3- Dimensional Modal Summation simulation of 2003 Mw=6.6 Bam Earthquake South Eastern Iran

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SUMMARY:

A new methodology for computing synthetic seismograms, complete of the main direct, refracted, converted phases and surface waves, in three – dimensional anelastic lateral heterogeneous media is applied to the modeling of the local records of the 2003 Bam Mw 6.6 Earthquake, South-Eastern Iran. The method is based on the combination of the Modal Summation (MS) technique with the Asymptotic Ray Theory (ART). The three – dimensional models are determined by a set of vertically heterogeneous sections (1D structures) that are juxtaposed on a regular grid. The distribution of these sections in the grid is done in such a way to satisfy the condition of applicability of the WKBJ approximation, i.e. the lateral variation of all the elastic parameters has to be small with respect to the prevailing wavelength. The procedure, described very briefly, has been applied for the area of destructive 26 December 2003 Bam earthquake Mw= 6.6.

Keywords: 3D Modal Summation, Ray Tracing, WKBJ Approximation, Bam Earthquake.

1. INTRODUCRION

The propagation of seismic waves in complex laterally varying 3D layered structures is a complicated process. Analytical solutions of the elastodynamic equations governing the phenomenon for such types of media are not known. As extensively described in PANZA et al. (2001), there are two main classes of methods that can be used for solving it, i.e. numerical and analytical methods. In the last decade many numerical methods for simulating wave propagation in 3D anelastic models have become available, e.g. the boundary integral equation method, finite elements, finite differences; the choice among these methods depends on the characteristics of the physical model assumed for describing the medium the wave propagates in (BIELAK et al., 2003). As pointed out by PANZA et al. (2001), when the equations of motion in flat, laterally heterogeneous media are solved with the modal approach, a distinction can be made between two classes of analytical methods: methods based on mode coupling and methods based on ray theory. In LA MURA (2009) the choice between the two classes of methods is discussed in depth, on the basis of the extent of the heterogeneity shown by the physical model. In this paper we used a new fully analytical procedure for determining the seismic wavefield in a 3D anelastic model based on coupling of the ray theory and the modal summation method. Starting from the expression of the wave field represented by a sum of normal modes in a half-space with weak lateral heterogeneity, under the assumption of validity of the WKBJ approximation (LA MURA, 2009; AA.VV. 1989; PANZA et al., 2001).

2. THE 2003, MW=6.6, BAM EARTHQUAKE

The 2003 Bam earthquake has been one of the most important earthquakes during recent decades of Iran Plate. The earthquake was particularly destructive with the death toll amounting to 26000 people and injuring an additional 50,000. The effects of the earthquake were exacerbated by the use of mud brick as the standard construction medium. More than 70% of buildings have been destroyed in the Bam area during this earthquake as estimated by Iranian government. Different national and international institute which monitor earthquakes, had different report on location and source mechanism related to the earthquake. Figure 1 shows 3 different locations reported by NEIC, Harvard and IIEES (International Institute of Earthquake Engineering and Seismology).

