Comparative Study Of
Stiffness Reduction And Damping Curves

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SUMMARY
Three families of shear modulus reduction and damping ratio curves, developed by Vucetic & Dobry (1991), Ishibashi & Zhang (1993) and Darendeli (2001), are reviewed. A comparative analysis of all families is presented for two soil groups, non plastic and plastic soils. The influence of five important factors in cyclic behaviour (number of cycles, loading frequency, overconsolidation ratio, mean confining pressure and plasticity index) is studied for trend, rate and magnitude of variation. A direct comparison with third party laboratory data not used in the development of the empirical curves (soil sampled in Turkey, following the 1999 Kocaeli earthquake) is also undertaken. The main advantages of the different curves, their range of applicability and shortcomings are identified and recommendations for best usage of each set are given.

Keywords: dynamic soil properties, empirical curve, shear modulus reduction, damping ratio

1. INTRODUCTION

Dynamic soil behaviour resulting from earthquake-induced ground motion can be hysteretic, highly nonlinear and plastic. Ground response prediction methods should ideally address all those aspects, while still maintaining a balance between efficiency and accuracy adequate for the problem under consideration. The recommended practice is to determine dynamic soil properties from laboratory and field testing of site materials. However, depending on the location and complexity of a project this approach might not be practical or viable. Current earthquake geotechnical engineering often resorts to worldwide or region specific empirical relations (e.g. shear modulus reduction and damping ratio variation with cyclic shear strain) to estimate dynamic soil properties and tackle various problems. For instance, site response analysis is usually carried out using equivalent linear models or simple cyclic nonlinear constitutive models calibrated against these empirical relations.

This paper presents a critical review of the Vucetic & Dobry (1991), Ishibashi & Zhang (1993) and Darendeli (2001) empirical sets of normalized shear modulus, G/G_{max}, and material damping ratio, ξ, curves. A comparative parametric study is then performed for two soil groups, non plastic and plastic, based on work from Guerreiro (2008). The influence of five important factors in cyclic shear stress-strain behaviour is examined. These are, in increasing order of significance for common geotechnical applications, number of cycles N, loading frequency f, overconsolidation ratio OCR, mean confining pressure p' and plasticity index I_p. The parametric study is followed by a direct comparison with third party laboratory data not used in the development of any of the previously assessed empirical curves.

2 EMPIRICAL CURVES

Several studies have been undertaken to develop laboratory-based empirical relations for dynamic soil properties, predominantly in the USA and Japan. Three common empirical G/G_{max} and ξ sets of curves are introduced in the following sections. None of them should be applied to sensitive or cemented natural soils, as these were never tested nor considered in their respective studies.

2.1 Vucetic & Dobry (1991) Curves

Vucetic & Dobry (1991) carried out a compilation of the results of numerous studies, which included different soil types, testing apparatuses and cyclic test types. Representative (not mean) G/G_{max} and ξ curves were graphically fitted, after which the well known family of modulus reduction and damping ratio curves were developed. These are reproduced in Fig. 1.

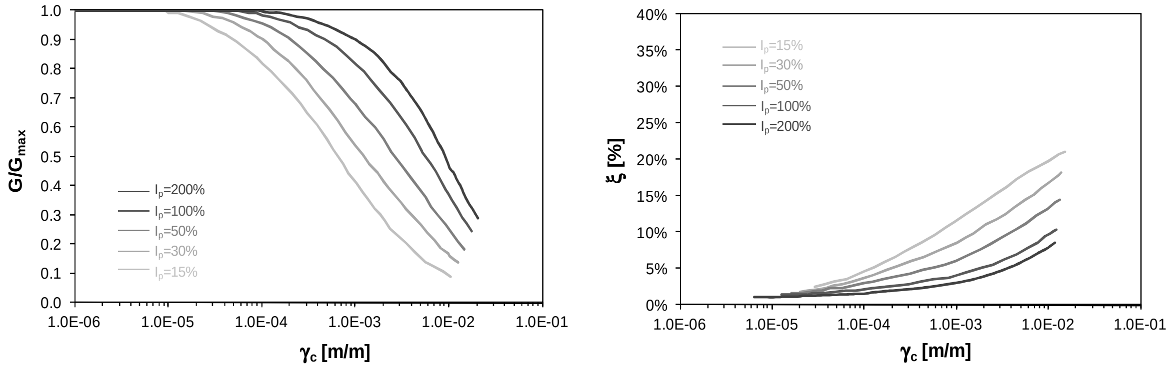

Figure 1. Vucetic & Dobry (1991) G/G_{max} - γ_c and ξ - γ_c curves equations

According to Vucetic & Dobry (1991), the charts are recommended only for preliminary studies due to high data scatter. It is also stated that soil data ranges from clean sands to clays, encompassing wide overconsolidation ratios (1-15) and plasticity indexes (0-175%). Nevertheless, it should be noted that only values of OCR of 1, 2 and 4 and $I_p \leq 60\%$ are well represented in terms of number of independent studies. The remainder of the values appear only once. Also, the compiled confining pressures did not exceed 400kPa. As such, any comparison with higher p′ values should be considered with caution.

2.2 Ishibashi & Zhang (1993) Curves

Khouri (1984) compiled data from numerous studies on sandy soils and proposed hyperbolic equations for G/G_{max}, as a function of cyclic shear strain and confining pressure, and for ξ, as a function of G/G_{max}. Based on these functions, Ishibashi & Zhang (1993) developed analytical expressions that also included the effects of plasticity index. These new equations were calibrated based on additional, though somewhat limited ($I_p \leq 50\%$) and scattered, laboratory data from several researchers. Fig. 2 shows representative G/G_{max} - γ_c and ξ - γ_c curves based on the Ishibashi & Zhang (1993) equations.

