Nonlinear Time-History Seismic Analysis of Bridge Frame Structures

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SUMMARY:
A new method for finding member forces for statically indeterminate bridge frames has recently been published. The unique aspect of this new method is that it produces exact member-end-moments for statically indeterminate bridge frames from simple closed-form equations, without the need to setup and solve a system of simultaneous equations, as required in the stiffness method. The result is that the new method is 1000s of times faster than any of the currently available methods that depend on matrix manipulation.

In the paper the incremental form of the closed-form approach is presented and nonlinear time-history results for a typical bridge frame subjected to earthquake loading are compared to results from the stiffness method. The new closed-form approach is based on modified classical analysis techniques and is so fast and stable that bridge designers may now consider using nonlinear time-history analysis for the seismic design of typical highway bridge structures.

Keywords: Fast earthquake analysis, nonlinear, bridge, frame, plastic hinge

1. INTRODUCTION

Since the widespread availability of computers, the stiffness method has been the preferred approach to solve statically indeterminate bridge frame structures for a given applied loading. It is a numerically intensive solution scheme that involves solving many simultaneous equations couched in the language of matrix mathematics. When the structure remains linear-elastic and there are only a few different loading cases this matrix-based approach has demonstrated to be very effective for bridge frame structures, including flexural, shear and axial deformations for all the members that make up the frame. However, the stiffness method does not work so well for nonlinear time-history response under earthquake loading which requires the entire solution to be repeated, possibly 10,000 times in a row, representing measured ground accelerations at small time intervals for the duration of the earthquake. This is because the stiffness matrix changes under nonlinear response, requiring the solution of many simultaneous equations at each time interval, slowing down the solution time and causing severe iteration at large nonlinear excursions that can delay or completely stop the analysis before the earthquake record is completed.

Using the stiffness method it may take several hours to run one time-history seismic analysis of a multi-span bridge frame with plastic hinges at the column ends. If, say, 10 different earthquake records are to be run for design purposes then this approach becomes prohibitive on the design floor, resulting in the current state of using linear spectral analysis for seismic demands. In this paper a new approach is presented for nonlinear time-history analysis of bridge frames, requiring no simultaneous equations and no iterations. The incremental closed-form method has been programed by the author in FORTRAN. This provides a stable solution scheme that is 1000s of times faster than the stiffness method, with analysis completed in seconds rather than hours. Furthermore, the results of the new method are identical to the stiffness method.

Due to the shear speed and stability of the new approach, multiple earthquake motions can be performed in seconds on a laptop computer, making this method a viable option for everyday bridge design to replace linear spectral analysis. Currently, Caltrans bridge design engineers use nonlinear pushover analysis to assess displacement capacity but linear spectral analysis to determine displacement demands (Caltrans, 2011), and then verify that capacity is greater than demand. One advantage of the spectral approach is that all possible earthquake motions have been considered in developing the smooth design acceleration response spectrum (ARS) curve. Clearly one earthquake motion is not enough to verify that maximum demands have been found, as the structure may fall in a valley of an ARS curve that is developed from a single earthquake motion, underrepresenting what would happen if a different earthquake with more resonant frequency content were to shake the structure. And so it is with time-history analysis, where it is not reasonable to run just one earthquake motion.

The new nonlinear time-history method is based on a solution scheme presented in a series of recently published papers by Dowell (2009) and by Dowell and Johnson (2011; 2012). The closed-form approach was derived from classical methods and has been converted here to incremental form for nonlinear seismic analysis. At each time increment all final member-end-moments are found from closed-form equations, providing exact results with no need for simultaneous equations. The general approach was developed considering only flexural deformations (Dowell, 2009). This was modified (Dowell and Johnson, 2011) to also include shear deformations. Shear flow from torsion in multi-cell box-girder superstructures was solved exactly using a similar approach (Dowell and Johnson, 2012). There is an on-going effort to modify the closed-form equations to also allow for axial deformations. At this point in the development of the new method 2-D frame structures can be analyzed. In the future 3-D bridge frames will be considered.