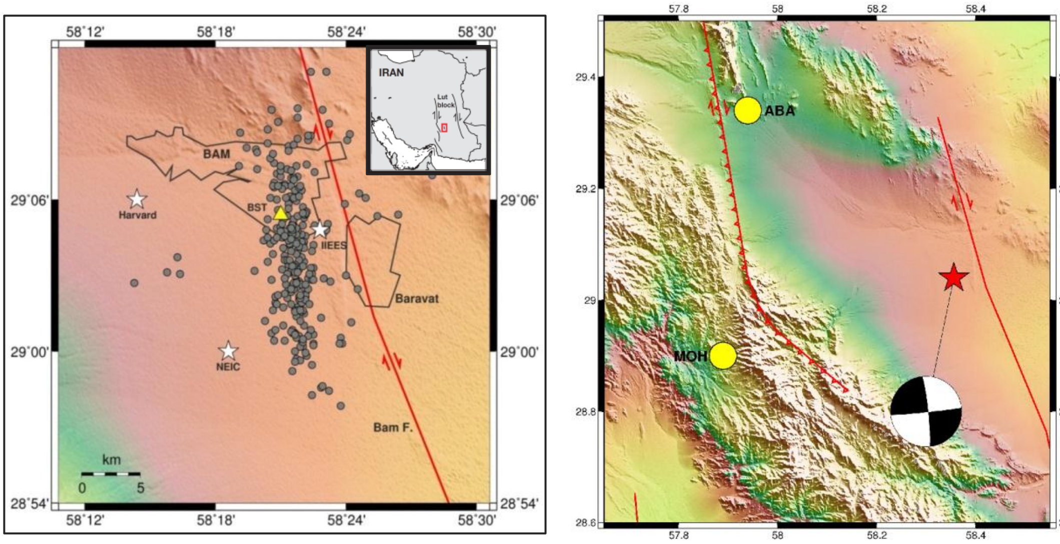

Fig. 1 (left) Different reports on Bam epicenter (white stars) & distribution of well-located aftershocks Recorded from the 3rd day after main shock (gray circles). Yellow triangle shows the strong motion station of Bam city. (right) epicentre and 2 stations used for simulation.

Majority texture of houses in the city consisted of poor construction of the unreinforced mud-brick and it simply could be one of the main reasons of such a devastating damage beside all other seismological aspects. Even so, the damage was disproportionately and unexpectedly large given the magnitude of the earthquake. Maximum acceleration of 0.98g was recorded in the vertical component at the Bam accelerograph station in the center of Bam city by the Building and Housing Research Center of Iran (BHRC; http://www.bhrc.gov.ir/) at distance of about 6 km from fine simulated location of epicenter.
Distribution of very well-located (Tatar et al. 2005) aftershocks which collected by a temporary dense network right after the main shock (3 days after mainshock) is also shown in figure 1.

3. SIMULATION METHOD

We treat the problem of the derivation of the surface wave field far, compared with the wavelength, from the seismic source in a laterally varying model. If the heterogeneity is not so severe, it can be reviewed as a small perturbation, i.e. within a wavelength, of a reference lateral homogeneous model and a procedure based on the ray method can be used to construct an approximate solution responding to the wave field (WOODHOUSE, 1974; AA.VV. 1989; DAHLEN and TROMP, 1998). Assuming as the reference lateral homogeneous model a vertical heterogeneous halfspace modeled with a series of homogeneous flat layers, parallel to the free surface, a lateral variation in the thicknesses of the layers, and in the elastic parameters characterizing each layer, can be expressed as a perturbation by a small parameter, e, of the original properties in such a way that, if e = 0 the medium is a laterally homogeneous layered structure. When a perturbation in e is small within a wavelength, an approximate solution corresponding to the surface wave field can be obtained by mean of a procedure based on the ray method (WOODHOUSE, 1974; AA.VV. 1989). If the lateral heterogeneous model is made up of two or more vertically heterogeneous structures juxtaposed each other, this means that two adjacent structures have to be very close in the parameter's space where they are determined by a point, or, analogously, they have to be very similar, in the sense of the elastic parameters, to allow the application of the WKBJ approximation. If we focus our attention on a model made up by only two structures, this problem is solved introducing between them a set of sub-structures that have the objective to ''smooth'' the gradient of the lateral variation, so that the new laterally varying model presents weak lateral heterogeneities, where weak is meant in the sense of the wavelength. More rigorously, in the framework of the WKBJ approximation the strongest smoothness condition that is imposed on the lateral variation of the elastic parameters can be written in the form:

