

Figure 2.1. Characteristic function (a) versus proposed membership function (b)

3. PROPOSED FUZZY-RANDOM MODEL

The proposal focuses on the membership function in Eqn. 2.1, assumed as follows (Colangelo, 2012)

$$m(r,s)=\begin{cases}0 & s\le(1-\gamma)r\\ \dfrac{1}{2}\left[\dfrac{s-(1-\gamma)r}{\gamma r}\right]^2 & (1-\gamma)r\le s\le r\\ 1-\dfrac{1}{2}\left[\dfrac{(1+\gamma)r-s}{\gamma r}\right]^2 & r\le s\le(1+\gamma)r\\ 1 & (1+\gamma)r\le s\end{cases} \tag{3.1}$$

The idea is to introduce quadratic increase between the straight lines $s = (1 \pm \gamma)\, r$ (Fig. 2.1b), that is, around the discontinuity of the characteristic function (Fig. 2.1a). γ is the fuzziness parameter; the greater γ, the flatter the transition into the damage state, the greater the fuzziness. In the author's knowledge, there is only one similar proposal of fuzzy-random fragility model (Gu and Lu, 2005). Relevant comparison can be found elsewhere (Colangelo, 2012).

3.1. First and Second Moment

Capacity and demand are assumed to be lognormal and independent of each other. With the proposed membership function, Eqn. 3.1, the integration in Eqn. 2.1 can be evaluated explicitly. One obtains

$$\mu_M = 1+\frac{1}{\gamma^2}\sum_{i=1}^{9}\alpha_i\Phi(\beta_i) \tag{3.2}$$

where Φ() is the standard normal cumulative distribution function (CDF) and it has been introduced

$$\begin{array}{lll}\alpha_1=1 & \alpha_5=-\alpha_4(1-\gamma)/2 & \beta_1=\mu_{\ln Z}/\sigma_{\ln Z}\\ \alpha_2=-(1-\gamma)^2/2 & \alpha_6=-\alpha_4(1+\gamma)/2 & \beta_2=\beta_1+\ln(1-\gamma)/\sigma_{\ln Z}\\ \alpha_3=-(1+\gamma)^2/2 & \alpha_7=\exp\left[2\left(\sigma^2_{\ln Z}-\mu_{\ln Z}\right)\right] & \beta_3=\beta_1+\ln(1+\gamma)/\sigma_{\ln Z}\\ \alpha_4=-2\exp\left[\sigma^2_{\ln Z}/2-\mu_{\ln Z}\right] & \alpha_8=\alpha_9=-\alpha_7/2 & \beta_i=\beta_{i-3}-\sigma_{\ln Z}\quad i=4,\ldots,9\end{array}$$

$\mu_{\ln Z}$ and $\sigma_{\ln Z}$ respectively are the logarithmic mean and standard deviation of the (lognormal) safety factor $Z = R / S$. Notice that β_1 is the classical reliability index, thus $\mu_C = \Phi(-\beta_1)$. In order to appraise the effect of fuzziness, the effective reliability index $\beta_e = -\Phi^{-1}(\mu_M)$ is introduced. Eqn. 3.2 shows that this index depends on: (i) the ratio of the mean values of capacity and demand, μ_R / μ_S; (ii) their coefficients of variation (CoVs), δ_R and δ_S; and (iii) the fuzziness parameter γ (Fig. 3.1).