In this paper the flexural response of a bridge frame with plastic hinges at the column ends is investigated. Shear flexibility could have been included as outlined by Dowell and Johnson (2011), with little change in speed or stability of the solution scheme. If only a continuous beam was being analyzed here then both flexural and shear terms would have been used. However, since a frame structure has bending, shear and axial deformations, it would have been disingenuous to include shear deformations while ignoring axial deformations. Although shear and axial deformations are typically small compared to flexural deformations, they should be included for completeness of the method in the future. In the oral presentation at the conference by the author, dynamic response of the new method will be compared directly to the stiffness method. Unfortunately, at the time this paper was submitted the final programming in FORTRAN of the dynamic behavior of bridge frames had not yet been finalized, but was close enough to determine the analysis speed. Therefore the following discusses the approach being developed and provides results for quasi-static monotonic and cyclic behavior of bridge frames using the new incremental closed-form method.

2. THEORY OF NEW METHOD

The engine of the new incremental approach presented here for bridge frames is the closed-form method developed by the author (Dowell, 2009). In this closed-form method, final equations are provided that give identical member-end-moment results to the stiffness method for a loaded bridge structure that has any number of spans. From these moments and statics, shears and axial forces are found. Of critical importance in the closed-form method is that no simultaneous equations are required for highly redundant bridge frames. The original paper (Dowell, 2009) considered only flexural deformations since shear and axial displacements are known to be small for civil engineering structures. There are some instances when shear deformations affect final member-end-moments for bridge structures and should be included in the analysis, such as for deep superstructures or wide columns. Shear flexibility has been successfully added to the closed-form method (Dowell and Johnson, 2011) and provides exact results that match the stiffness method when it includes bending and shear flexibility.

For continuous beams that have no axial load, the modified closed-form method gives all possible results that are provided by the stiffness method. And in this sense the new method can replace the stiffness method for these structures. However, members of a frame develop axial forces, and not just in the columns but also in the spans of the superstructure, indicating that axial deformations may be important for bridge frames. Regardless of their significance, for completeness of the new method all three types of deformations should be included. Thus a significant effort is currently underway to add axial flexibility to the closed-form method for bridge frames. With this addition the closed-form approach will provide all possible behaviors that the stiffness method captures, but without solving any simultaneous equations. Therefore if this inclusion of axial flexibility is successful, all force effects and deformations will be identical to the stiffness method. Due to the significantly reduced effort of the closed-form approach there may be no further need for the stiffness method for bridge frame structures.

2.1 Incremental Analysis

To perform nonlinear bridge analysis the closed-form approach by Dowell (2009) was modified to incremental form and programmed by the author in FORTRAN. The first step is a vertical gravity load analysis that solves for all final end moments with no sway of the frame followed by a sway analysis that balances the base shears. This is the state the bridge is in at the time of a future earthquake. An incremental lateral displacement is applied, resulting in fixed-end-moments at the column ends. Final member-end-moments that are associated with this incremental displacement are found directly from the closed-form equations. A second displacement increment is applied and again incremental member-end-moments are determined. Total end moments for the members are computed at each increment by summing all prior incremental results. Columns are modeled as elastic members with nonlinear moment-rotation springs at both ends, representing plastic hinges under cyclic loading. As proof-of-concept here, simple elasto-plastic hinges are used in this presentation, but more detailed hysteretic behaviors that realistically capture reinforced concrete columns under large cyclic seismic displacements, such as the Pivot Model developed by Dowell, Seible and Wilson (1998) can be readily included in the future.

For new seismic bridge design the column ends are detailed for plastic behavior as required by Caltrans (2011) and all other members are capacity-protected as discussed by Priestley, Seible and Calvi (1996) to remain essentially linear-elastic by providing more strength than maximum possible forces induced from full column plastic moments. Unwanted modes of column failure such as from shear are also capacity-protected from occurring. This ensures that the only nonlinear responses of a bridge frame that need to be considered are from plastic hinging at the column ends.

At some point in the incremental bridge frame analysis the moment demand at the top or bottom of one of the columns will exceed its moment capacity, and unless corrected the results will no longer represent the physical behavior of the bridge. One could change the stiffness at this point in the analysis and continue marching forward with more displacement increments, but the overshoot from the prior increment in moments and forces throughout the structure will not be corrected. In the incremental closed-form approach presented here, overshoot is corrected at the time that it occurs by backing up all member-end-moments using an event-scaling analysis (Priestley, Seible and Calvi, 1996). Thus the moment demand at the first plastic hinge location is returned to its plastic moment capacity and the same return ratio allows all other end moments in the columns and superstructure to be moved back to their proper values at the time the first nonlinear event occurred. Also, the frame displacement is returned to the point when the first nonlinear hinge develops. The program automatically adds a new increment at the first hinge location in the analysis. Therefore if only one nonlinear event occurred, the total number of increments would be one more than originally prescribed. If 50 nonlinear events develop then 50 more increments are automatically added to the analysis.