$$|\nabla_{\perp} p| \ll \frac{\omega}{c} p \tag{1}$$

Where p is the elastic parameter (i.e. P and S wave velocities, VP and VS, and the density, ρ) and C = C(x, y) phase velocity. Details of the procedure used for determination of the structures used for smoothing of the horizontal gradient can be found in LA MURA (2009), where Eq. (1) has been studied in depth and, by analysis of the inequality, a less strict rule that enables construction of a weak lateral heterogeneous model has been derived.
The 3D models are determined by a set of vertically heterogeneous sections (1D structures) that are juxtaposed on a regular grid. In each knot of the grid a vertically heterogeneous section is located; hence, the values of the phase velocities, of the phase attenuation and of the group velocities are assigned once and for all. Inside the grid the source and the receiver are located, assigning their coordinates by means of a Cartesian reference system introduced in the grid itself. By this way a vertically heterogeneous structure, hence one-dimensional structure, is associated to the source and another to the receiver. The eigenfunctions of these two structures do contribute to the seismogram. The computational scheme is based, besides on the WKBJ - approximation for weak lateral heterogeneities, on the two point ray – tracing algorithm, by means of the bi - dimensional shooting method. It can be summarized as follows: at first the ray connecting two points, the source and the receiver, is computed solving the Cauchy problem for the system of ordinary differential equations governing the phenomenon of the evolution of the ray itself; the system is solved employing the numerical fourth – order Runge – Kutta method. Once the ray is determined, the attenuation is computed along it, solving, once again using the fourth – order Runge – Kutta method, the Cauchy problem for a system of ordinary differential equations that is made up of four equations: three equations for the ray and one equation governing the evolution of the attenuation along the ray itself. Finally, the geometrical spreading is computed considering two more rays that leave the source with an azimuth that is determined increasing and decreasing the azimuth of the characteristic curve of the ray – tracing system (the true ray) by a fixed quantity. for the validation of technique see (La Mura et al., 2011).

4. MODAL SUMMATION AND RAY TRACING

Spectrum of the wave field represented by a sum of normal modes in a half-space with weak lateral heterogeneity in the WKBJ approximation (3DMS) is expressed by the formula (LEVSHIN, 1985; AA.VV. 1989):

$$\mathbf{U}(x,y,z;\omega) = \sum_k \frac{exp(-i\pi/4)}{\sqrt{8\pi}} \frac{\exp(-i\omega\tau_k - \omega\boldsymbol{\Gamma}_k)}{\sqrt{J_k\omega}} \left.\frac{\boldsymbol{V}_k(z,\omega)}{\sqrt{u_k I_{0k}}}\right|_R \left.\frac{\boldsymbol{W}_k(h,\omega)}{\sqrt{c_k u_k I_{0k}}}\right|_S \tag{2}$$

Where k is the index identifying the mode, c_k is phase velocity, u_k is group velocity, J_k is geometrical spreading, τ_k is the phase travel time and it is given by

$$\tau = \int_S^R c^{-1}(x,y)ds, \tag{3}$$

I_{0k} is the energy integral, Γ_k is the attenuation factor and it is given by

$$\Gamma = \int_S^R \eta(x,y)ds, \tag{4}$$

$\boldsymbol{V}_k$ is the eigenfunction of the wave (Rayleigh or Love), $\boldsymbol{W}_k$ is the source function depending on the source mechanism and source spectrum, R and S indicate receiver and source sites, h is the source depth and z is the receiver depth.

5. PARAMETER'S ASSIGNMENT

The algorithm used in this work is based on the two-point ray tracing performed by the shooting method. Two-point ray tracing begins reading the following data: a threshold parameter representing the tolerance of the shooting method, δ, in fact it is the precision of the end point, i.e. the distance between the true station point and the computed one; the step and the accuracy of the Runge-Kutta integration; source and receiver coordinates, in kilometers; a parameter, α_Z, for the computation of geometrical spreading, that will be extensively explained later, in this same section. As far as concerns the values of the parameters needed for computation of ray, the following observations have to be taken into account:

- tolerance of the shooting method, δ: its value is assigned considering the grid step, and a reasonable choice is to assign it a value of $\frac{1}{20}min(dx,dy)$ where dx and dy are the grid steps;
- step and accuracy of the Runge – Kutta algorithm: the step should be much greater than the precision, in our simulations we used 0.01 for the step and 0.01*10^2 for the accuracy;
- Parameter for computing geometrical spreading, α_Z: it is assumed very small, the value used for all the simulations we performed is 0.001.

