Estimating the Annual Probability of failure Using Improved Progressive Incremental Dynamic Analysis of Structural Systems

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SUMMARY:

A methodology based on the progressive incremental dynamic analysis has been introduced in this paper to estimate the structural response and the corresponding annual probability of failure. The proposed methodology employs the genetic algorithm optimisation technique and an equivalent single degree of freedom system corresponding to the first mode period of a considered structure. The proposed methodology can significantly reduce the number of ground motion records needed for estimating the annual probability of failure. The numerical results indicate that the proposed method can effectively reduce the computational effort needed for computation of probability of failure for the first-mode dominated structures which is advantageous as the structure becomes larger. It has been shown that the probability of failure can be estimated within ±15% error with 95% confidence. The proposed method can speed up the decision-making process in the probability based seismic performance assessment of structures.

Keywords: Progressive IDA, fragility curve, genetic algorithm, probability of failure

1. INTRODUCTION

Structural analysis often involves large uncertainties, especially when the input is highly uncertain, as is the case of seismic loading. The probability-based methods attempt to deal with this uncertainty in the seismic design and assessment of structures. The performance evaluation of structures is often described in terms of demand and capacity, where the demand can be any structural response of interest (shear, moment, drift, etc.) and the capacity is the maximum structural response in which the structural behaviour is acceptable. The seismic demand and capacity and their distributions can be calculated by means of Incremental Dynamic Analysis (IDA), which is commonly used for different nonlinear analysis applications [Vamvatsikos and Cornell, 2002, Liao et.al, 2007, Tagawa et al., 2008]. IDA employs several response history analyses for a given ground motion record by increasing the intensity measure until the collapse occurs. This process is repeated for a sufficient number of ground motion records to determine the median collapse capacity and the record-to-record variability. A comprehensive review of some analytical methods can be found in a state of the art article by Villaverde [2007].

One of the most well-known methodologies for the probability assessment of structures, was developed for the SAC2000 project [Cornell et al., 2002]. The SAC2000 methodology provides a closed form solution for determining required values, but there are some shortcomings in the closed form solution rooted in the simplifying assumptions, e.g. a fixed value for dispersion, structural type limits and etc., but these can be avoided by means of the direct IDA analysis.

One of the most challenging issues in IDA is the significant computational effort which is needed for the nonlinear response-history analyses. This issue even gets more complicated as the structure grows taller in terms of extensive computational effort. To reduce the this effort required in IDA calculation, different approximate methods have been introduced which can be summarized in seven categories. (1) Vamvatsikos and Cornell [2005, 2006] presented SPO2IDA to reduce the required time to obtain IDA curves; (2) Cornell and Baker [2005] introduced the epsilon-based filtration approach to select the ground motion records, which employs the epsilon advantages for reducing the number of ground motion records; (3) Dolšek and Fajfar [2005] showed that the N2 method can also be used for the determination of approximate summarised IDA curves; (4) Han and Chopra [2006] proposed the approximate IDA using Modal Pushover Analysis of the multi degree of freedom (MDOF) system and nonlinear dynamic analysis of corresponding single degree of freedom (SDOF) systems, which can consider higher mode effects but may not be reliable in estimating IDA curves in the case of irregular structures [Vejdani-Noghreiyan and Shooshtari,2008]; (5) Ghafory-Ashtiany et al. [2010] tried to classify ground motion records for different structural groups by incorporating the multivariate statistical analysis with the principal component analysis. They classified a wide range of SDOF systems into six different groups and have proposed eight ground motion records for each group to reliably estimate the mean structural response; (6) Azarbakht and Dolšek [2007, 2011] introduced the Progressive Incremental Dynamic Analysis (PIDA), which involves a precedence list of Strong Ground Motion Records (SGMRs) and is capable of reducing the computational efforts needed to obtain the summarised IDA curves (16th, 50th and 84th fractiles) with reasonable approximation for MDOF systems. The proposed methodology takes advantage of the analysis of a first mode equivalent SDOF system and optimisation concept using the Genetic Algorithm (GA). The proposed method is obviously limited to the first-mode dominated structures in its current form.

In this research, an attempt has been made to modify the progressive IDA optimisation method to estimate P_{PL}. The proposed method was applied to MDOF structures for a given hazard condition to estimate the annual probability of failure. The results are described in Section (4).