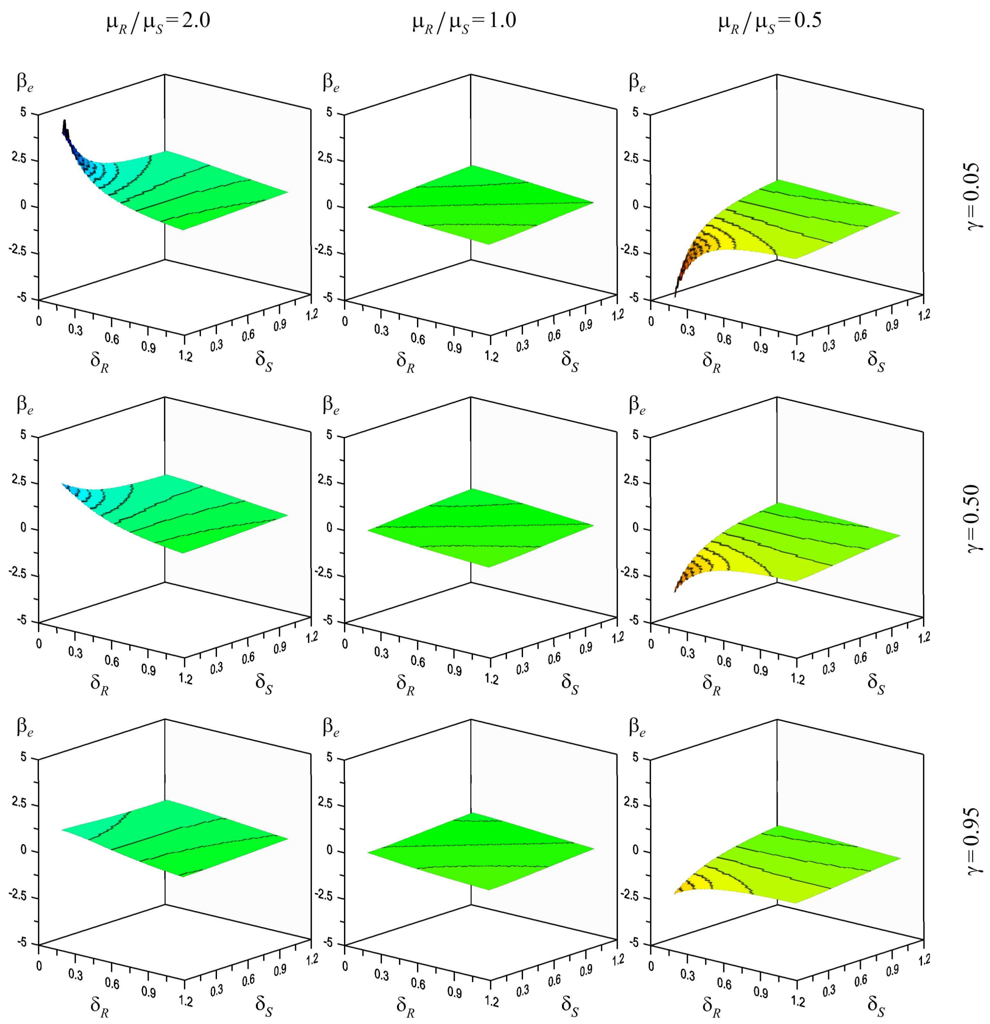

Figure 3.1. Effective reliability index

When fuzziness is small (γ = 0.05, top graphs), the effective reliability index is close to the classical reliability index, as can be shown. Where the mean value of capacity is greater than that of demand (top left graph), the reliability indices decrease with increase of any CoV, and fragility increases, as well known. However, where the mean value of capacity is less than that of demand (top right graph), the greater any CoV, the greater the reliability indices, the lesser the fragility. Finally, where the mean values of capacity and demand are similar (top middle graph), the reliability indices are slightly sensitive to any CoV. Whichever their values may be, a fragility value around 0.5 results. It follows that the degree of randomness has an effect on the trend of the fragility curve. Since μ_R and δ_R are constant, the fragility curve rises with increase of seismic intensity, that is, with increase of μ_S and decrease of μ_R / μ_S. A greater value of any CoV causes the fragility increase where seismic intensity is low, but the fragility decrease where intensity is high. A flatter curve results.

The effect of fuzziness is similar to the effect of randomness. The middle and bottom graphs refer to intermediate fuzziness (γ = 0.50) and high fuzziness (γ = 0.95), respectively. It is apparent that where seismic intensity is low (left graphs), the greater the fuzziness, the smaller the effective reliability index, the greater the fragility. Conversely, where seismic intensity is high (right graphs), the greater the fuzziness, the greater the index, the smaller the fragility. Considering the trend of the fragility curve, the greater the fuzziness, the flatter the curve. However, one also notices that the effective reliability index becomes less and less sensitive to any CoV with increase of γ. Therefore, the greater the fuzziness, the smaller the influence of randomness. Conversely, as any CoV increases, the effective reliability index approaches 0, similar to the classical index, whichever the γ value may be. Therefore, the greater the randomness, the smaller the influence of fuzziness.