6. COMPUTATION OF THE RAY

The ray-tracing problem is formalized as a Cauchy problem of ordinary nonlinear Differential equations:

$$\begin{cases} \dfrac{dx}{dt} = c(x,y)cos\alpha \\ \dfrac{dy}{dt} = c(x,y)sin\alpha \\ \dfrac{d\alpha}{dt} = c_x sin\alpha - c_y cos\alpha \end{cases} \tag{5}$$

With the initial conditions $x(0) = X_S$, $y(0) = Y_S$, $\alpha(0) = \alpha_S^{(0)}$

The integration is performed by means of the Runge – Kutta algorithm to determine the value of the azimuth corresponding to the ray that effectively does reach the receiver point. The solution of the system defines a ray at the discrete points (dots) shown in Fig 2. To ascertain that the wavefront passed through the receiver point we proceed as follows:

1. at the point of the ray corresponding to the length r, the angle γ_1 is defined, and at the next point the angle γ_2 is defined;
2. if the cosines of γ_1 and γ_2 have the same sign, the procedure goes on to the next pair of points;
3. if the cosines result to be opposite in sign (this circumstance occurs when $\gamma_1 < \frac{\pi}{2}$ and $\gamma_2 > \frac{\pi}{2}$, as shown in Fig. 2), this means that the wavefront passed the receiver point and the Runge-Kutta computation of the ray can stop;
4. if the tolerance threshold, δ , is exceeded, the azimuth, $\alpha_{A'}$, of the line connecting the point A' and the source is computed; a new Cauchy problem is solved with a new initial condition for the azimuth given by $\alpha_S^{(1)} = \alpha_{A'}$, (new ray); for the new ray steps 1 to 3 are repeated until convergence is reached, i.e:

5. the iterative computation of the ray is terminated when the distance between the receiver and the end point of the ray, defined at 3 (see point A' in Fig. 2) is less than the tolerance parameter.

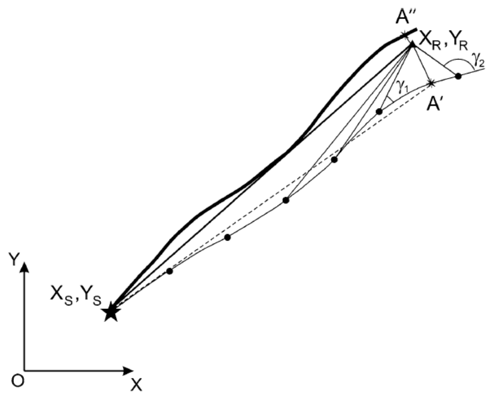

Fig. 2 A schematic representation of the construction of the ray by successive points (black dots) and angles.

7. SOURCE PARAMETERS

Table 1 shows different reports and different parameters of the source used for simulation by various studies. Collecting all data from some various studies and satellite images processed for the event for the starting step of the 3D simulation the location of the earthquake used finally from Funning et al. (2005) as listed in table 2.

Table 1: Different Reports of Source Parameters of the Bam Earthquake

Organization / Researcher	Segment	Long	Lat	Dep	Str	Dip	Rake
NEIC	1	58.266	29.010	14	174	88	178
Harvard CMT	1	58.240	29.100	15	172	59	167
Talebian et al. [2004]	2	58.294	28.972	6	357	88	-166
		58.294	28.864	5	180	90	30
Funning et al. [2005] uniform slip	1	58.353	29.037	5.2	354	84	-177
Funning et al. [2005] Variable slip and rake	1	58.356	29.040	8.1	354	85	Variable
Funning et al. [2005] uniform slip	2	58.357	29.038	5.5	354	85	-178
		58.406	29.988	6.7	180	64	149
Funning et al. [2005] Variable slip	2	58.356	29.040	6.4	354	85	-178
		58.406	28.988	5.9	180	64	149

Table 2: Source Parameters of the source used from published results

Source Parameters	Values
Strike	354^o
Dip	85^o
Rake	178^o
Depth	~7 km
Magnitude (Mw)	6.6

8. CONSTRUCTION OF THE 3D MODEL

The 3D model (Fig. 3) is determined by distributing a set of vertically heterogeneous sections (Fig. 4) on a regular grid: it can be rectangular or square.