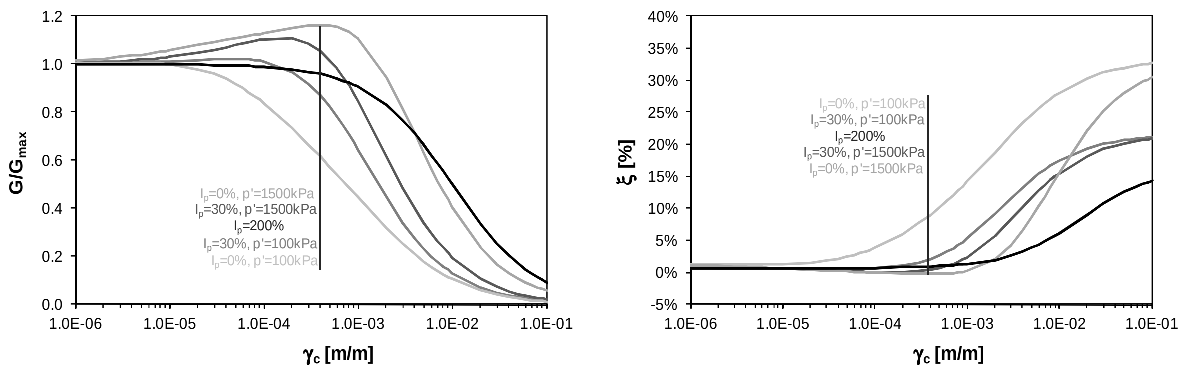

Figure 2. G/G_{max}-γ_c and ξ - γ_c curves after Ishibashi & Zhang (1993) equations

Fig. 2 reveals that the normalized shear modulus equation returns values above 1.0 at low and intermediate strains. This inconsistency increases with p′ but decreases with I_p (not observable above I_p=100%). Note that, for I_p values above 100%, all curves practically collapse into a single relationship, which is independent of p′. Similarly, the damping ratio equation returns negative values at intermediate strains when p′ values are sufficiently large, although outside the typical range relevant for engineering practice (e.g. 1100kPa for I_p=15%; 3000kPa for I_p=30%). Therefore, it is important to retain that additional restrictions may be needed for G/G_{max} and ξ values.

2.3 Darendeli (2001) Curves

Darendeli (2001) used an extensive database from various research projects to develop the most recent of the analysed families. The database consisted of combined Resonant Column and Cyclic Torsional Shear (RCTS) tests, sequentially performed on intact samples from soils described as having low void ratio and not liquefiable during seismic activity. The tested soil samples ranged from natural clean sands to clays, characterised by broad intervals of sampling depth (3-263 m), confining pressure (0.3-27.2atm), I_p (0-132%) and OCR (1-8). A statistical analysis of the database was undertaken to calibrate all of the required parameters. It is stated that the five key factors influencing dynamic soil properties were taken into consideration. Also, due to test data collection limitations, the equations are recommended only for $\gamma_c \leq 1.0\times10^{-2}$. The resulting curves are presented in Fig. 3 for a range of I_p and p' values. It should be noted, however, that I_p lacks representativeness for values above 30% (10 samples) when compared to the remainder (100 samples). This has very likely introduced a bias in the development of the curves, as they exhibit a narrower breath for varying values of I_p, when compared with the proposals by Vucetic & Dobry (1991) and Ishibashi & Zhang (1993). OCR values are also not very well represented above 2 (9 samples against 101 samples for OCR $\leq$ 2).

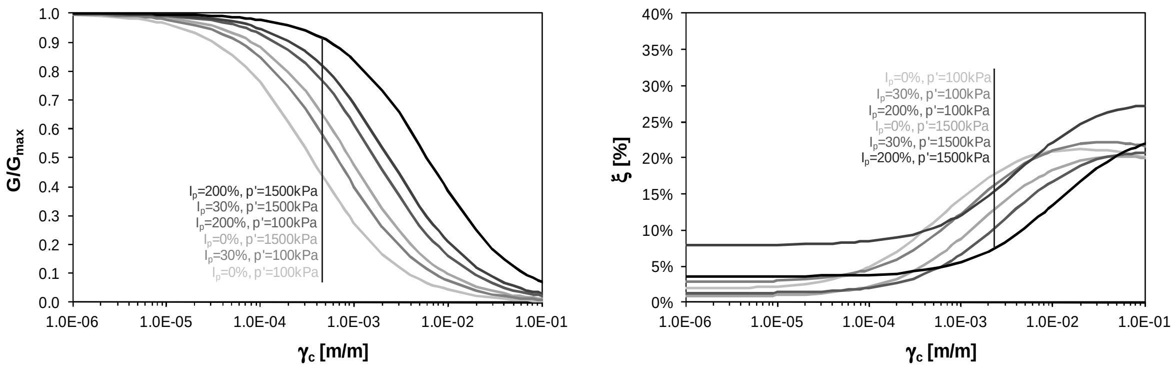

Figure 3. G/G_{max}-γ_c and ξ - γ_c curves after Darendeli (2001) equations

Fig. 3 shows that Darendeli (2001) ξ curves reach a peak at large strains and then start to drop, particularly for lower p' and/or lower I_p values. Although this trend has so far not been observed during testing, the values of strain for which it occurs are clearly outside the recommended range of applicability of the expressions proposed by Darendeli (2001)..