In addition to returning the end moments and frame displacements to the correct values at the time of the first nonlinear event, the rotational stiffness of the first plastic hinge is changed. In this case it is reduced to zero for perfectly elasto-plastic behavior. Therefore, no additional moment develops while the hinge rotates in future increments. Prior to reaching the plastic moment the hinges are rigid, resulting in no additional deformations to the columns and structure. This is reasonable as the column members properly capture the elastic behavior from the superstructure centroid to the top of footing.

Once the results have been correctly moved back to the first nonlinear event, a new displacement increment is applied and incremental moments found. This continues until a second plastic hinge develops. As with the first event, moment and displacement results are scaled back to the values of the second event and the rotational stiffness of the second plastic hinge is changed to zero. This process continues throughout the loading sequence, with added increments provided automatically at all nonlinear events. For a bridge frame with three single-column-bents, a total of six events will occur in a monotonic push, developing plastic hinges at the tops and bottoms of all columns. As shown later, one advantage of the elasto-plastic column ends is that the total force capacity of a bridge frame can be found by hand as simply the sum of the plastic column shears, each determined by adding the end column plastic moments and dividing by the column length. Note that this works even if the columns have initial moments and shears from gravity loads, as the column shears must cancel under this load case.

3. MONOTONIC PUSHOVER ANALYSIS

To demonstrate the new incremental approach the example bridge frame given in Fig. 3.1 is analyzed. All members are reinforced concrete with modulus of elasticity E = 4,000 ksi and moment of inertia I = 100 ft^4. In keeping with new bridge design practice, plastic hinges can form only at the column ends, with moment capacity at each hinge given in Fig. 3.2. Plastic moment capacities are the same for the top and bottom of the left column while they are different for the right column, as indicated in the figure. An initial gravity load of 5 kips/ft is given along the center span, as shown in the figure below. Thus there are member-end-moments at the column ends prior to lateral loading, which reduces moment capacities for one column while increasing moment capacities in the other column.