2. METHODOLOGY

The Maximum Inter-story Drift Ratio (MIDR) was selected as the Engineering Demand Parameter (EDP). The capacity (or the ultimate limit state), which is the acceptable structural behaviour limit (here selected as the global dynamic instability), should also be represented on the same basis as the demand parameter, MIDR, to make the comparison possible. This methodology uses the progressive IDA concept, for which a detailed step by step procedure can be found in Azarbakht and Dolšek [2011]. Probability of failure in IM-based approach, P_{PL} can be computed as:

$$P_{PL} = \int P[S_{a,C} \le x].\left|dH_{S_a}(x)\right| = \int F(s_a).\left|dH_{S_a}(x)\right| \qquad (1)$$

where $F(s_a)$ is the fragility function at spectral acceleration (s_a) and $dH_{Sa}(x)$ is the differential of the seismic hazard curve. Different studies on steel and concrete frames have shown that the lognormal CDF provides a good fragility model in the inelastic range of response [Hwang and Jaw, 1990, Singhal and Kiremedjian,1996, Song and Ellingwood,1999]. The multiplication of failure fragility curve and hazard derivative is referred to, herein, as the "Hazard derivative-Fragility product".

The original Error function introduced by Azarbakht and Dolšek [2007, 2011] is shown in Equation (2). In this equation, s is the number of selected ground motion subsets to estimate the fractiles, which is a factor of three as three fractiles are to be estimated, EDP is the engineering demand parameter of the simple model, IM is the intensity measure for the IDA, and $\Delta IM\ (s, f\,)$ is the difference in the IM corresponding to the "original" and "estimated" f^{th} summarised IDA curves. The "or" as in $IM_{or}\ (f\,)$ refers to original values, and "f" refers to the f^{th} summarized IDA curve of interest (16% , 50% etc.). $EDP_{\max}\ (s, f\,)$ is the maximum of the engineering demand parameters corresponding to the global dynamic instability of the "approximate" or "original" f^{th} summarised IDA curve, and $EDP_{\max,or}\,(f\,)$ is the engineering demand parameter corresponding to the capacity point of the "original" f^{th}

summarised IDA curve. The parameters ΔIM (s, f) and $EDP_{\max}$ (s, f) depend on the s selected subsets of the ground motion records, which were used in determining the "approximate" f^{th} summarised IDA curve.

$$Error_{or}(s,f) = 100\left[\frac{\int_0^{EDP_{\max}(s,f)} \left|\Delta IM(s,f)\right| d(EDP)}{\int_0^{EDP_{\max or}(f)} IM_{or}(f)\, d(EDP)}\right] \tag{2}$$

But given Equation (1), a better estimation of fragility for an assumed hazard could lead to a better approximation of P_{PL}. This additional constraint can be effectively included in the original fitness function by including some additional terms as shown in Equation (3) which hereafter is referred to as the improved error function:

$$Error_{I}(s,f) = 100\left[\frac{\int_0^{EDP_{\max}(s,f)} \left|\Delta IM(s,f)\right| d(EDP)}{\int_0^{EDP_{\max or}(f)} IM_{or}(f)\, d(EDP)} + \frac{\left|\mu_{Ln(IM)_{or}} - \mu_{LnIM}(s)\right|}{\mu_{Ln(IM)_{or}}} + \frac{\left|\beta_{IM_{or}} - \beta_{IM}(s)\right|}{\beta_{IM_{or}}}\right] \tag{3}$$

Here, $\mu_{Ln(IM)_{or}}$ is the "original" logarithmic mean value of the collapse capacity, $\mu_{Ln(IM)}(s)$ is the "estimated" logarithmic mean value of the collapse capacity based on selected SGMRs and β is logarithmic standard deviation considering a lognormal distribution of the collapse capacity. The final improved fitness function (Z) that was used in the GA can be defined as:

$$Z = \frac{1}{m}\sum_{s=1}^{m}\sum_{f=1}^{3} Error_{I}(s,f) \tag{4}$$

Figure 1 summarises the steps involved in the determination of the adequacy of the proposed method, which is investigated numerically in section (3). Steps can be described as follows:

