Lateral Stiffness of Reinforced Concrete Moment Frames with Haunched Beams

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SUMMARY
Tapered elements in general and haunched beams in particular have been traditionally difficult to model in a practical manner. Leading commercial software worldwide for structural analysis such as ETABS or STAAD-Pro started to include them in their element libraries near year 2000; however, often the software´s manual does not describe the details of the numerical modelling. As many structural engineers worldwide use this software, it is of paramount importance to evaluate how accurate the solutions obtained with commercial software are, particularly for building in seismic zones, where reasonable estimates are crucial for displacement-based design methods. In this paper the approximations obtained with commercial software for a set of RC frames with symmetric haunched beams under lateral loading are reported when compared to those obtained with a traditional beam theory. It is shown that the modelling used in commercial software is in general reasonable, but it leads to an underestimation of the lateral displacements.

Keywords: reinforced concrete haunched beams, moment frames, lateral displacements.

1. INTRODUCTION

Haunched beams are used in bridges and buildings for many reasons (i.e., Tena-Colunga 1994), among them as they favor a more efficient use of materials to clear a given span or to provide a reasonable clear height for the stories of buildings.

Structural engineers devoted to the design of real-life structures have a need for practical but accurate enough tools to help them design properly complex structures. However, for many years, the most practical aid that they had for the elastic analysis of haunched beams was the handbook of frame constants for nonprismatic members ("Handbook" 1958) published by the Portland Cement Association (PCA), where some hypotheses were taken to simplify the problem. Despite their usefulness, it was later found that using the frame constants of this handbook could lead to significant errors, especially for deep haunches (El-Mezaini *et al.* 1991, Tena-Colunga 1996).

Likewise, many researchers worldwide have worked in the past four decades with the goal of providing an accurate elastic modeling of tapered beams, although few of them kept in mind that their proposed formulations must be practical enough in order to be implemented later in the software used by design engineers. Among the proposed solutions worth mentioning are those based upon classical beam theory (i.e., Just 1977, Schreyer 1978, Eisenberger 1985, Tena-Colunga 1996), the calculus of variations (i.e., Medwadovski 1984, Brown 1984), the transfer stiffness method (Luo *et al.* 2007), or the finite element method (i.e., Rajasekaran 1994, Shooshtari and Khajavi 2010, Failla and Impollonia 2012).

Some recent studies have focused in trying to further improve existing finite element formulations for Bernoulli-Euler and Timoshenko beams (Shooshtari and Khajavi 2010, Failla and Impollonia 2012). Nevertheless, their numerical examples, primarily single tapered beams with complex variations along their longitudinal axis, are only compared to standard finite element solutions. On this regard, it has been previously shown that the approximations obtained with formulations based upon classical beam theory (i.e., Tena-Colunga 1996) for T-haunched beams in single frame models (Tena-Colunga 2003) are good enough for practical purposes when compared to the results obtained by others (Balkaya 2001) with three-dimensional finite element models.

Indeed, tapered elements in general and haunched beams in particular have been traditionally difficult to model in a practical manner and that was the main reason that most commercial software did not include them in their elements libraries for many years. In fact, it was until near year 2000 when leading commercial software worldwide for structural analysis such as ETABS (since version 6) or STAAD-Pro started to include them in their element libraries. However, in some cases, the minimal technical information provided within the software´s manual (i.e., Bentley-2008 2008) does not describe the details of the numerical modeling. It is not completely clear to the user whether the modeling is based on rigorous traditional methods or is it just an approximation based upon the calculus of variations. As many structural engineers worldwide use this software, it is of paramount importance to evaluate how accurate the solutions obtained with commercial software are when compared to those obtained with a recognized method already proposed in the literature. It is particularly important for building in seismic zones, where reasonable estimates are crucial for displacement-based design methods.

Therefore, in this paper the approximations obtained with commercial software for a set of RC frames with symmetric haunched beams under lateral loading are reported when compared to those obtained with a traditional beam theory when shear deformations are included as presented by Tena-Colunga (1996). The parametric study is reported in detail in Martínez-Becerril (2011) and it is briefly described in following sections.