3 EFFECTS OF CYCLIC LOADING FREQUENCY AND NUMBER OF CYCLES

Generally, frequency and number of cycles are related. The typical applied frequencies and number of cycles of the Cyclic Triaxial and/or Torsional Shear [*TS*] tests resemble a strong ground motion, i.e. a vibration problem with low *f* (1-10Hz) and N (1-10 cycles). Conversely, the Resonant Column [*RC*] test nearly represents a fatigue problem, i.e. high *f* (100Hz) and N (1000 cycles). The combined effects of frequency and number of cycles are therefore assessed for the Darendeli (2001) curves. Vucetic et al. (1991) and Ishibashi & Zhang (1993) curves do not account for these two factors. They have, however, been included in Fig. 4 for reference.

Darendeli (2001) found that, for his database, G/G_{max} curves were not very sensitive to frequency while ξ curves could shift noticeably depending on soil type and strain level. Similarly, it was found that changes in G/G_{max} curves were negligible for different values of the number of cycles, while ξ curves changed slightly at large strains. The equations were developed accordingly, i.e., G/G_{max} values are neither affected by *f* nor N, whereas ξ values increase with increasing *f* and decrease at large strains with increasing N.

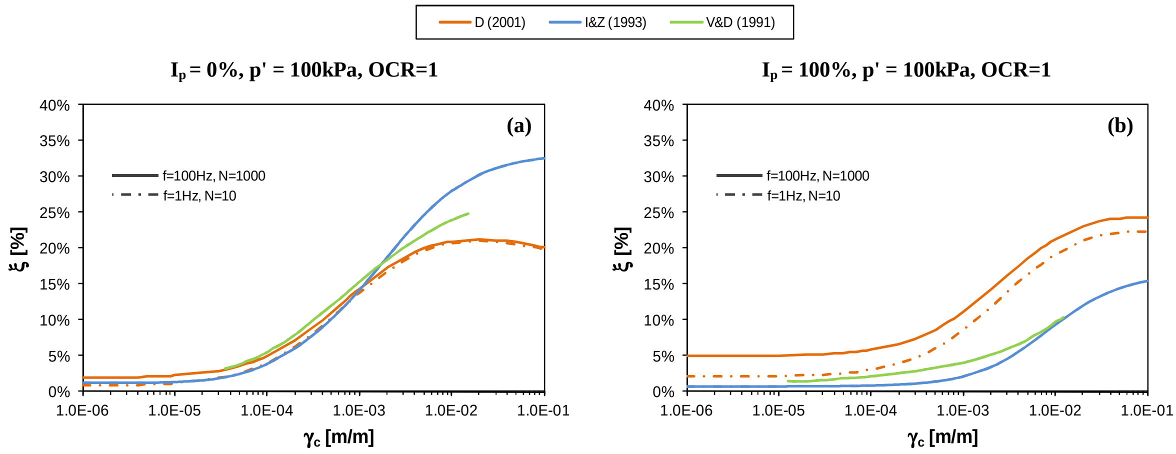

Figure 4. Damping ratio prediction for non plastic (a) and plastic (b) soils in TS and RC type tests

Fig. 4(a) shows that the combined effects of *f* and N, when changing from a *TS* type to a *RC* type test, have a minor impact on the damping curves for non plastic (I_p=0%) soils. For low I_p ($I_p \leq 30\%$) soils (not shown herein for brevity), both factors tend to nearly cancel out at intermediate to large strains, while slightly increasing damping at small strains. For plastic soils (see Fig. 4(b)), the curves display a nearly uniform increase in damping (growing with increasing I_p). This occurs irrespectively of p′ and OCR values, with frequency playing a more important role than the number of cycles.

Being less significant (compared to the other factors), the effects of loading frequency and number of cycles are not considered in the remainder of this study. In all subsequent comparisons *f*=100Hz and N=1000 are assumed, which correspond to a *RC* type test. This test provides a clearer benchmark due to the widest availability of results amongst the examined laboratory data.

4 COMPARISON OF EMPIRICAL CURVES FOR NON-PLASTIC SOILS

Ishibashi & Zhang (1993) and Darendeli (2001) curves (see Fig. 5) show the commonly observed trends of increasing normalised shear modulus and reducing damping ratio with increasing confining pressure. Furthermore, both sets of curves show that these effects become less important for higher confining pressures. Vucetic & Dobry (1991) family of curves do not directly account for effects of confining pressure, but are included in Fig. 5 for reference.

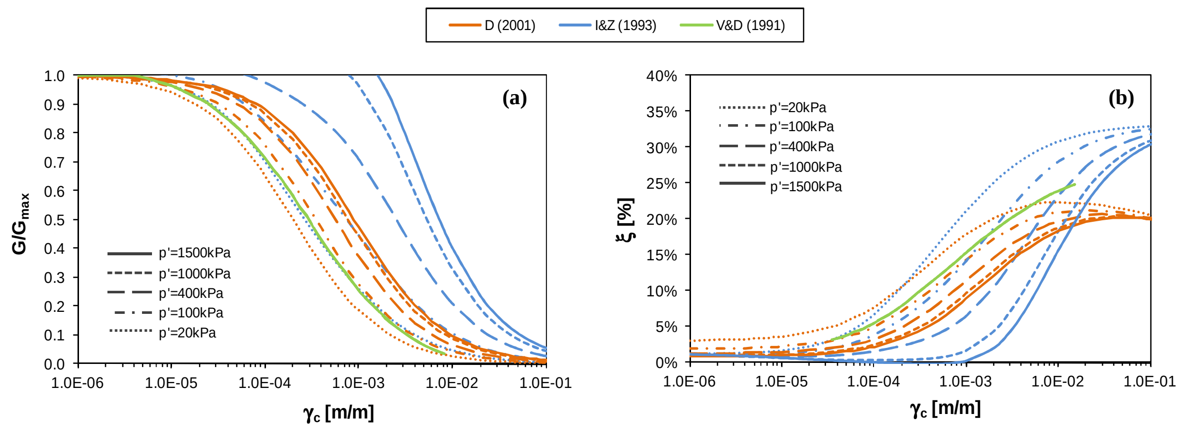

Figure 5. Effect of confining pressure on G/G_{max} and ξ predictions for I_p=0%.