The second non-central moment of the membership indicator can be evaluated by symbolic integration similar to that for the mean value, with the proposed membership function squared. One obtains

$$E\left(M^2\right)=1+\frac{1}{\gamma^4}\sum_{i=1}^{14}\zeta_i\Phi(\beta_i) \tag{3.3}$$

where it has been introduced

$$\begin{array}{lll}
\zeta_1=\gamma(\gamma-2) & \zeta_6=(1+\gamma)(\gamma^2-2\gamma-1)e_1 & \zeta_{11}=(1-\gamma)e_3 \\
\zeta_2=-(1-\gamma)^4/4 & \zeta_7=\gamma(\gamma-6)e_2 & \zeta_{12}=-(1+\gamma)e_3 \\
\zeta_3=(1+\gamma)^2(1-\gamma)(1+3\gamma)/4 & \zeta_8=-3(1-\gamma)^2 e_2/2 & \zeta_{13}=-e_4/4 \\
\zeta_4=2\gamma(3-\gamma)e_1 & \zeta_9=(3+6\gamma+\gamma^2)e_2/2 & \zeta_{14}=e_4/4 \\
\zeta_5=(1-\gamma)^3 e_1 & \zeta_{10}=2\gamma e_3 & e_k=\exp\left[k\left(k\,\sigma_{\ln Z}^2/2-\mu_{\ln Z}\right)\right]
\end{array}$$

The arguments β_i in Eqn. 3.3 additional to those in Eqn. 3.2 are

$$\beta_i=\begin{cases}\beta_{i-3}-\sigma_{\ln Z} & i=10,11,12\\ \beta_{i-2}-\sigma_{\ln Z} & i=13,14\end{cases}$$

Eqns. 3.2 and 3.3 yield the standard deviation of the membership indicator, $\sigma_M=\sqrt{E\left(M^2\right)-\mu_M^2}$. This is related to the standard deviation σ_C of the state indicator. It holds

$$\begin{aligned}
E\left(C^2\right)&=E\left[c^2(r,s)\right]=0+\iint_{s>r}1^2\cdot f_{RS}(r,s)\,\mathrm{d}r\,\mathrm{d}s=\mu_C \\
\sigma_C&=\sqrt{E\left(C^2\right)-\mu_C^2}=\sqrt{\mu_C(1-\mu_C)}
\end{aligned} \tag{3.4}$$

The difference $\Delta\sigma=\sigma_M-\sigma_C$ is plotted in Fig. 3.2. Obviously, the greater the fuzziness, the greater $\Delta\sigma$ (see the graphs in each column from top to bottom). When μ_M and μ_C are different (left and right graphs), local positive maxima appear where the CoVs are, say, 0.2. This means that with such a degree of randomness, σ_M is greater than σ_C. However, the domain over which $\Delta\sigma$ is negative results to be much wider. In general, the membership indicator is less scattered than the state indicator. All the more so when μ_M and μ_C are equal (middle graphs). The positive maxima disappear, $\Delta\sigma$ remains negative, and it becomes as small as –0.45 with increase of γ and decrease of the CoVs. In fact, such a case is an almost deterministic one along the straight line $s = r$ (Fig. 2.1). The state indicator, valued either 0 or 1, changes dramatically there because of (small) dispersion. Conversely, the membership indicator remains around 0.5, all the more so with increase of γ. σ_M is expected to be quite less than σ_C.

3.2. Distribution and Percentiles

With the proposed membership function, Eqn. 3.1, basic theory on the functions of random variables (Papoulis, 1991) yields the CDF of the membership indicator explicitly

$$F_M(x)=\iint_{m(r,s)\le x}f_{RS}(r,s)\,\mathrm{d}r\,\mathrm{d}s=\begin{cases}\Phi\left\{\beta+\dfrac{\ln\left[1-\gamma\left(1-\sqrt{2x}\right)\right]}{\sigma_{\ln Z}}\right\} & 0\le x\le\dfrac{1}{2}\\ \Phi\left\{\beta+\dfrac{\ln\left[1+\gamma\left(1-\sqrt{2(1-x)}\right)\right]}{\sigma_{\ln Z}}\right\} & \dfrac{1}{2}\le x<1\\ 1 & x=1\end{cases} \tag{3.5}$$