2. MODELING OF NONPRISMATIC BEAMS WITH COMMERCIAL SOFTWARE

As mentioned earlier, two commercial programs for structural analysis were evaluated in this study: ETABS as per version 9.6.0 and STAAD-Pro as per release 2007. Whereas the information provided in the reference manual for ETABS is reasonable to understand how the modeling of non-prismatic sections is done, the information available in the reference manual for STAAD-Pro gives no clue about the selected modeling.

According to the analysis reference manual provided for ETABS version 9.6.0 (CSI-2005 2005), one can model non-prismatic beams by dividing the element length into any number of segments; these do not need to be of equal length. Non-prismatic section properties are interpolated along the length of each segment from the values at the two ends. The variation of the bending stiffness may be linear, parabolic, or cubic over each segment of length. The axial, shear and torsional properties all vary linearly over each segment. Section properties may change discontinuously from one segment to the next. If a shear area is zero at either end, it is taken to be zero along the full segment, thus eliminating all shear deformation in the corresponding bending plane for that segment. Therefore, as described by the analysis reference manual of ETABS, the modeling of non-prismatic beams is an approximation based upon the calculus of variations.

According to the technical reference manual for STAAD-Pro release 2007 (Bentley-2008 2008), cross-sectional properties of tapered I-sections are calculated from the key section dimensions, and these properties are subsequently used in analysis. The user must enter the depths of the web section; the depth of the web section at starting node should always be greater than the depth of section at ending node, then, the user must provide the member incidences accordingly. Therefore, a linear tapering of the web of an I section is apparently rigorously modeled in STAAD-Pro, according to a classical beam theory, but the provided information in the technical reference manual is not clear on this regard. It is worth noting that uniformly distributed moments cannot be assigned to tapered members for analysis in STAAD-Pro. STAAD-Pro has the limitation of modeling strictly tapered elements for I sections only, but approximations for T and rectangular sections can also be achieved, as the user is allowed to provide different thicknesses and widths for the top and bottom flanges.

3. DESCRIPTION OF THE MODELS OF STUDY

Symmetric haunched beams are frequently used in reinforced concrete moment-resisting framed (RC-MRF) office buildings in Mexico City (Fig. 3.1). Given that the earthquake hazard of Mexico City is high, it is important to evaluate the approximations obtained with commercial software for RC-MRFs with haunched beams under lateral loading.

Figure 3.1. Reinforced concrete buildings with haunched beams under construction in Mexico City

In order to evaluate such approximations, a parametric study was designed considering the most common dimensions and characteristics currently used in Mexico City for such office buildings. Therefore, the following was considered for the regular and symmetric RC-MRFs with haunched beams under study (linear tapering of the web depth), as schematically depicted in Fig. 3.2: (a) three building heights: 5, 10 and 15 stories, with a typical story height of 3.5 meters (11.48 ft), (b) 2-bay frames and 3-bay frames, (c) four different lengths or span (L) for the bays: 7.0, 8.5, 10 and 12 meters (22.97, 27.89, 32.81 and 39.37 feet), (d) two different proportions of the haunching length (L_h) with respect to the beam span (L): L_h/L=1/3 and L_h/L=1/5 (Fig. 3.3) and, (e) two different proportions of the haunching depth (h_{max}) with respect to the minimum depth of the beam (h_o): h_{max}/h_o=2 and h_{max}/h_0=3.