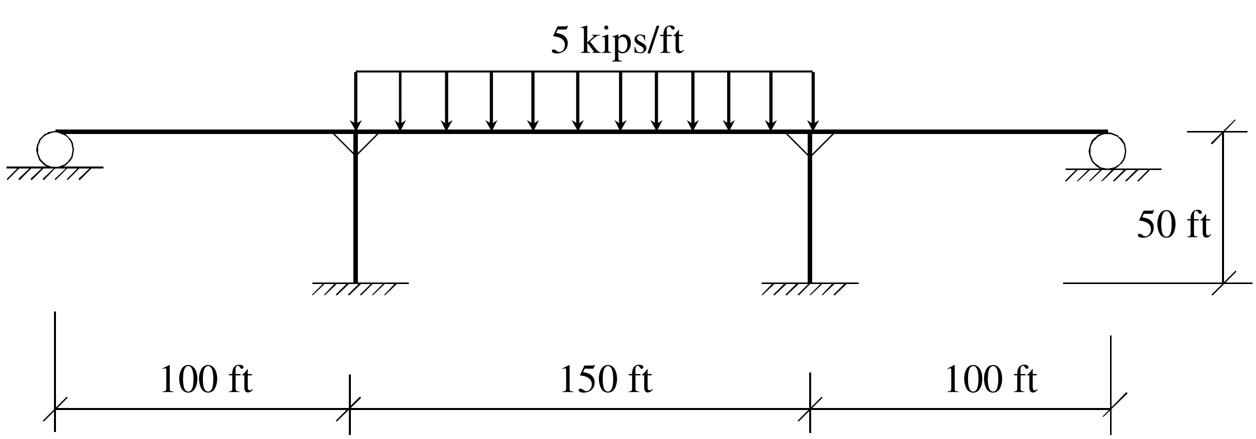

Figure 3.1. Bridge frame for example

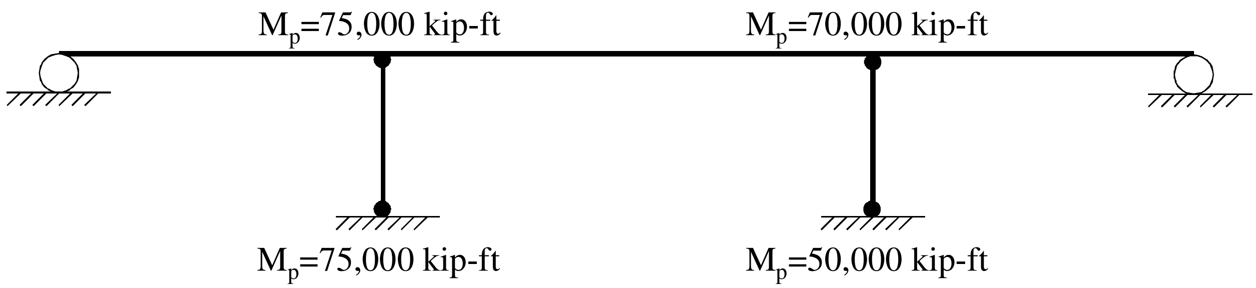

Figure 3.2. Plastic moment capacities at the four column plastic hinges

A monotonic pushover analysis shows that all four plastic hinges develop on the push to three ft of lateral displacement (Fig. 3.3). As each event occurs, the slope of the force-displacement graph suddenly changes due to the softening of the structure. Only a few increments are prescribed since the program automatically finds the event locations and backs up results to those points. The program adds an extra increment for each nonlinear event that occurs. Combining the closed-form approach of Dowell (2009) with an event-scaling analysis there are no simultaneous equations to be setup and solved, no overshoot of results and no required iterations, making this approach extremely fast. Furthermore all values are exactly correct. Development of the last plastic hinge indicates a mechanism has formed at 5,400 kips, with no additional lateral force capacity. This ultimate force is quickly verified by hand by summing the plastic shear capacities of the two columns; the plastic shear capacity of the left column is V_p = (75,000 +75,000)/50 = 3,000 kips and the plastic shear capacity for the right column is V_p = (70,000 +50,000)/50 = 2,400 kips. Summing these plastic shears gives the same ultimate force of 5,400 kips found from the incremental analysis (Fig. 3.3). This demonstrates that there is no overshoot in the monotonic incremental analysis.