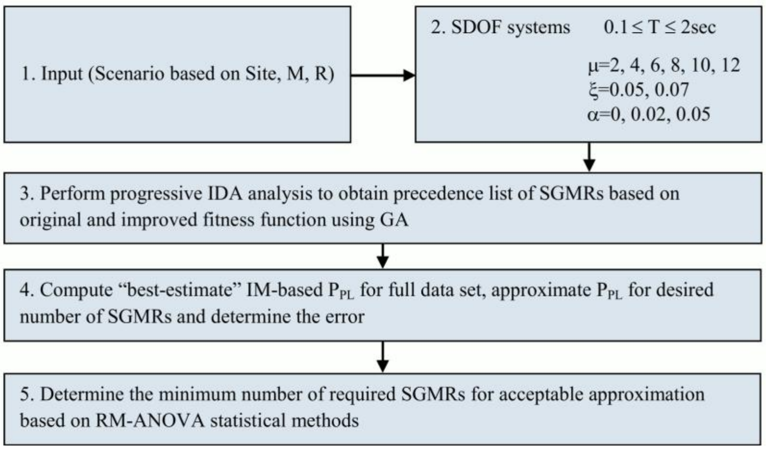

Figure 1. Process of the improved progressive IDA for the purpose of P_{PL} computation.

The Analysis of Variances (ANOVA), which can compare the central tendencies of the different groups of observations, was used as the statistical approach to determine the minimum number of required SGMRs. ANOVA has some restrictive conditions, and violating them could result in unreliable outcomes. Due to lack of normality and independence, Repeated Measures ANOVA (RM-ANOVA) was chosen to be used [Davis, 2002].

Finally, by the statistical tests for the amount of the error in P_{PL} computation, it is proposed that six ground motion records out of the pre-determined precedence list can be employed for an appropriate estimation of the P_{PL}.

3. ANNUAL PROBABILITY OF FAILURE (P_{PL}) ESTIMATION BASED ON IMPROVED PROGRESSIVE IDA FOR SDOF SYSTEM SET

To study the efficiency of the proposed methodology and provide a basis for MDOF application, IDA analysis using the Hunt and Fill method [Vamvatsikos and Cornell, 2002] for the considered SGMRs database was performed on a set of SDOF systems.

3.1. Seismic Hazard function and Strong Ground Motion Records

In the first step, a simple source, which is capable of producing only a specific magnitude at a specific distance, was considered. For the purpose of sensitivity analysis, different M_w and $R_{rupture}$ values were assumed, which are summarised along with a sample hazard curve as shown in Figure 2. The Campbell and Bozorgnia 2008 (CB 08) [2008] attenuation relationship has been used to determine s_a. Considering s_a, $\lambda_{S_a}(s_a)$ can be computed as: $\lambda(S_a > s_a) = \nu P[S_a > s_a \mid M, R_{rupture}]$ for a desired Return Period (TR) and ν=1/TR.

Also, a general far-field ground motion set [FEMA P695, 2009], consisting of 22 ground motion pairs recorded at sites located more than 10 Km from the fault rupture, was selected from [PEER, 2005] to calculate IDA. The SGMRs details are listed in Table 4 in Appendix A.

3.2. SDOF systems properties

As the second step, a set of SDOF systems consisting of 27 periods ranging from T=0.1 sec to 2 sec (from T= 0.1 to 1 with 0.05 increments and 1.15, 1.25, 1.35, 1.5, 1.65, 1.75, 1.85 and 2sec), six ductility ratios (μ= 2, 4, 6, 8, 10, 12), two damping ratios (ξ=5%, 7%) and three strain-hardening stiffness ratios (α= 0, 0.02, 0.05). A total of 972 combinations of SDOF systems were considered. The P-Δ effects and cyclic deterioration were not included in the analysis for the purpose of simplicity.

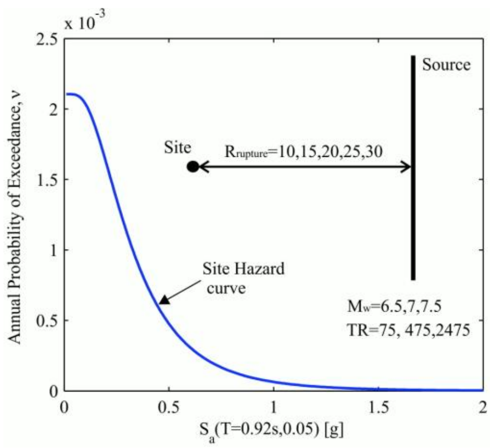

Figure 2 Different parameters used in sensitivity analysis and sample hazard curve for T=0.92sec, M=7 and TR=475

3.3. Progressive IDA for SDOF systems

In the third step, IDA curves of SDOF systems were computed. To analyse the SDOF systems, assuming a fixed mass value, the system stiffness can easily be calculated with regard to the selected period of the system. Using ground motion properties and R_y-μ-T equations, consistent with Newmark-Hall inelastic design spectra [Chopra, 2001], the yield strength (F_y), yield deformation (D_y) and other parameters required to perform the analyses of the SDOF systems were computed. Figure 3(a) shows IDA curves and SDOF backbone curve for one of the SDOF systems. The probability density function of the collapse capacity and the corresponding fitted lognormal function are shown in Figure 3 (b).