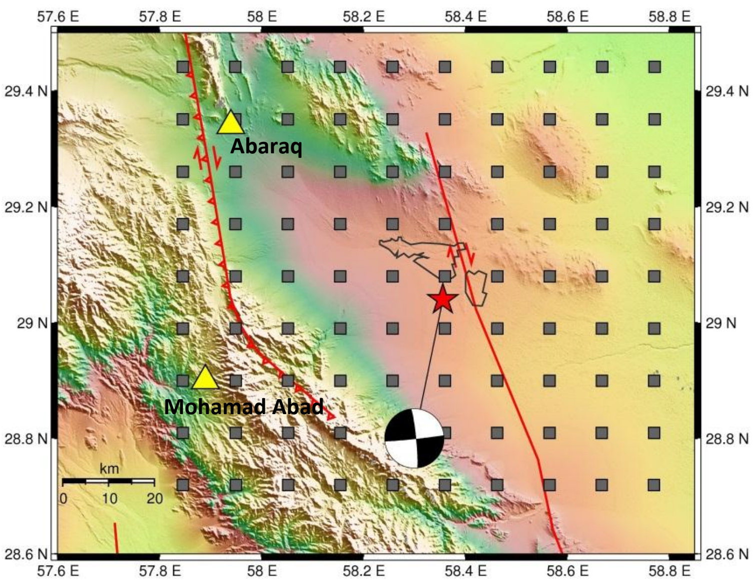

Fig. 3 3D grid: the star stands for the source, the triangle stands for the receiver; the squares stand for the different vertically heterogeneous sections.

A Cartesian reference frame is associated with the grid itself. The grid step is determined in such a way that the WKBJ approximation is satisfied. So, the grid step is chosen as the maximum length needed to allow the lateral gradient of the elastic parameters, characterizing the structures in each node of the grid, to be small within a wavelength.

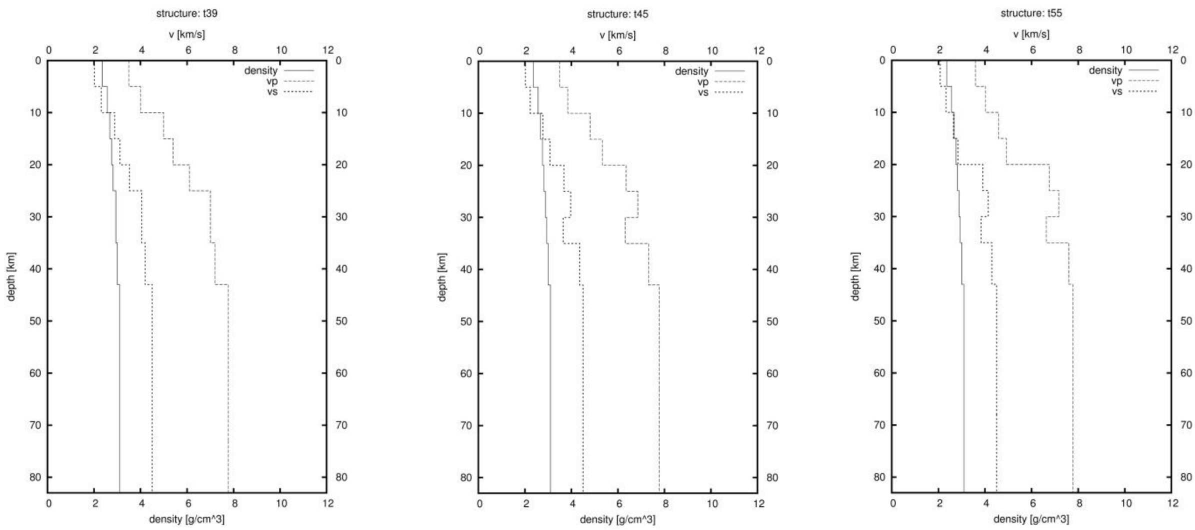

Fig. 4 Velocity and Density Structures for some of nodes. Name of each structure written on structures. (subscripts start from 11 from corner bottom right).