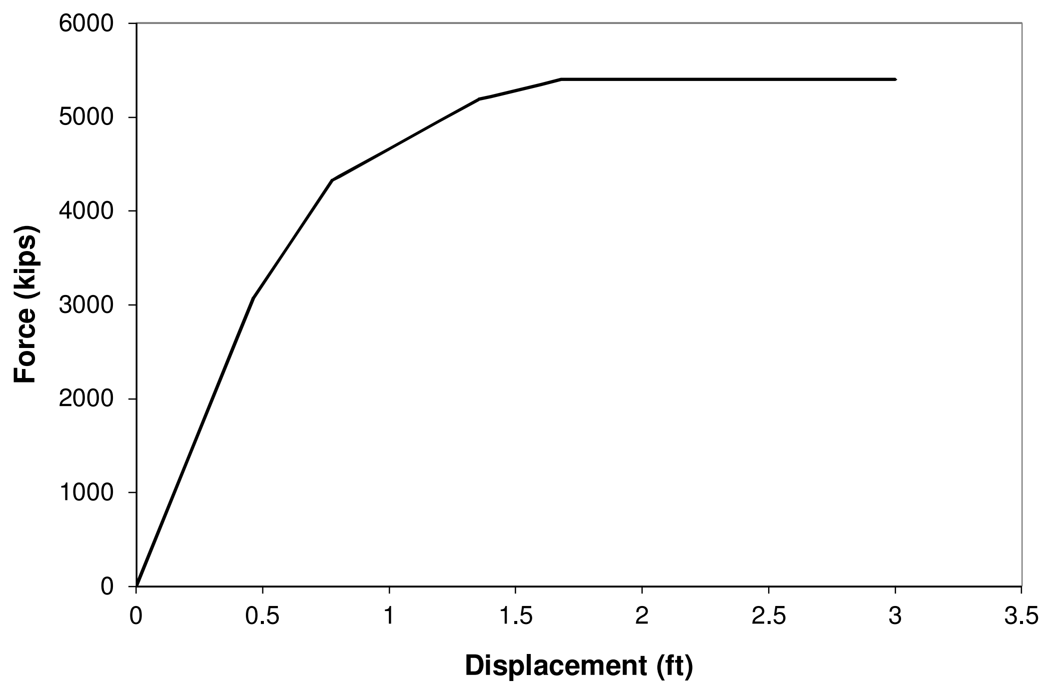

Figure 3.3. Monotonic push to 3 ft with all plastic hinges forming

4. CYCLIC PUSHOVER ANAYSIS

The same example bridge and plastic hinge capacities given in Figs. 3.1 and 3.2 for monotonic loading are used here to demonstrate the cyclic capabilities of the new incremental closed-form approach. Various lateral displacement loading patterns are applied to the structure with force-displacement hysteretic results for the bridge frame shown in Figs. 4.1 through 4.4. These figures clearly demonstrate that there is no overshoot in the cyclic response, with maximum force capacity of the frame not exceeded in either direction. Exact return to the prior force value when cycled to maximum displacements from earlier cycles also shows the elimination of force overshoot. The displacement pattern for each analysis is indicated in the figure caption. Total lateral force capacity for all the chosen loading patterns equals the monotonic force capacity of 5,400 kips in each direction (see Figs. 4.1 through 4.4). Fig. 4.2 shows that upon unloading from a plastic state and reloading the force-deformation response stays on the same initial stiffness line when no plastic hinge locations are currently plastic. Fig. 4.3 has a symmetrically increasing cyclic displacement pattern with initial displacement reversals that occur prior to all of the hinges forming, as indicated by the lower force levels at these early displacement cycles.

For the displacement loading pattern given in Fig. 4.4 there were 10 increments input for each push displacement. Increments are indicated by a symbol in Fig. 4.4. As discussed before, for every nonlinear event the program automatically creates a new increment, with the result that more increments are present in the final output than asked for in the input. There would be 4 additional increments for each loading direction if all four plastic hinges develop for the example bridge given in Fig. 3.1. A total of 80 increments for six displacement pushes are shown from the analysis in Fig. 4.4.