Fig. 5(a) shows that the Ishibashi & Zhang (1993) G/G_{max} curves always plot above the Vucetic & Dobry (1991) and Darendeli (2001) curves, i.e. predict slower shear modulus reduction, with this difference increasing largely with increasing confining pressure. Darendeli (2001) G/G_{max} curves compare well with those proposed by Vucetic & Dobry (1991) for p′ values lower than 200kPa.

At small strains, Fig. 5(b) shows that the Ishibashi & Zhang (1993) ξ curves compare well with the Darendeli (2001) curves except for very low p′ values. In contrast, at large strains they compare very poorly for all values of p′. At intermediate strains, the Ishibashi & Zhang (1993) ξ curves generally predict less damping, with the difference increasing with confining pressure. Fig. 5(b) also shows that the Vucetic & Dobry (1991) ξ curves compare reasonably well with the Ishibashi & Zhang (1993) curves, but only for p′ values lower than 200kPa and for small and intermediate strains. Conversely, the comparison with Darendeli (2001) curves is fairly good between p′=50kPa and 400kPa.

5 COMPARISON OF EMPIRICAL CURVES FOR PLASTIC SOILS

For the following comparisons, overconsolidation ratio, confining pressure and plasticity index are examined for values ranging from 1 to 10, 10kPa to 1500kPa and 1% to 200%, respectively.

5.1 Effect of Overconsolidation Ratio

Darendeli (2001) is the only family of curves that considers the influence of overconsolidation ratio. Whilst having no effect for non plastic soils, Fig. 6 shows that, for plastic soils, there is an increase in the normalised shear modulus and decrease in damping ratio with increasing overconsolidation ratio. This effect is slightly more significant for higher I_p values, as illustrated by the changes from Fig. 6(a) to (c) and Fig. 6(b) to (d) (wider breath of the curves at intermediate strains). Nevertheless, the impact of overconsolidation ratio is minimal at both small and large strains and becomes increasingly less important for higher OCR values.

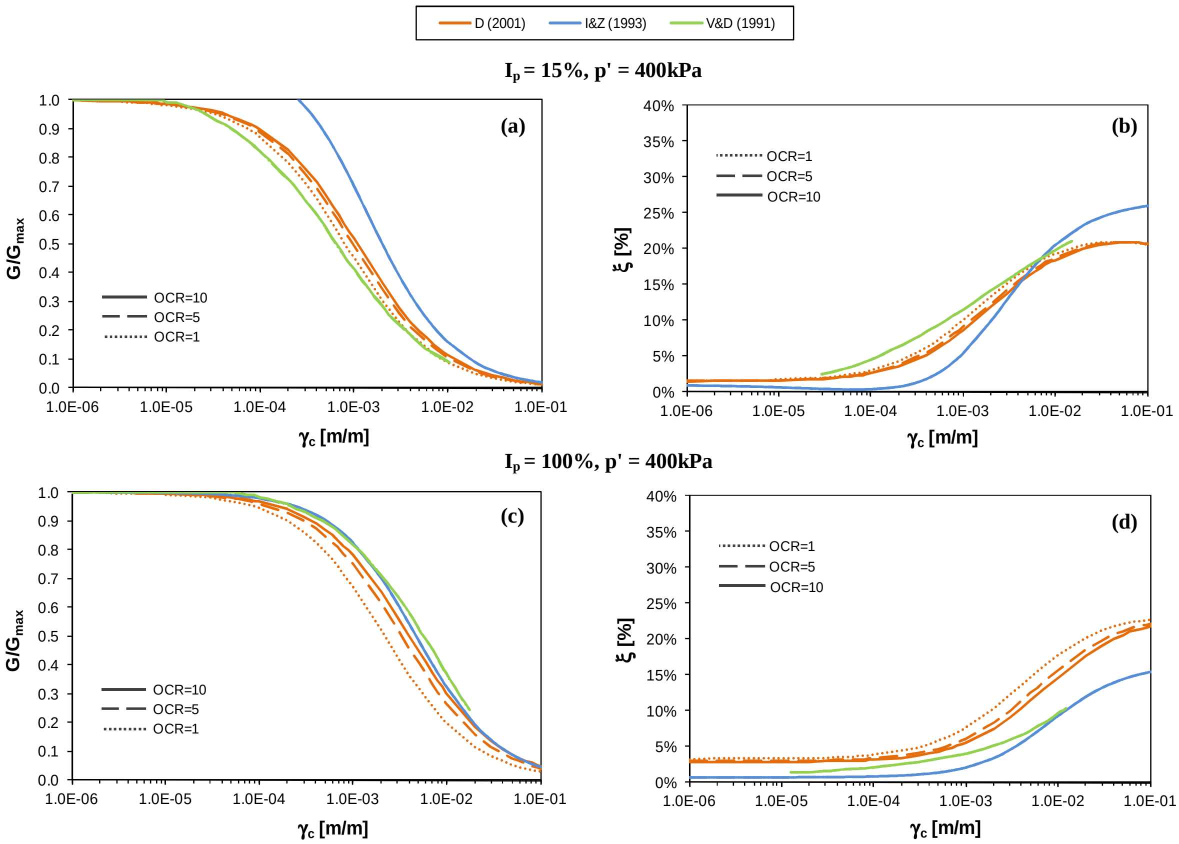

Figure 6. Effect of OCR on G/G_{max} and ξ predictions for low and high plasticity indexes

In contrast, confining pressure barely modifies the effect of OCR (not shown in the figure). Increasing p′ values basically shifts all curves to the right and either upward for G/G_{max} or downward for ξ.