To construct 3D structures a combination of studies of Jeddi and Tatar (2011) and Rahimi (2010) is used. Velocity Structures at the first 35 km is choose base on study of Jeddi and Tatar (2011) and for deeper part of models we used study of Rahimi (2010). All the models under the depth 35 km have an identical structure. Attenuation and density parameters of the area (even for the first 35 km of the model which the 3D velocity structure was available) are assigned based on Anelastic parameters resolved by Rahimi 2010.

9. RESULTS

Because of the limitation of the 3D grid area, the simulation was done for two (Abaraq and Mohamad Abad) stations. For each station earthquake ground motion acceleration, velocity and displacement have been generated as time domain parameters and response spectra calculated as frequency domain parameter. Computations are done for low (1Hz) and high (6Hz) frequencies. Figure 5 shows the synthetic signals for radial component of each station (Other components are not shown because of the limitation of the abstract). On figure 5, the first 2 rows are for Abaraq station for various type of 1D, 2D and 3D simulations which 3DMS is assigned for 3D simulation. For 1D a single structure is assigned for all the area of grid, we tested 2 classes of 1D modeling, one with the structure at the place of source (1Dsrc) and the next with the structure at the place of receiver (1Drec). 2DMS is 2Dimentional Modal Summation which has done by reducing the structures from 3D to 2D. To do this reduction the structures at the direct connection from source to receiver is extended in the area. 2DFD also is a result for a hybrid simulation of Finite Difference and Modal Summation which has been done as a separate study. The second 2 rows on figure 5 are as the first 2 rows but for Mohamad Abad station. The third 2 rows are again for Abaraq for high frequency (6Hz) simulation and the last 2 rows are the same for Mohamad Abad station. Table 3 shows the peak values of acceleration for each station. Figures 6 and 7 are the response spectra calculated at 1Hz (fig. 6) and 6Hz (fig. 7) for Abaraq Station for various type of modeling described above.

Table 3: Observed and 3D simulated acceleration peak amplitude values for Abaraq (ABA) and Mohamad Abad (MOH) stations.

Station / Frequency	**ABA (1Hz)**	**MOH (1Hz)**	**ABA (6Hz)**	**MOH (6Hz)**
Component				
Rad (Cm/S^2) - Obs	6.47	9.09	39.53	38.99
Rad (Cm/S^2) - Syn	6.70	10.70	39.56	35.68
Tra (Cm/S^2) - Obs	6.74	25.63	37.93	98.17
Tra (Cm/S^2) - Syn	6.17	27.52	34.01	93.84
Ver (Cm/S^2) - Obs	5.70	4.04	20.33	43.10
Ver (Cm/S^2) - Syn	5.28	5.25	20.35	28.60

10. DISCUSSION

A new methodology for computing synthetic seismograms in three-dimensional anelastic lateral heterogeneous media is used. Comparison of the 3DMS and other synthetics and observed data at the 2 stations available throughout the Bam earthquake revealed that the 3DMS synthetics reasonably match the early part of the recorded waveforms, and it is clear that the modeling does not reproduce all the details seen in the recorded waveforms, for both sets of synthetic signals at low and high frequencies. Some later arrivals at observed waveforms can be related to this reality that the Bam had a small event right after the main shock which affect the later arrivals of the event. Other discrepancies are because of a localized shallow velocity structure not modeled in the 3DMS velocity model from which we derived our own model. It has to be stressed that, once the study region is chosen and the 3D model is constructed, the spectral quantities are computed once and for all. The improvement of waveform and spectra can easily seen while using 3D modeling. The subsequent computation at each station of the three components synthetic signal takes less than 3 h on a 2 Ghz CPU that provides a useful tool for rapid and sophisticated modeling.