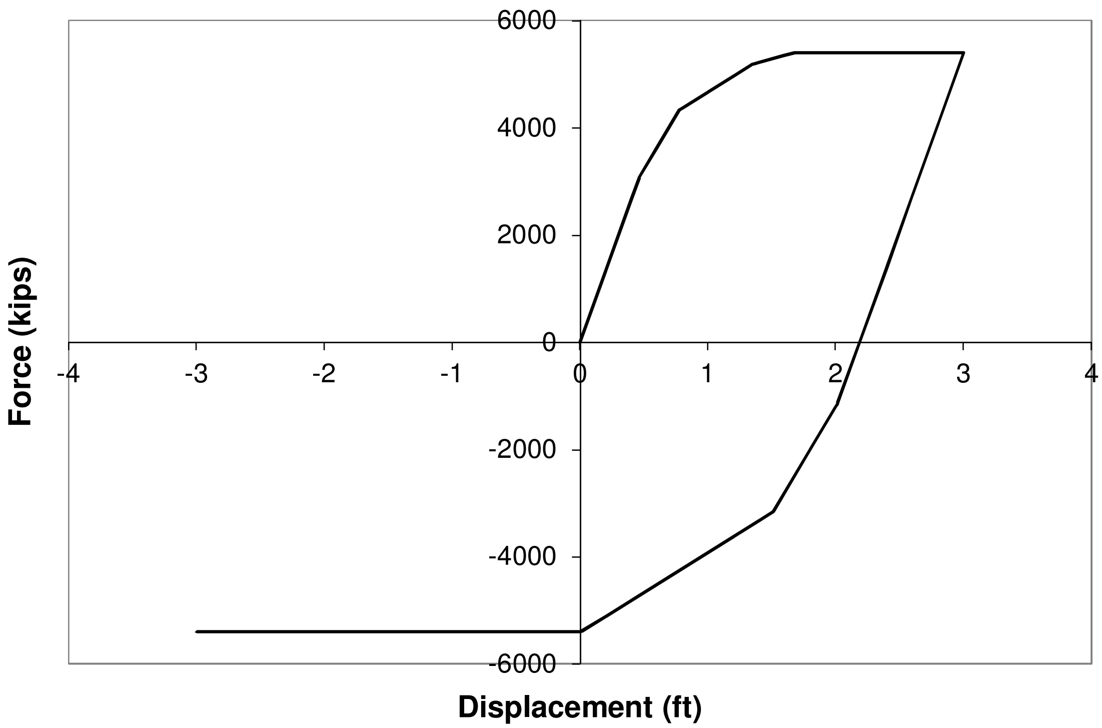

Figure 4.1. Frame displacements (in ft) of 3, -3

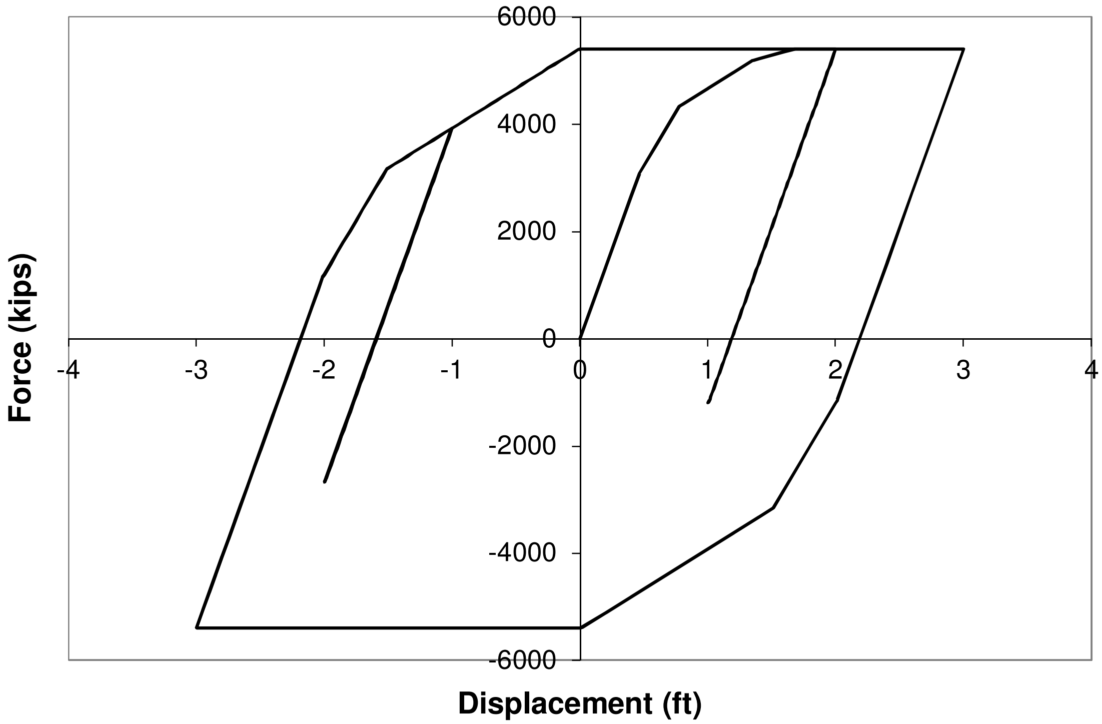

Figure 4.2. Frame displacements (in ft) of 3, -3, -1, -2, 2, 1

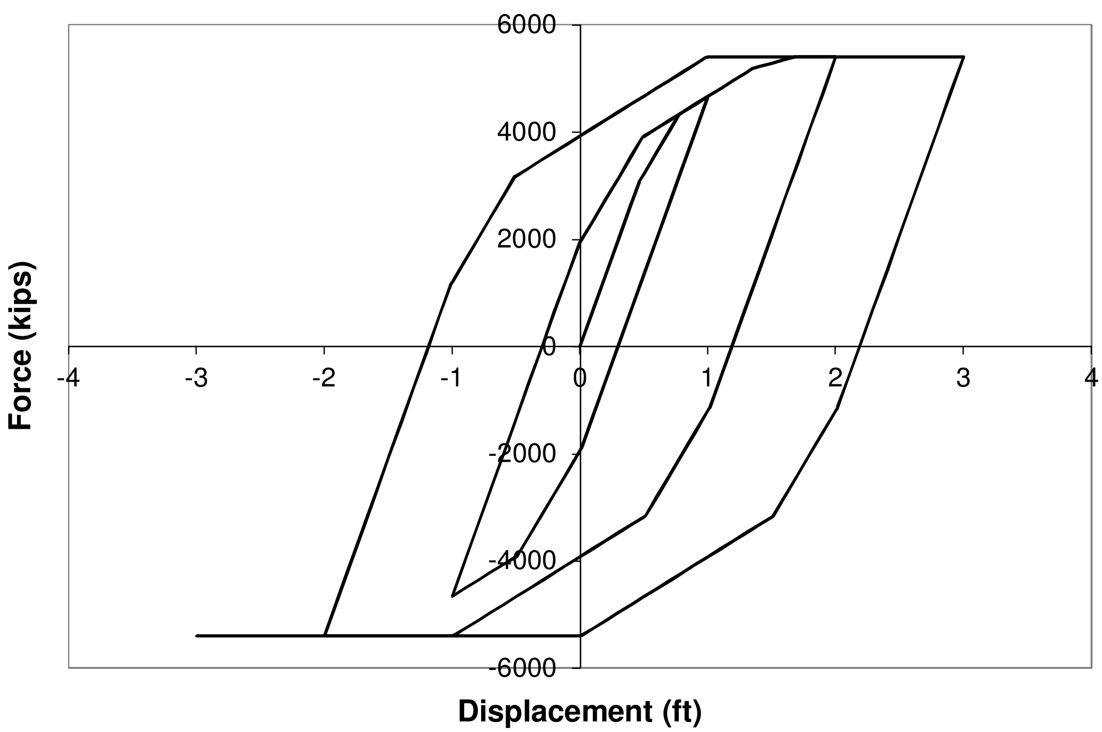

Figure 4.3. Frame displacements (in ft) of 1, -1, 2, -2, 3, -3

With 10 prescribed increments for each of the 6 displacement pushes, for a total of 60, the program added a total of 20 more increments, one for each nonlinear event. If all 4 hinges had developed in each of the 6 displacement pushes then 24 increments would have been automatically added by the program. Since only 20 increments were added, there were 4 times that a plastic hinge did not form by the time of displacement reversal. This is confirmed by viewing the total force levels which did not reach the mechanism capacity of 5,400 kips at some displacement pushes in Fig. 4.4.

The ability of the new incremental closed-form method to determine exact results with only a few increments in each displacement direction is a major success. Iteration is not required for the program to find the exact position of a nonlinear event, with only a single point added for each occurrence. The program then continues on with the chosen displacement pattern, adding required points only at nonlinear events. For seismic loading the number of predefined increments is much greater than given here and is typically defined by the number of measured incremental acceleration values at small time intervals.

For a measured earthquake record there are typically from 2,000 to 10,000 loading increments depending on duration and time increment. As with the cyclic quasi-static behavior of a structure presented here in displacement control, the nonlinear dynamic response of a bridge frame from earthquake loading uses a combination of the closed-form approach of Dowell (2009) and event-scaling analysis. Since no iteration and no simultaneous equations are required, the early comparisons show that the solution time for earthquake loading is 1000s of times faster than from the stiffness method.