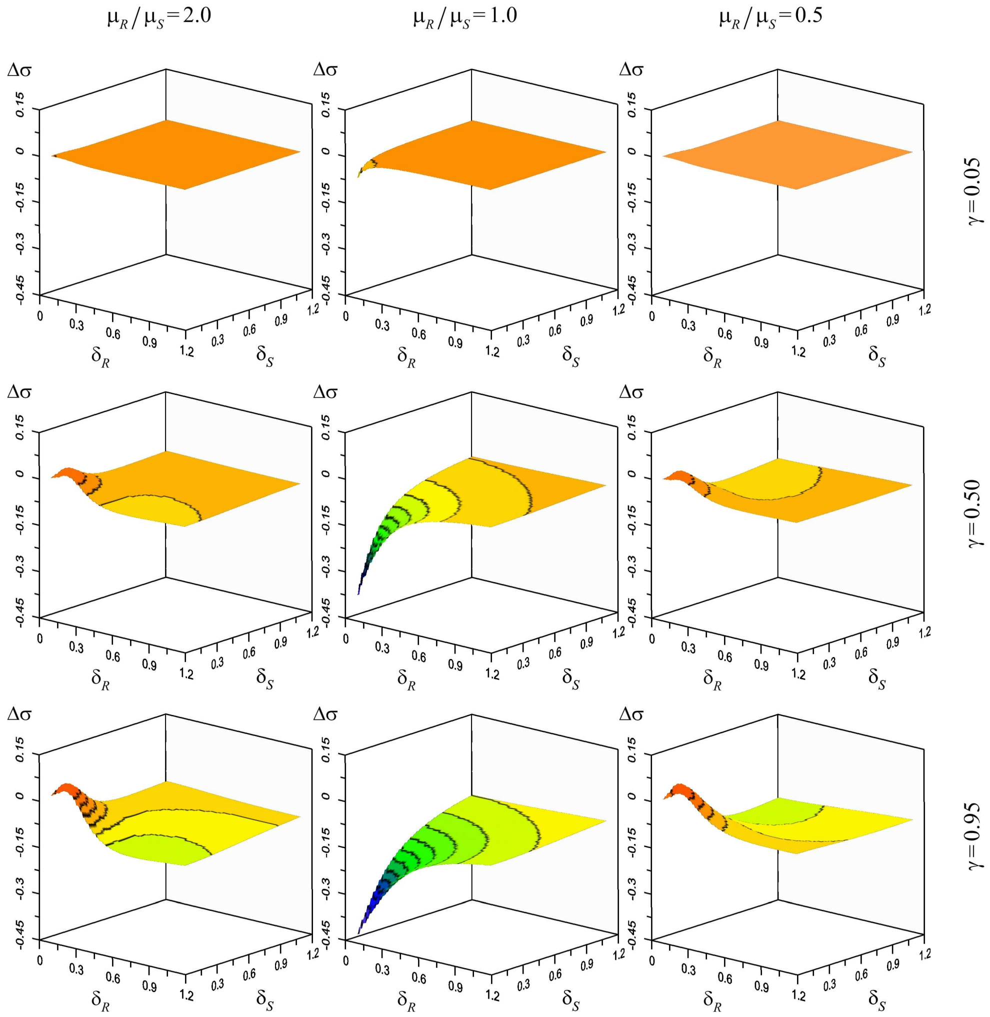

Figure 3.2. Difference of standard deviations of the membership and state indicators

Contrary to the CDF of the state indicator, which is discrete, this is a mixed-type distribution. In fact, there is discrete probability of M being 0, corresponding to the domain $s < (1 - \gamma)\, r$ (Fig. 2.1b). There is also discrete probability of M being 1, corresponding to the domain $s > (1 + \gamma)\, r$ (Fig. 2.1b). The former probability vanishes in the case of full fuzziness ($\gamma = 1$). In any case, there remains the latter probability and, most important, the continuous probability associated with the transition domain between the straight lines $s = (1 \pm \gamma)\, r$, which is missing with the two-valued state indicator.