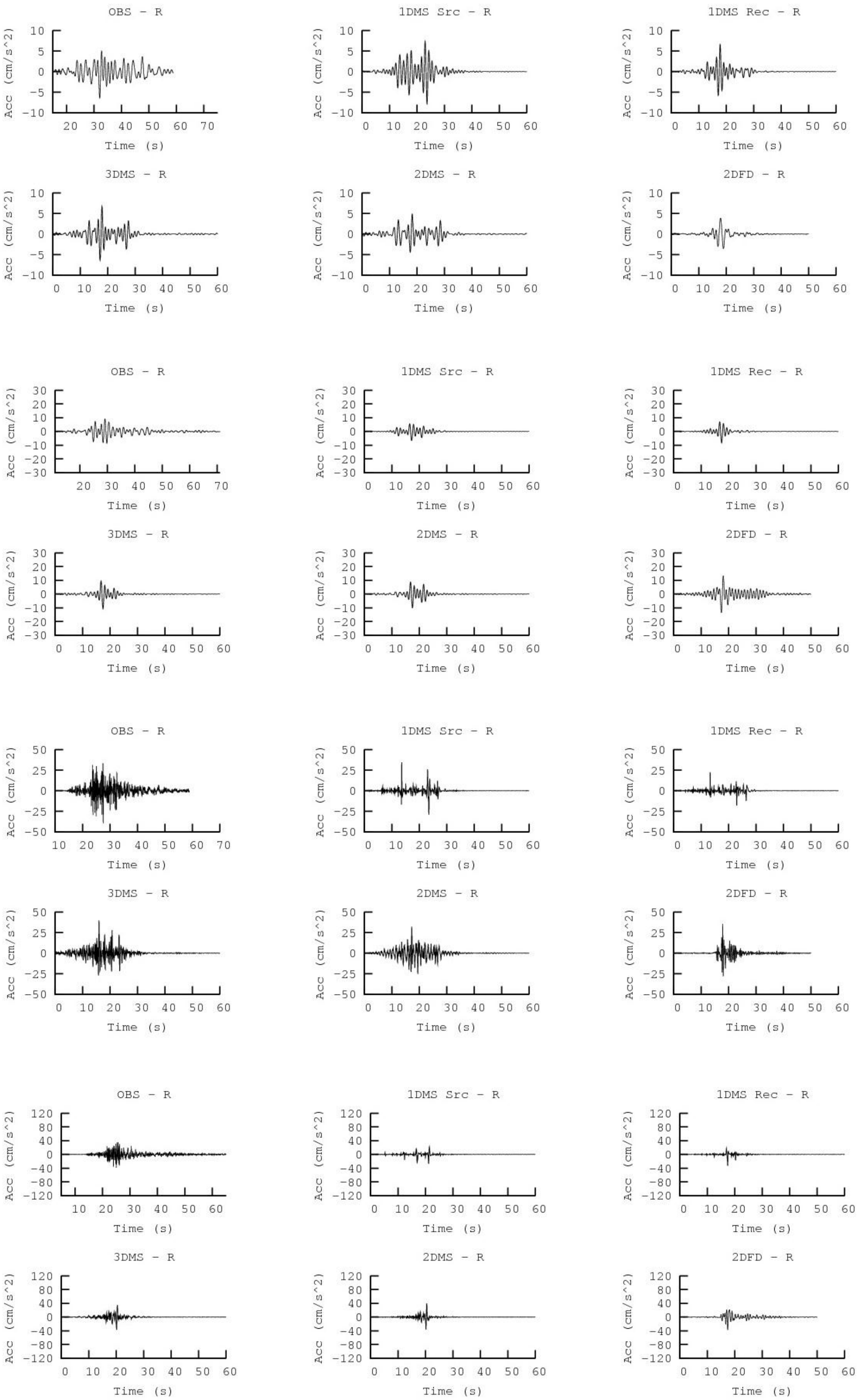

Fig. 5 Observed and simulated waveforms for radial (R) component. The first 2 rows for Abaraq station at 1hz, the second 2 rows for Mohamad Abad at 1hz. The third 2 rows for Abaraq at 6 Hz and the last 2 rows for Mohamad Abad at 6Hz.