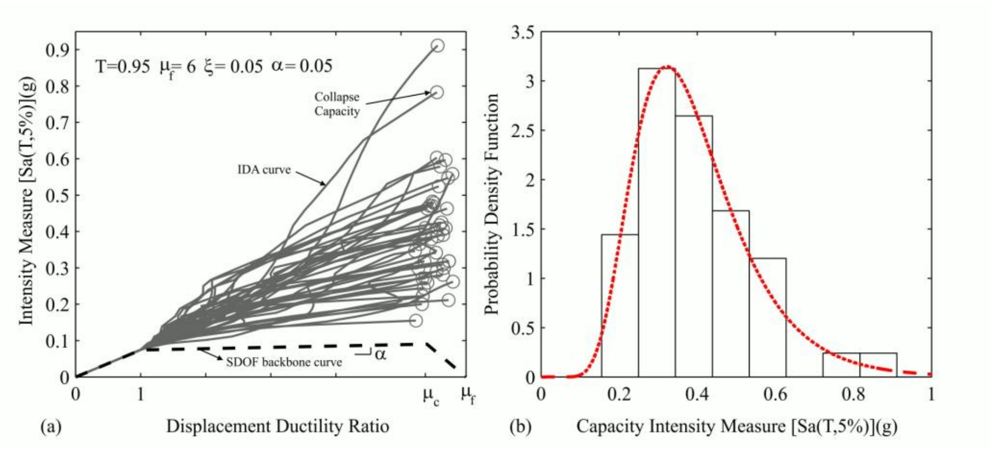

Figure 3. (a) IDA curves for a sample SDOF system with T=0.95sec, μ=6, ξ=0.05, α=0.05 and (b) The distribution of collapse capacity.

By employing the progressive IDA, the precedence list for any given system can be calculated. Having the precedence list of SGMRs for each SDOF system, P_{PL} was obtained for "Full data" using all the records for one specific structure (P_{PLf}) and for the "selected" number of SGMRs based on the precedence list (P_{PLr}). The error in the computed P_{PL} values was defined with respect to P_{PLf} as $(P_{PLf} - P_{PLr}) / P_{PLf}$; therefore, a negative error value implies overestimation of P_{PLf}.

3.4. P_{PL} computation and the respective errors

As the fourth step, P_{PLf}, P_{PLr} and their respective errors were computed. Figure 4 (a) and (b) show the P_{PLf} and P_{PLr} for ξ=0.05 and ξ=0.07 with all other parameters fixed (six SGMRs, $R_{rupture}$=10 Km, M_w=6.5, α=0.02 and TR=475 years). P_{PLf} is shown using surface and P_{PLr} with the mesh. As stated earlier, using a limited number of SGMRs may lead to overestimation or underestimation of P_{PL}. The light regions in Figure 7 (a) and (b) imply the overestimation of P_{PL} while the dark regions imply that a reduced number of SGMRs has led to an underestimation of P_{PL}.

Figure 4(a) and (b) show the error bar diagram for comparison of error values at 95% confidence level for different numbers of selected SGMRs and different fitness functions. The error bars represent the mean ± 1.96× (standard deviation) of the 14,580 computed error values for each number of SGMRs. It can be seen that at least six SGMRs were needed to be used in the improved method to keep the errors relatively low (less than 15%), but, the error range is relatively higher in the case of the original fitness function as shown in Figure 4 (a).