It is easy to invert Eqn. 3.5 in order to obtain the π percentile of the membership indicator

$$m_\pi = \begin{cases} 0 & 0 \le \pi \le \Phi[\beta + \ln(1-\gamma)/\sigma_{\ln Z}] \\ \dfrac{1}{2}\left\{\dfrac{\exp\{\sigma_{\ln Z}[\Phi^{-1}(\pi) - \beta]\} - 1 + \gamma}{\gamma}\right\}^2 & \Phi[\beta + \ln(1-\gamma)/\sigma_{\ln Z}] \le \pi \le \Phi(\beta) \\ 1 - \dfrac{1}{2}\left\{\dfrac{\exp\{\sigma_{\ln Z}[\Phi^{-1}(\pi) - \beta]\} - 1 - \gamma}{\gamma}\right\}^2 & \Phi(\beta) \le \pi \le \Phi[\beta + \ln(1+\gamma)/\sigma_{\ln Z}] \\ 1 & \Phi[\beta + \ln(1+\gamma)/\sigma_{\ln Z}] \le \pi \le 1 \end{cases}$$

The percentiles yield the fragility band, that is, the interval $[m_\pi, m_{1-\pi}]$ within which the membership indicator shall be with probability $1 - 2\pi$. Any similar interval does not exist for the two-valued state indicator. Consistent with decrease in the standard deviation, the greater the fuzziness, the narrower the interval, as can be readily shown.

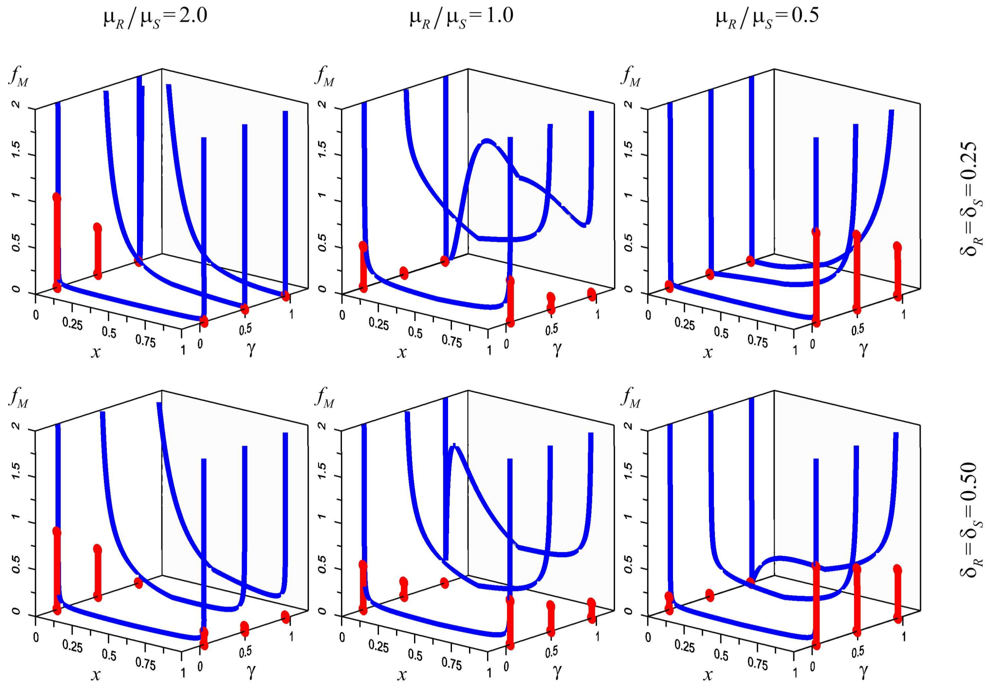

Figure 3.3. Probability mass (red segment) and density function (blue line) of the membership indicator

3.3. Mass and Density Function

Eqn. 3.5 immediately gives the probability mass function (PMF) of the membership indicator at the extremes x = 0 and 1, which equals F_M (0) and 1 – F_M (1^-) respectively. The PDF of the membership indicator over the interval [0,1] results from analytical differentiation of F_M (x). The PMF and PDF pertaining to γ = 0.05, 0.50, and 0.95 are illustrated in Fig. 3.3.