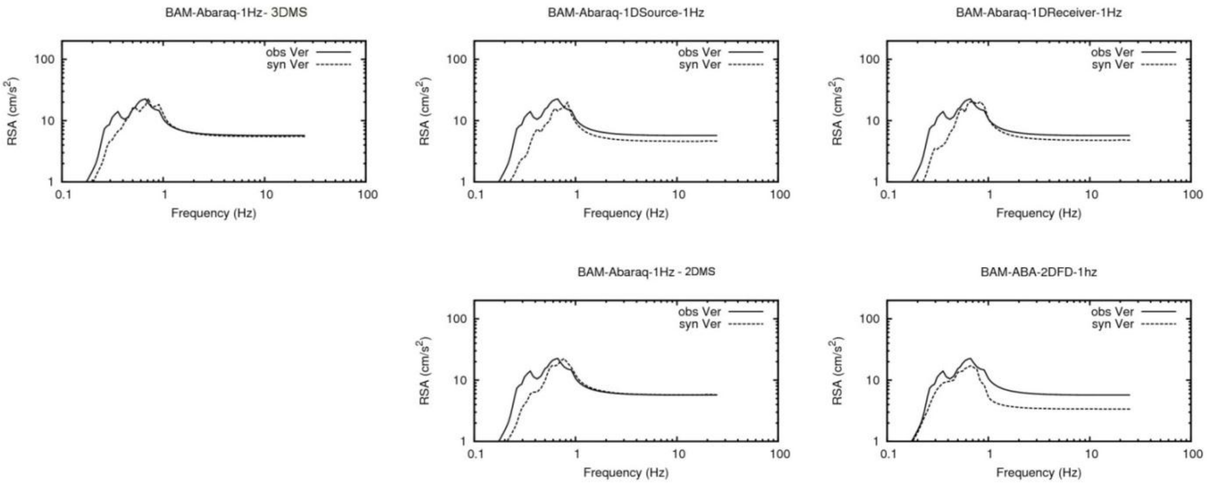

Fig. 6 Response Spectra of Observed and Synthetic Signals for Abaraq Station - 1Hz Simulation. 3DMS (top left), 1Dsrc (top middle), 1Drec (top right), 2DMS (bottom middle) and 2DFD (bottom right).

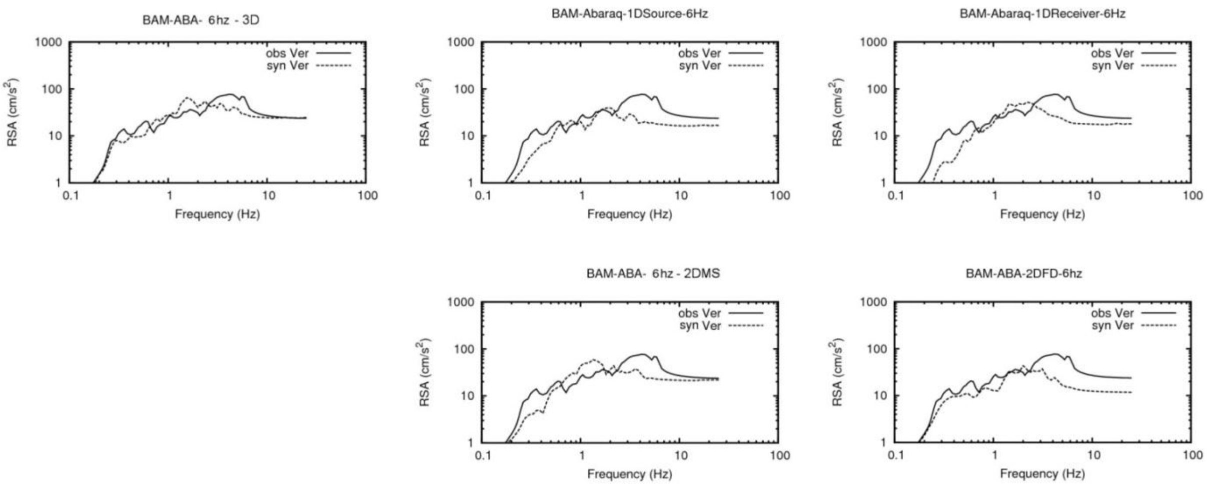

Fig. 7 Response Spectra of Observed and Synthetic Signals for Abaraq Station - 6Hz Simulation. 3DMS (top left), 1Dsrc (top middle), 1Drec (top right), 2DMS (bottom middle) and 2DFD (bottom right).

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