Comparisons between the three families of curves range from poor to fair, particularly at intermediate strains. It is worth noting that, overall, the influence of OCR appears to be considerably less significant than that of p’ and I_p. Therefore, for clarity of interpretation between plastic and non-plastic soils, a value of OCR=1 is assumed in the next two subsections.

5.2 Effect of Confining Pressure

Similarly to non-plastic soils, Fig. 7 shows that Darendeli (2001) and Ishibashi & Zhang (1993) curves predict increasing normalised shear modulus and decreasing damping ratio with increasing confining pressure. Both sets of curves also show that these effects are less important for higher confining pressures. Further analysing the Darendeli (2001) G/G_{max} curves, Fig. 7(a) and (c) reveal that different I_p values do not alter the effect of confining pressure (curves merely appear to shift uniformly). Fig. 7(b) and (d), however, show that larger variations on the ξ curves are found for higher I_p values. On the contrary, Ishibashi & Zhang (1993) curves show an increasing impact of confining pressure with decreasing plasticity (see Fig. 5 and Fig. 7). Also, confining pressure changes have no impact for I_p values above 60%, as all Ishibashi & Zhang (1993) curves practically collapse into a single relationship (see Fig. 7(c) and (d)). As previously mentioned, the Vucetic & Dobry (1991) family of curves is included only for reference.

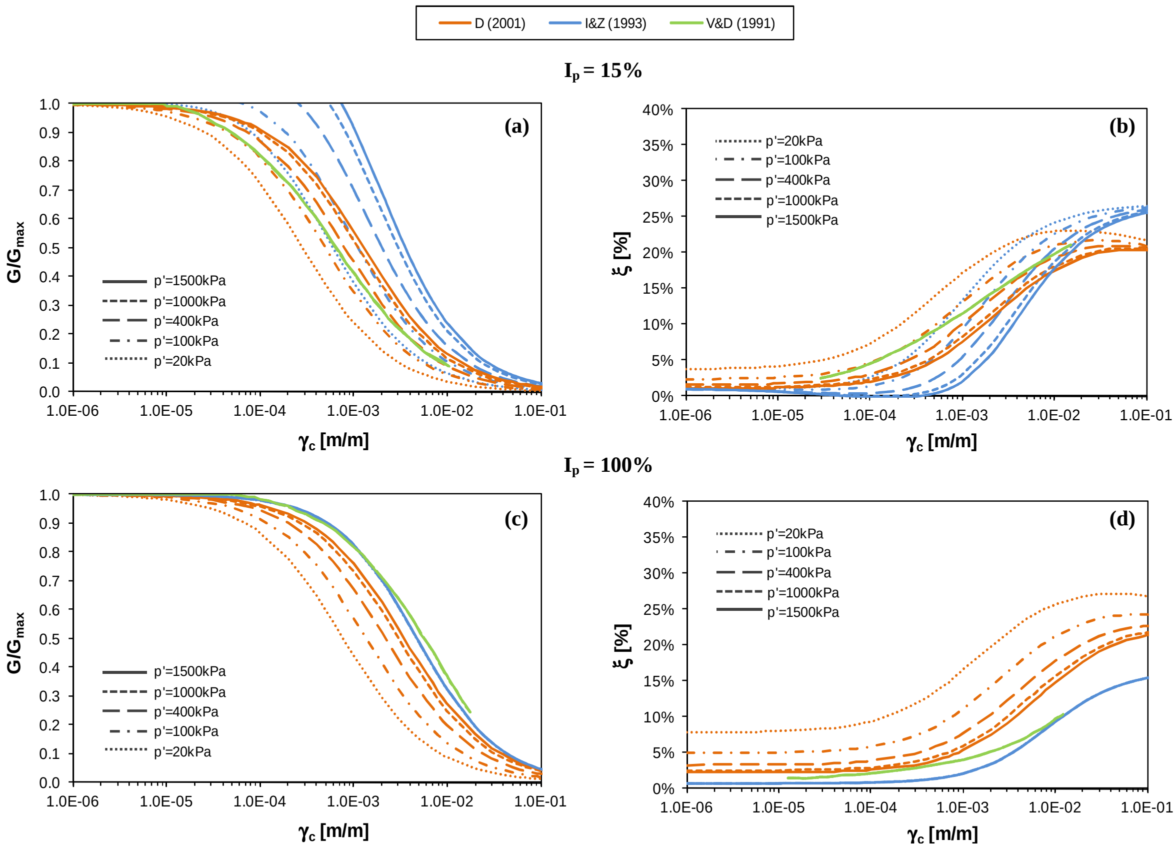

Figure 7. Effect of confining pressure on G/G_{max} and ξ predictions for low and high plasticity indexes

At low I_p values (I_p≤30%), Fig. 7(a) and (b) show that the Ishibashi & Zhang (1993) and Darendeli (2001) G/G_{max} and ξ curves share the same comparative behaviour as for non-plastic soils (see Fig. 5). However, note that, while the trends are similar, the actual values vary with plasticity index. The Vucetic & Dobry (1991) G/G_{max} curves compare better with the Ishibashi & Zhang (1993) or the

Darendeli (2001) curves for lower (Fig. 7(a)) or higher (Fig. 7(c)) p′ values, respectively. In contrast, the ξ curves compare poorly with both the Ishibashi & Zhang (1993) and Darendeli (2001) curves.