When fuzziness is small (γ = 0.05), most probability is associated with the PMF at the extremes x = 0 and 1, rather than with the PDF. In fact, the transition domain between the straight lines $s = (1 \pm \gamma)\, r$ is small. In the case of no fuzziness, the PDF would vanish. If μ_R is large with respect to μ_S (left graphs), then the PMF at 0 is much greater than the PMF at 1. Indeed, the membership indicator is expected to be small. If μ_R is small with respect to μ_S (right graphs), the contrary holds. If μ_R and μ_S are equal (middle graphs), the PMF at 0 is similar to that at 1. Moreover, the PDF increases, with respect to the cases of different mean values. Once again one may think of being around the straight line $s = r$, thus the probability of the membership indicator being between 0 and 1 is greater. The PDF increase is also expected with greater fuzziness, because the transition domain becomes wider. In fact, considering intermediate fuzziness (γ = 0.50) and high fuzziness (γ = 0.95), the PMF decreases while the PDF rises further on. If capacity is large (left graphs), the most noticeable increase of PDF is beside 0, but if capacity is small (right graphs), this increase is beside 1. Once again this is what is expected.

4. APPLICATION

A reinforced-concrete planar frame infilled by non-structural masonry, typical of residential buildings, is considered (Fig. 4.1). The structure is designed to Eurocode (CEN, 2004a; 2004b) with peak ground acceleration (PGA) equal to 0.25 g, medium seismic ductility class, and rock or rock-like ground. Additional information can be found elsewhere (Colangelo, 2012). The frame members are modelled one-to-one as linearly elastic beam elements provided with rigid-plastic springs at the end, following the Takeda model as simplified by Otani and by Litton (CEB, 1996). Each infill wall is modelled as two diagonal struts behaving according to Panagiotakos and Fardis (1997). The first cracking, strength decrease, pinching, and deterioration of both stiffness and envelope curve, is modelled. Experimental pseudo-dynamic test results (Colangelo, 2005) were used to calibrate this model (Colangelo, 2003).

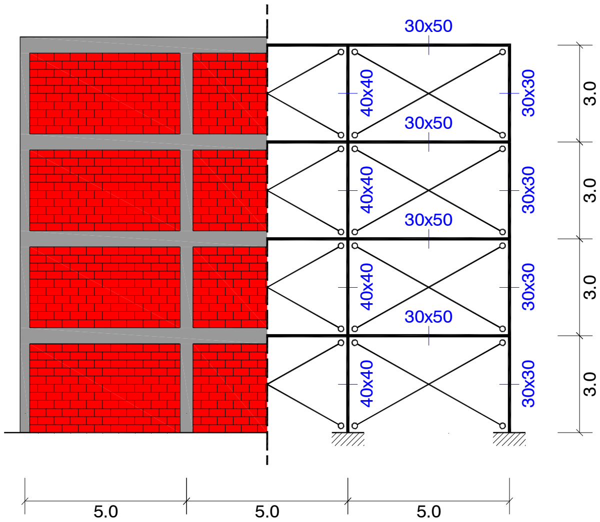

Figure 4.1. Masonry infilled reinforced-concrete frame

4.1. Demand Estimation

The demand is measured by the peak value of the inter-storey drift ratio, whose estimate derives from inelastic time-history analyses of the infilled-frame model subjected to artificial, spectrum-compatible accelerograms. The PGA is assumed as indicative of seismic intensity; 100 realizations at every PGA from 0.05 to 0.95 *g* by step 0.05 *g* are performed. First, the mechanical properties are assumed to be deterministic, thus randomness is due to the ground motion only. Second, the mechanical properties of the infill walls are assumed to be random, in order to introduce a greater degree of randomness. In detail, the strength of all infill walls on each storey is considered as a single lognormal random variable, its mean value and CoV being the same as those at the other stories. Correlation among the four stories is considered (Colangelo, 2012).

4.2. Definition of Fuzzy Damage States

The non-structural damage states are defined on the basis of the same experimental test results used to calibrate the behaviour model (Colangelo, 2005). The infill damage observed during the tests is descriptively classified into four degrees, referred to as slight damage (SD), moderate damage (MD), extensive damage (ED), and complete damage (CD). The SD state is the first cracking of the infill wall. The MD state consists of extended, wide cracks in the infill wall, before its peak strength is reached. The ED state is attained as a few bricks exhibit splitting and falling out of their outer layer; repairing of the infill seems to be still reasonable. Finally, in the CD state, so many bricks have been broken that attempting to repair the infill is unreasonable; replacement is necessary.