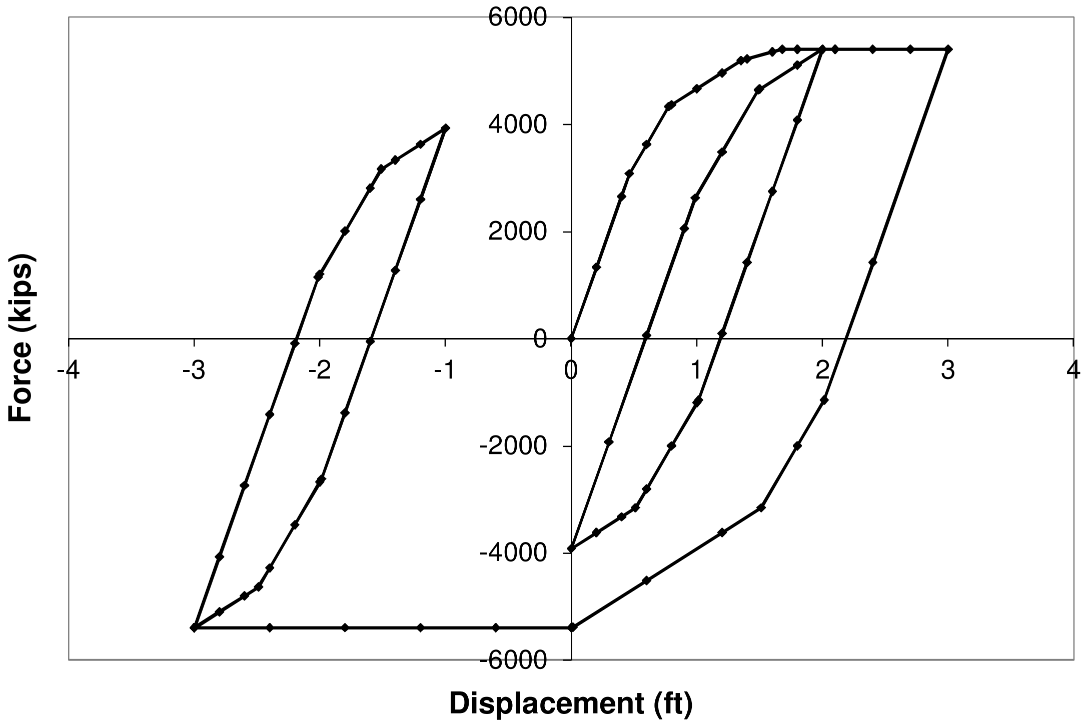

Figure 4.4. Frame displacements (in ft) of 2, 0, 3, -3, -1, -3 with each increment indicated

5. CONCLUSIONS

A new method has been developed for cyclic quasi-static and nonlinear time-history analyses using the closed-form approach developed by the author (Dowell, 2009) at each increment of loading. The solution at each loading increment is based on a linear-elastic analysis of a bridge frame. However, rather than setting up and solving the many simultaneous equations at each step as required in the stiffness method, the incremental results from the closed-from approach are provided in simple expressions. This makes the new method extremely fast compared to all other methods that depend on matrix manipulation at each increment for the 1000s of load increments used in nonlinear time-history seismic analysis. For example, a bridge frame model that has 100 degrees of freedom and 10,000 base acceleration time steps has one million simultaneous equations to be solved (not all at the same time). Using the new approach there are no simultaneous equations for the same problem and yet the results are identical.

At this point in the development of the new method, simple plastic hinge models have been provided at the column ends for 2-D bridge frames. Currently the FORTRAN program considers only flexural deformations. Adding shear flexibility to the computer program would be straight-forward as the original closed-form equations (Dowell, 2009) have been modified to allow for shear (Dowell and Johnson, 2011). However, to add shear deformations without also including axial deformations for a frame structure seems somewhat inconsistent. And since axial flexibility has not yet been fully developed for inclusion in the closed-form equations, shear and axial deformations have not been added to the program.

It is expected that the FORTRAN program will be fully functioning for nonlinear time-history dynamic analysis by the time of the conference and oral presentation by the author, and so earthquake results and time comparisons to the stiffness method will be presented at that time. Future, long-term additions to the method that will not be ready by the time of the conference may include (1) P-Delta effects that soften the structure, (2) transition from 2-D to 3-D analysis, (3) hysteretic models for different types of plastic hinges such as the Pivot Model by Dowell, Seible and Wilson (1998) and (4) interaction between axial load and moment-rotation behavior of the plastic hinges.

The new method combines classical analysis approaches with modern computing to create a super-fast analysis scheme that is stable and exact. Due to its speed, stability and ease of use, bridge design engineers will have the necessary tools to perform nonlinear time-history analysis for seismic design rather than the elastic spectral method currently employed. Multiple earthquakes can be scaled and run, and maximum demands determined in seconds using the new method compared to hours using the stiffness method. Furthermore the new method does not get stuck iterating which is often a problem in nonlinear incremental analysis using the stiffness method. Ultimately, seismic bridge design will be more realistic using nonlinear time-history analysis as the structure does physically respond nonlinearly under moderate to large seismic attack, and will result in safer bridge structures. For some time, Caltrans has used nonlinear analysis to assess displacement capacity of a bridge frame but linear-elastic tools to determine displacement demands. This is clearly an orange and apple comparison that needs to be resolved now that the appropriate tools are available.

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