For high I_p soils, Fig. 7(c) and (d) show that the Vucetic & Dobry (1991) and Ishibashi & Zhang (1993) curves compare fairly well and practically coincide for $I_p \geq 60\%$. The Darendeli (2001) curves predict, for values of p' typically encountered in engineering practice, a sharper modulus reduction and higher damping ratio than the other two sets. Only for very high confining pressures (p′≥3000kPa) the curves invert their relative positions (not shown in the figure). It should be noted that increasing OCR causes this threshold to occur sooner.

5.3 Effect of Plasticity Index

Plasticity index is the factor traditionally mentioned as having the highest impact on cyclic soil behaviour and all three analysed sets take it into consideration as demonstrated in Fig. 8.

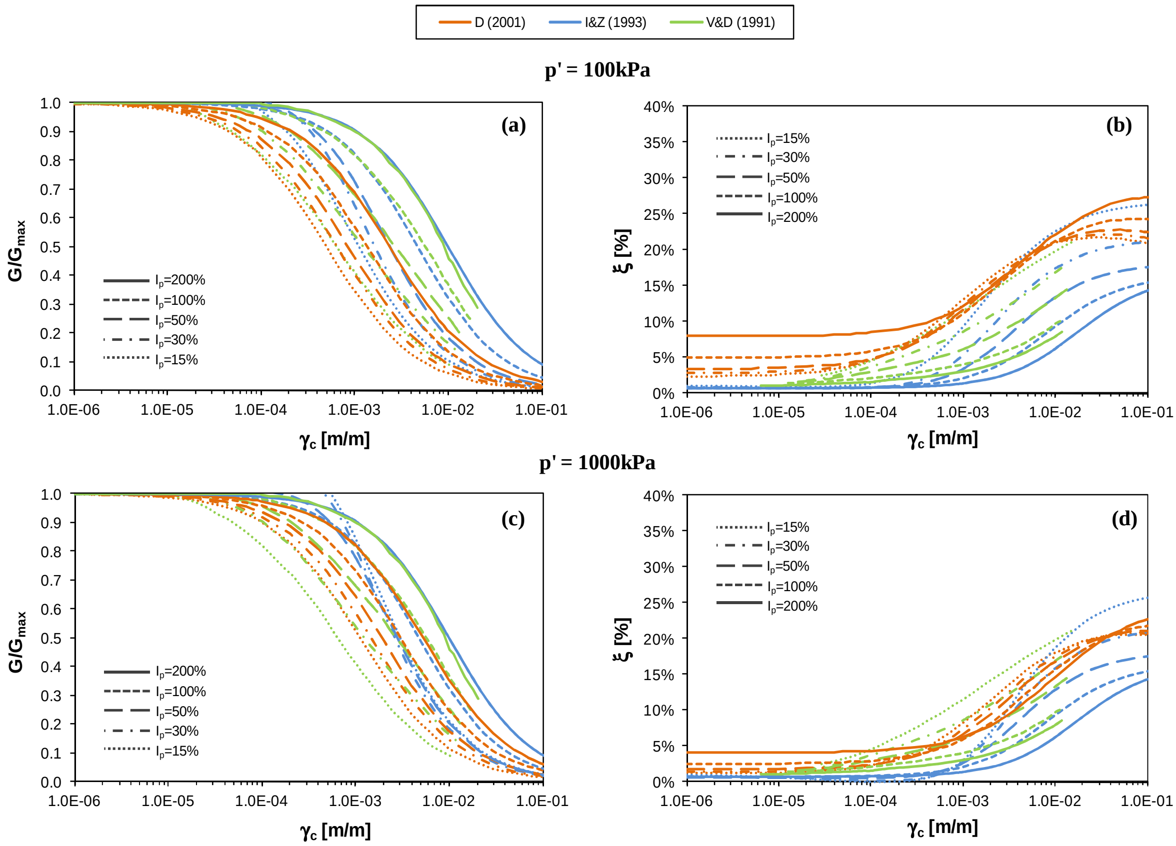

Figure 8. Effect of plasticity index on G/G_{max} and ξ predictions for low and high confining pressures

Fig. 8(a) and (c) display, for all G/G_{max} sets, the widely recognized trend of higher normalised shear modulus with increasing plasticity index. This behaviour becomes less significant for higher I_p values. On the contrary, for damping ratio (see Fig. 8(b) and (d)), the three sets present varying responses at different strain levels. Vucetic & Dobry (1991) ξ curves predict the well known trend of decreasing damping with plasticity index. This effect also diminishes for higher I_p values. Ishibashi & Zhang (1993) ξ curves generally predict the same behaviour as Vucetic & Dobry (1991) curves, with one exception. Fig. 8(d) illustrates a trend reversal, whereby damping increases with plasticity index, present only at small and intermediate strains and for p′ values higher than 200kPa. Finally, Darendeli (2001) ξ curves always predict two similar reversals at small and large strains. The trend reversal at small strains accurately reproduces EPRI (1993) and Vucetic (1998) results. However, it remains unclear whether a trend reversal is realistic at large strains, as this behaviour has so far not been observed in laboratory tests. It is important to note that the breath of Darendeli (2001) G/G_{max} and ξ curves is about half that of Vucetic & Dobry (1991) curves. This difference is probably due to the fact that the calibration was performed with a limited I_p database and it results in curves that should be used with care for I_p values above 30%.