4.3. Capacity Estimation

As soon as every damage state defined above was deemed to be reached by each specimen under pseudo-dynamic testing, the peak drift was registered. The corresponding drift values are also taken from any respective time history by the behaviour model. The parameter estimation of both capacity and fuzziness is based on such drift values. Relevant measurements of the membership in each damage state are derived by pairwise comparison (Saaty, 2008) and treated as samples of the membership indicator, each following a given sample of the demand. The optimum values of γ, μ_R, and δ_R, are identified by minimizing an objective function involving the sample mean and standard deviation of the membership values on the one hand, and the same statistics of the membership indicator on the other hand (Eqn. 3.2 with $\delta_S = 0$ and μ_S = drift values at attaining the damage states). Precisely, this function is the squared difference between the sample means plus the squared difference between the sample standard deviations. The estimated parameter values are in Table 4.1, yielding the PDFs and membership functions plotted in Fig. 4.2 together with the drift and membership values, respectively.

Table 4.1. Estimated parameter values of the fuzziness and randomness of capacity

Parameter	SD state	MD state	ED state	CD state
γ	0.43	0.45	0.44	0.24
μ_R (%)	0.04	0.49	1.07	1.78
δ_R (%)	40.3	53.5	46.3	12.8

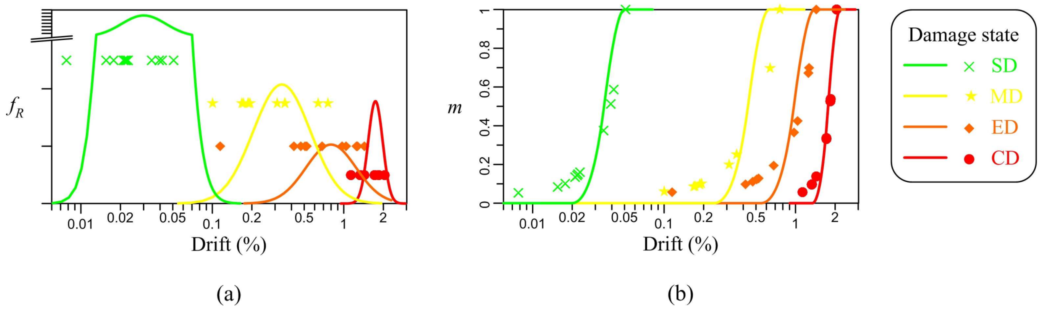

Figure 4.2. PDFs of capacity (a) and membership functions with r = median value (b)

4.4. Fragility Curves

Referring to the first storey, the fragility mean values and bands based on the first two moments are plotted in Fig. 4.3. All in all, it is noted that the membership indicator yields a lower, narrower band, as opposed to the state indicator. The lesser mean value can be shown to be related to the parameter identification. The narrower band reflects the difference between the second moments discussed in Sec. 3.1, that is, the lesser standard deviation because of the fuzziness. In the case of infill walls with random properties (bottom graphs) all bands become wider. This result is consistent with the CoV of demand being greater than that in the case of deterministic properties (Colangelo, 2012). Moreover, at the same time the greater randomness of demand belittles the effect of fuzziness, which is narrowing the bands in terms of standard deviation (Sec. 3.1).

The fragility bands based on 5th and 25th percentiles (exclusive to the membership indicator, Sec. 3.2) are plotted in Fig. 4.4. Noticeable difference from the previous bands may result, in terms of steepness especially. In the case of infill walls with random properties (bottom graphs), once again all bands become wider, because the dispersion of demand is greater as before. Moreover, at the same time this greater randomness of demand belittles the effect of fuzziness, which is narrowing the bands in terms of percentiles as well (Sec. 3.2).