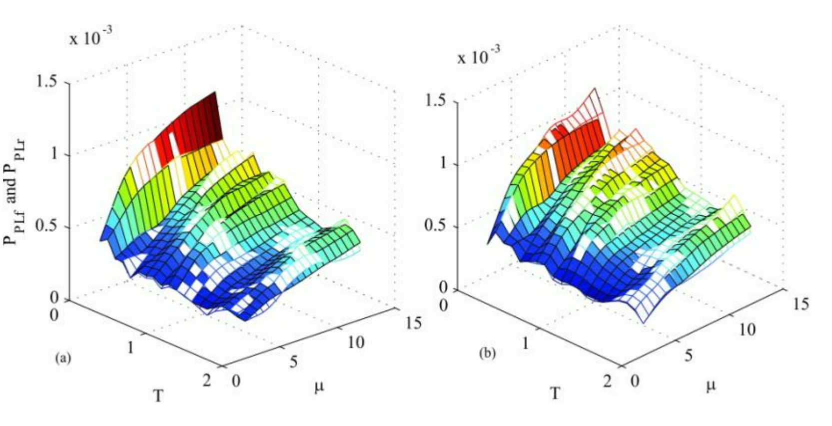

Figure 4. Computed values of P_{PLf} and P_{PLr} (a) ξ=0.05 (b) ξ=0.07 mesh indicates P_{PLr} and suface shows the P_{PLf}; regions with white colour means P_{PLr} is overestimating the P_{PLf}.

3.5. Statistical analysis for determining the minimum number of SGMRs

It is worth emphasising that, as a whole, the mean of error, using any number of SGMRs greater than or equal to six, reaches an appropriate value of less than 6%. However, to determine the existence of meaningful differences in the mean values of P_{PL} errors using different numbers of SGMRs, these groups were compared using Repeated Measures ANOVA (RM-ANOVA) [Davis, 2002]. Figure 5 shows the comparison of mean error values employing different numbers of SGMRs. According to this comparison, using nine SGMRs in the improved method would increase the mean value of error to the extent that, at 5% significance, it is considered higher than using six SGMRs.

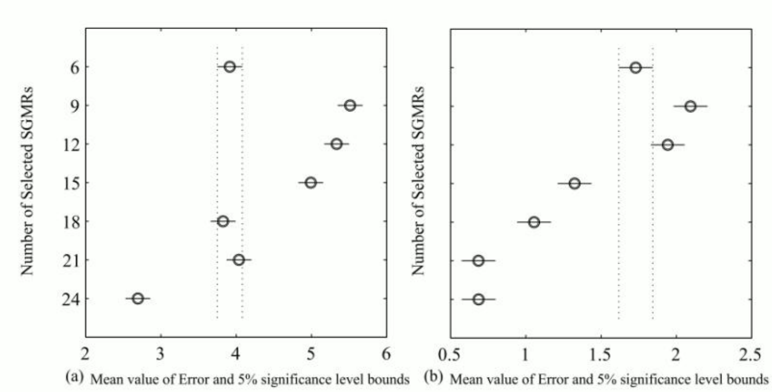

Figure 5. Comparison of mean error value at 5% significance level considering different number of SGMRs (a) Original fitness function (b) Improved fitness function

4. APPLICATION OF THE PROPOSED METHOD ON MDOF STRUCTURAL SYSTEMS

In this section the improved method and the original method (by using both GA and simple optimisation techniques [Azarbakht and Dolšek 2011]) were employed on three different MDOF systems, namely, a 3-storey, an 8-storey and a 12-storey structure to compare their behaviour. First general definitions and assumptions are presented, then, they are numerically investigated. Figure 6 shows the steps involved to determine the P_{PL} for the MDOF system.

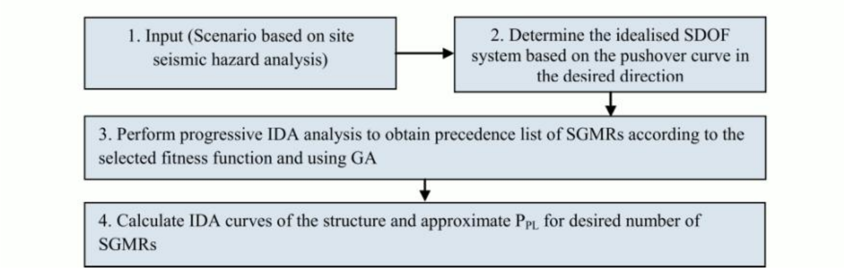

Figure 6 Steps involved in determination of P_{PL} using progressive IDA

4.1. General definitions

In order to determine the P_{PL} for MDOF systems a hazard curve based on the Probabilistic Seismic Hazard Analysis (PSHA) has been considered. In this hazard curve, S_a (1sec) for 50% in 50 yrs, 10% in 50 yrs and 2% in 50 yrs equals to [0.36, 0.59, 0.87]g respectively. The site has been located at 20 km from an active fault on stiff soil ($V_{s\text{-}30}$=350 m/sec, NEHRP site class D). It is usually helpful to estimate the hazard especially in the region of interest by a power-law relationship: $H_{Sa}=k_0\ (s_a)^k$ [Cornell et al., 2002]. Damping ratio of 5% has been assumed for analyses.