For low confining pressures, Fig. 8(a) shows that the Vucetic & Dobry (1991) and Ishibashi & Zhang (1993) G/G_{max} curves always predict a slower modulus reduction than the Darendeli (2001) curves. Similarly, Fig. 8(b) shows that the latter nearly always predicts higher damping ratio. The exception occurs at large strains and for very low I_p values. Note also that the difference between the former two sets and the latter one increases largely with plasticity index for both G/G_{max} and ξ curves. Furthermore, Fig. 8(a) shows that Vucetic & Dobry (1991) G/G_{max} curves tend to compare better with Darendeli (2001) curves at lower I_p values and with Ishibashi & Zhang (1993) at higher I_p values. In contrast, Fig. 8(b) displays a generally poor agreement between all three damping ratio sets.

At high confining pressures, agreements vary from poor to fair at different strain levels (see Fig. 8(c) and (d)) and clear relative trends are hard to establish. Nevertheless, Ishibashi & Zhang (1993) curves predict a slower modulus reduction and lower damping ratio when compared to Darendeli (2001) curves. As for low confining pressures, the only exception occurs in the ξ curves at large strains for very low I_p values. Furthermore, Fig. 8(c) and (d) show that the Vucetic & Dobry (1991) curves generally compare better with the Ishibashi & Zhang (1993) and the Darendeli (2001) curves for $I_p \geq 50$ and $I_p < 50$, respectively.

6 COMPARISON WITH THIRD PARTY LABORATORY DATA

Okur & Ansal (2007) produced modulus reduction and damping ratio curves from laboratory testing of soils sampled at several locations in Turkey, following the Kocaeli earthquake in 1999. The samples were taken at depths of 2.0m to 23.55m and consist of normally/slightly overconsolidated low and high plasticity silts and clays (I_p values from 9% to 40%). Multi-stage stress-controlled cyclic shear tests were undertaken with a cyclic triaxial apparatus, equipped for reliable small (1.0×10^{-5}) to large (1.0×10^{-1}) strain measurements.

In this study, the test data sets (not the produced curves) from three plasticity indexes are compared with the most adequate Vucetic & Dobry (1991) and Darendeli (2001) empirical curves. For the Darendeli (2001) curves, values of f =0.5Hz and N=3 are chosen for consistency with the testing procedure reported by Okur & Ansal (2007). Also, since information regarding overconsolidation ratio and confining pressure was not available for every sample, reported values of OCR=1 and p'=200kPa are adopted. Furthermore, the plasticity indexes are chosen amongst the values available for the Vucetic & Dobry (1991) curves. Similarly, test data from three distinct confining pressures, corresponding to the I_p=12% sample, are compared with the corresponding Darendeli (2001) curves.

6.1 Laboratory Data for Varying Plasticity Index

In Fig. 9, the test data reveals the expected trends of increasing shear modulus and decreasing damping ratio with increasing plasticity index. As also expected, this effect reduces for higher I_p values.

From Fig. 9(a), it is clear that Vucetic & Dobry (1991) G/G_{max} curves display accurate values for the I_p=12% and I_p=27% data range. For I_p=40%, shear stiffness is slightly overpredicted (i.e. slower modulus reduction) for intermediate to large strains. In contrast, Fig. 9(c) shows that the Darendeli (2001) G/G_{max} curves are only accurate for the lowest I_p data set, slightly underpredicting shear stiffnesses for higher plasticity samples.

Fig. 9(b) shows that Vucetic & Dobry (1991) ξ curves compare fairly in terms of trends and accuracy. Consequently, a considerable underdamping is predicted at large strains. Similarly to the modulus reduction curves, Fig. 9(d) shows that the Darendeli (2001) ξ curves reproduce well the laboratory data trends. However, the empirical curves fail to replicate the test data values at small and large strains, resulting in underdamping and overdamping, respectively. It is interesting to note that the test data does not confirm the trend reversals at small and large strains that are present, albeit not very noticeably, in the Darendeli (2001) ξ curves.