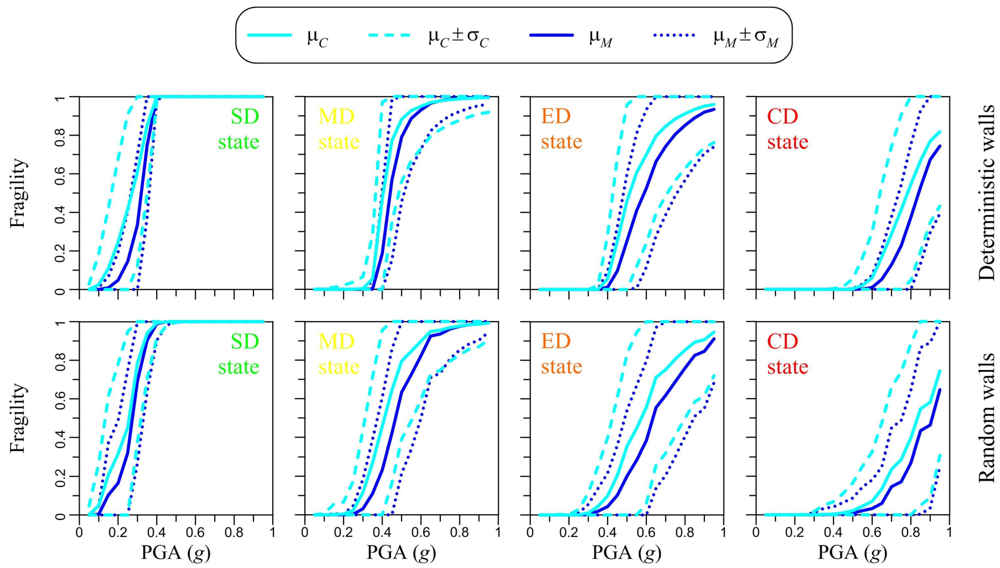

Figure 4.3. Classical fragility versus fuzzy fragility based on the first two moments

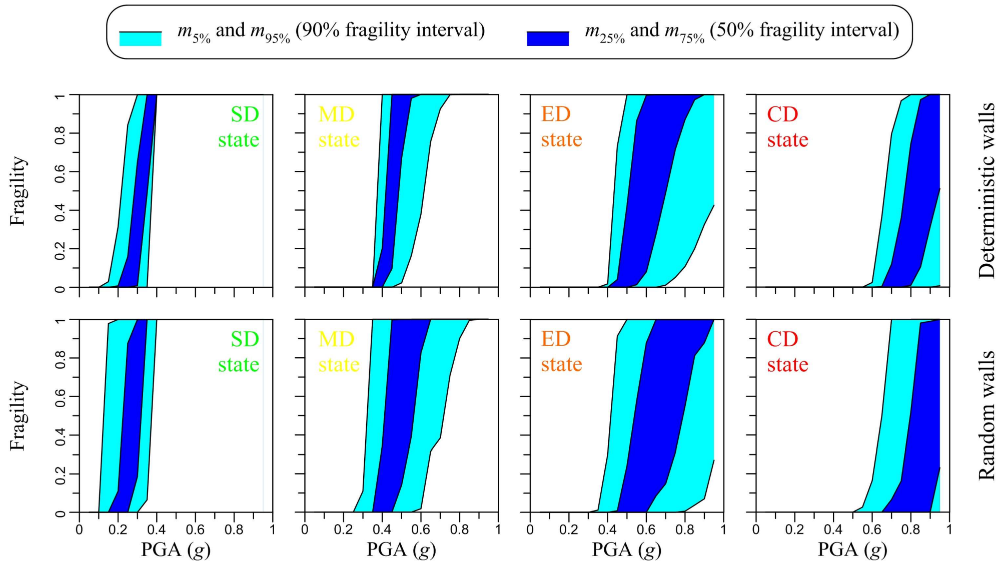

Figure 4.4. Fuzzy fragility based on percentiles

5. CONCLUSION

The fuzziness is inherent in the seismic limit states in that they are defined qualitatively. A simple analytical probabilistic model has been proposed to estimate fragility taking fuzziness into account. This model is based on a single-parameter membership function, so that the first two moments of fragility are expressed explicitly depending on the fuzziness parameter and the distribution parameters of capacity and demand. The model is also characterized in terms of distribution and percentiles. The fuzziness causes the mean value of fragility to increase at lower seismic intensity, but to decrease at higher intensity, which is the same effect of the randomness. In most cases, the fuzziness causes the fragility dispersion to decrease. The fragility percentiles may be used to estimate the fragility intervals. The proposed model has been implemented referring to seismic non-structural damage to typical building frames. The results are reasonable and encouraging. The model appears to be suitable for engineering application, provided that confidence in the parameter values is gained.

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