Furthermore a Confidence Level (CL) can be computed corresponding to an allowable probability noted as P_0 [Jalayer and Cornell, 2003]. In Equation (5), k_x is the standard Gaussian variate with the probability x of not being exceeded and β_U is the dispersion measure representing the total epistemic uncertainty in the IM-based approach.

$$e^{k_x} \leq \frac{P_0}{P_{PL}} e^{-k\beta_U} \tag{5}$$

By solving Equation (5), k_x and the corresponding CL can be computed from a normal distribution table. In calculation of the fragility curves, to determine the probability and the mean annual frequency of collapse, a dispersion of 0.34 has been considered and added to the randomness dispersion computed from IDA analyses to account for modeling uncertainty as suggested by Haselton [Haselton and Deierlein, 2007]. The error definition for CL is the same as the error defined previously for P_{PL} computation.

4.2. 3-storey rc structure

In this section a 3-storey 3D reinforced concrete structure designed by Fardis [2002] for which a pseudo-dynamic experiment was performed at full scale at the ELSA Laboratory, within the European research project SPEAR ("Seismic performance assessment and rehabilitation of existing buildings") [Negro et al., 2002] was investigated. The structure has T_1=0.85 sec, and the idealised period for the corresponding first mode equivalent SDOF system is 0.92 sec. A more detailed explanation of the model and comparison of experimental and numerical results can be found in [Fajfer et al., 2006]. The Nonlinear Response History Analyses (NLRHA) were performed on the weak (X) direction of the structure. Figure 7 shows the IDA curves, the pushover curve in the X direction and the equivalent SDOF backbone behaviour. The force-displacement envelope of the SDOF model was obtained by dividing the forces and displacements of the idealized pushover curve by a transformation factor Γ [Fajfar, 2000].

The original progressive IDA and Improved progressive IDA along with the simple method [Azarbakht and Dolšek, 2011] were applied to the MDOF test structure based on the first mode equivalent SDOF system and using the SGMRs database to obtain the precedence list.

Figure 8 shows that even with a small number of SGMRs, 6 out of 44, the improved progressive IDA can provide a good estimate of the collapse capacity distribution based on the analysis of the first mode equivalent SDOF system for the structure studied. Table 1 shows the comparison of obtained results using six SGMRs and different fitness functions. β_R is the dispersion measure representing randomness uncertainty (it is the logarithmic standard deviation of the collapse capacity).

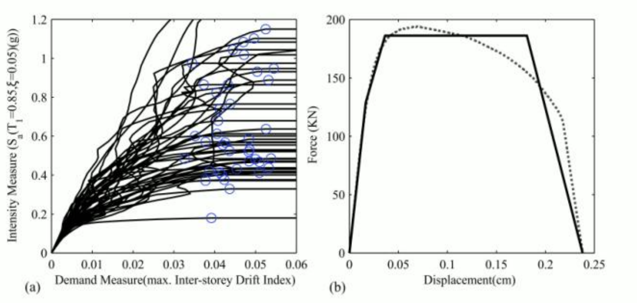

Figure 7. (a) IDA curves; (b) pushover curve in X direction and the equivalent idealized SDOF behaviour.