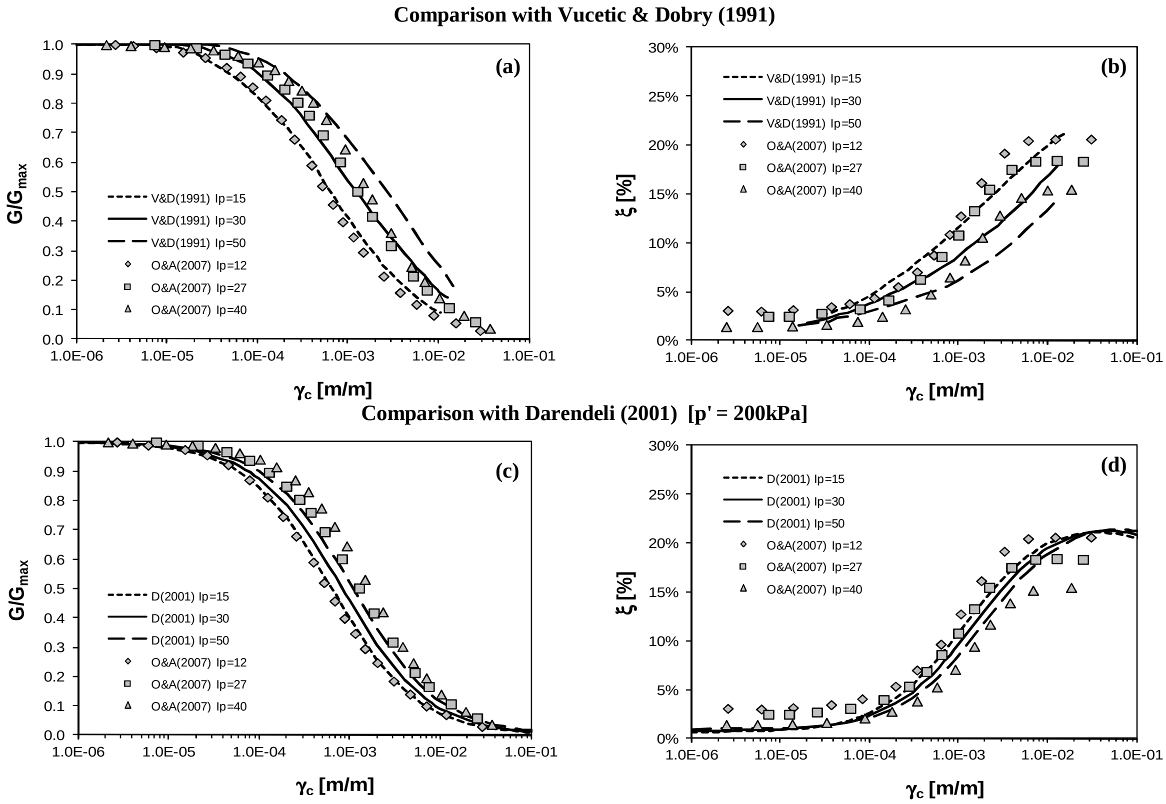

Figure 9. Empirical G/G_{max} and ξ curves comparison with laboratory data from Okur & Ansal (2007) Effect of I_p

Overall, the test data seems to suggest an impact of the plasticity index somewhere in between of what was proposed by Vucetic & Dobry (1991) and Darendeli (2001).

6.2 Laboratory Data for Varying Confining Pressure

Unfortunately, the information provided in Okur & Ansal (2007) did not allow for an extensive comparison of confining pressures. The reported data values were restricted to 150kPa, 200kPa and 300kPa and only for samples with a low I_p value (12%).

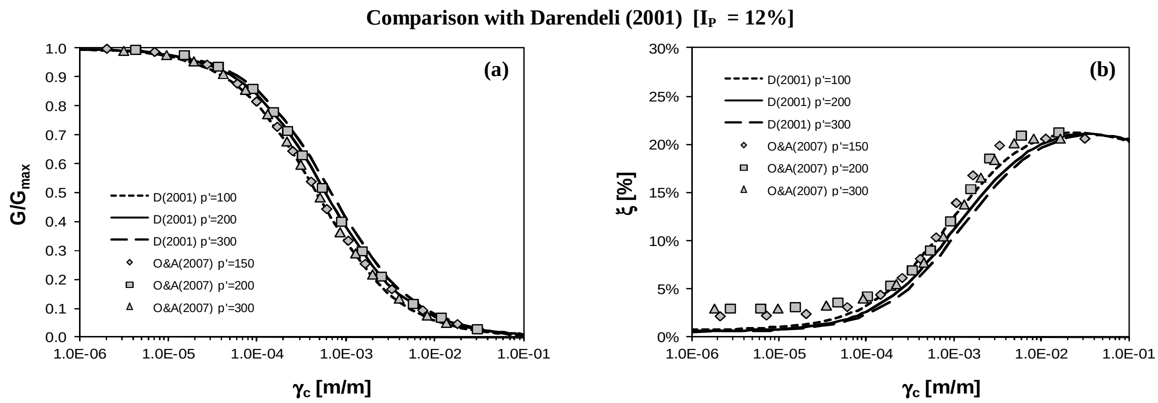

Figure 10. Empirical G/G_{max} and ξ curves comparison with laboratory data from Okur & Ansal (2007) Effect of p′

Fig. 10 shows that the impact of varying the confining pressure from 150kPa to 300kPa, at I_p=12%, is very small. This behaviour compares very well with the Darendeli (2001) curves. There is only some underdamping predicted at the small strain range (see Fig. 10(b)).

7 CONCLUSIONS

A critical review of three empirical families of curves was presented, followed by a comparative parametric study and a direct comparison with third party laboratory data.

It was shown that, under certain conditions, the Ishibashi & Zhang (1993) curves may require the adoption of additional restrictions in order not to violate two physical principles (i.e. $G/G_{max}>1$ and $\xi<0\%$). Therefore, for general application to engineering practice, Vucetic & Dobry (1991) and Darendeli (2001) curves appear to provide better alternatives. The Vucetic & Dobry (1991) family of curves only considers the influence of plasticity index. Moreover, it is based on laboratory data that is only representative for I_p values below 60%. Its applicability is restricted to a rather limited strain range and it does not capture all the trends in damping ratio observed under small strain amplitudes (later identified in Vucetic et al. (1998) and Lanzo & Vucetic (1999)). Therefore, and since confining pressure has a reduced impact at higher I_p values, these curves are probably better if applied to high plasticity soils (30%<Ip<60%) and, in the case of the damping ratio curves, only for intermediate to relatively large strains ($\leq 1.0\times10^{-2}$). The family of curves proposed by Darendeli (2001) seems to be able to capture all major effects across the entire strain range. However, the reduction in damping ratio curves observed at very large strains has never been captured so far in laboratory tests. Furthermore, the soil database used lacks representative data for values of I_p higher than 30% and for OCR larger than 1. As a result, these curves are more appropriate for non-plastic to medium plasticity soils.

This study equally showed that it is difficult to draw conclusions regarding the influence of I_p. Indeed, whether it results in a narrower (Darendeli (2001)) or a wider (Vucetic & Dobry (1991)) breath of curves, this issue needs to be confirmed by carrying out additional testing. This should take into account all the main factors (N, *f*, OCR, p', I_p) and should cover a representative selection of soils and in-situ stress states. Nevertheless, it is likely that plasticity index does have a larger impact than that proposed by Darendeli (2001), since this study was based on a lower I_p database. Additionally, the test data reported by Okur & Ansal (2007) appears to corroborate this hypothesis.

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