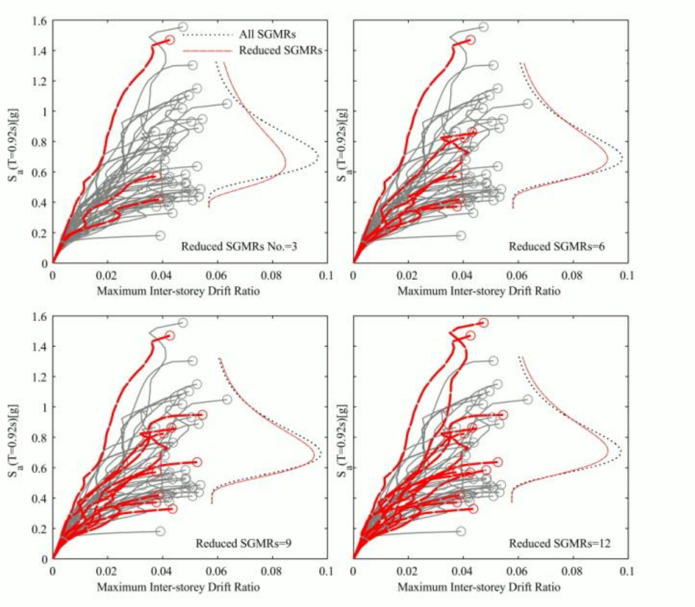

Figure 8. The effect of number of selected SGMRs on the probability density function of collapse capacity using the improved fitness function for the 3-storey RC structure.

Table 1: Comparison of obtained results using different fitness functions and six SGMRs for 3-storey RC structure

Method	**EQ. IDs**	β_R	$e^{\mu_{LnS_a(T_1,col)}}$	P_{PL}	**Error in** P_{PL}	**CL%***	**Error in CL%**
Best-estimate	All Data set records	0.45	0.6416	0.0104	-	1.0	-
Original PIDA using GA	22,33,30,23,17,38	0.56	0.6288	0.0148	-42.42	0.45	55.42
Original PIDA using simple method	22,33,30,27,13,17	0.61	0.7154	0.0125	-20.3	0.949	5.71
Improved PIDA using GA	17, 33, 22, 2, 20, 36	0.51	0.67	0.0109	-4.82	1.044	-3.71

*P_0=0.0004 corresponding to 2% in 50 yrs Hazard level ** without considering ε effects

5. CONCLUSION

An improved version of the Progressive Incremental Dynamic Analysis to estimate the annual probability of failure of structures has been proposed. This method offers much less computational effort, which is very important as the structure grows larger, and makes it possible to explicitly consider the randomness of the input SGMRs. It also provides a good approximation of P_{PL} value. The first-mode equivalent SDOF system for a given structure obtained by the pushover analysis and the GA optimisation technique were utilised to accurately determine the failure fragility curve and the corresponding annual frequency of failure. A sensitivity analysis using results of an SDOF database with different variables revealed that, at least for the selected SGMRs database and within the given assumptions, a good approximation for the probability of failure can be obtained by using only six SGMRs. The 95% error bound was between +15% and -11%. Analysis of MDOF systems showed that this method could very effectively predict the fragility curve and the annual probability of failure of these structures using a limited number of SGMRs.

APPENDIX A

Table 2: ID numbers of different SGMRs used.

ID	PEER NGA Rec. #	Event, Year	Mw	R_{ave}	ID	PEER-NGA Rec. #	Event, Year	Mw	R_{ave}
1	953	Northridge, 1994	6.7	13.3	**23**	848			19.85
2	1602	Duzce, Turkey, 1999	7.1	12.2	**24**	960	Northridge, 1994	6.7	11.9
3	1602			12.2	**25**	752	Loma Prieta, 1989	6.9	22.1
4	1787	Hector Mine, 1999	7.1	11.2	**26**	752			22.1
5	1787			11.2	**27**	767			12.5
6	169	Imperial Valley, 1979	6.5	22.25	**28**	767			12.5
7	169			22.25	**29**	1633	Manjil, Iran, 1990	7.4	12.8
8	174			13	**30**	1633			12.8
9	174			13	**31**	721	Superstition Hills, 1987	6.5	18.35
10	953	Northridge,1994	6.7	13.3	**32**	721			18.35
11	1111	Kobe, Japan,1995	6.9	16.15	**33**	725			11.45
12	1111			16.15	**34**	725			11.45
13	1116			23.8	**35**	829	Cape Mendocino, 1992	7	11.1
14	1116			23.8	**36**	829			11.1
15	960	Northridge, 1994	6.7	11.9	**37**	1244	Chi-Chi, Taiwan, 1999	7.6	12.75
16	1158	Kocaeli, Turkey, 1999	7.5	14.5	**38**	1244			12.75
17	1158			14.5	**39**	1485			26.4
18	1148			12.05	**40**	1485			26.4
19	1148			12.05	**41**	68	San Fernando, 1971	6.6	24.35
20	900	Landers, 1992	7.3	23.7	**42**	68			24.35
21	900			23.7	**43**	125	Friuli, Italy, 1976	6.5	15.4
22	848			19.85	**44**	125			15.4

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