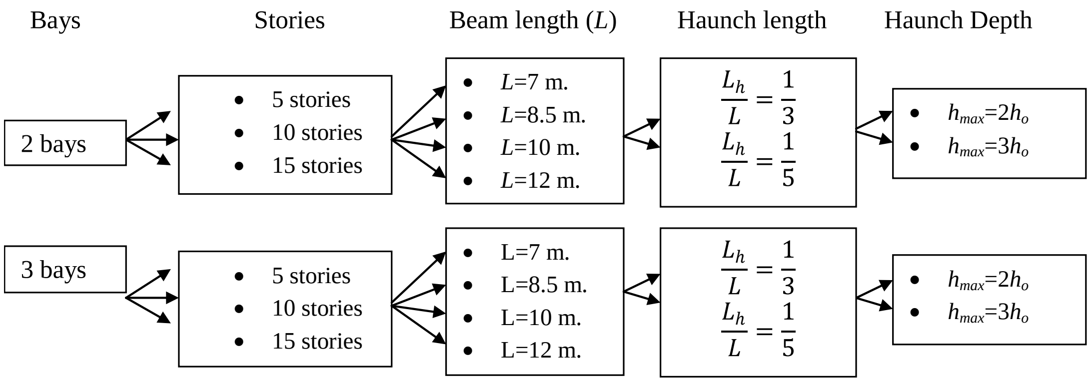

Figure 3.2. Summary of the conducted parametric study for the RC-MRFs with haunched beams

It is worth noting that taking into account recommendations from engineering practice, for simplicity, it was considered that $h_{max}=L/10$ for all models. All reinforced concrete beams were considered of having a T-cross section, with a web width $b=h_0$, a flange thickness t=10 cm (4 inches) and a flange width b_f=b+16t. It is worth noting that dimensions for the flange of the T cross section are based on considering the contribution of the slab as an equivalent flange. The width and thickness of the equivalent flange are those specified by the building code of Mexico City for stiffness modeling under lateral loading and to account indirectly for shear lag effects. For simplicity, all columns were assumed of square cross sections having a width equal to $h= h_{max}=L/10$. Therefore, the conducted parametric study involved 96 different frames (Fig. 3.2).

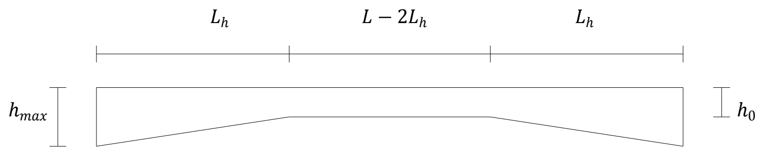

Figure 3.3. General geometry of the studied symmetric haunched beams

4. ANALYSES AND RESULTS

An inverted lateral load distribution was considered for the static analysis. For all frames, the total applied lateral load at the roof level was 100 Ton (220.26 kips) evenly distributed at both corner nodes (Fig. 4.1). Therefore, the applied base shear was 300 Ton (660.79 kips) for the five-story models (Fig. 4.1), 550 Ton (1,211.45 kips) for the ten-story models and 800 Ton (1762.11 kips) for the fifteen-story models.

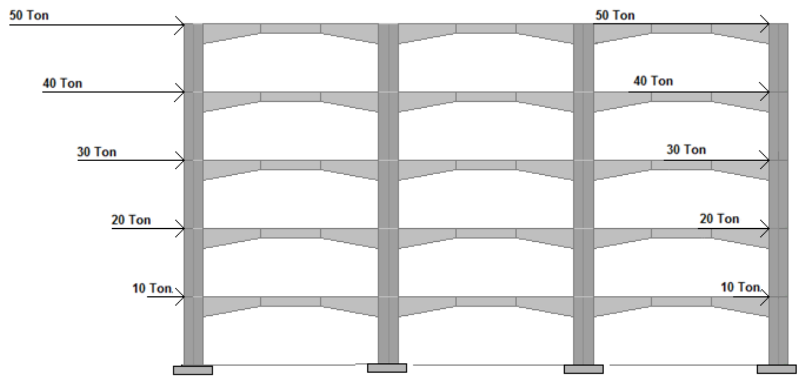

Figure 4.1. Applied lateral loads for the five-story models

Haunched beams of the studied RC-MRFs were modeled with ETABS using three segments per haunch beams. Two modeling options were used: a) linear variation of the bending stiffness ("ETABS-1") and, c) cubic variation of the bending stiffness ("ETABS-3"). This was done as one may assume *a-priori* that the best approximation would be obtained with the cubic variation of the bending stiffness, as it is a closer modeling to the way the bending stiffness varies in a linearly tapered haunched beam, and the worst approximation would be obtained with the linear variation.

Because STAAD-Pro as per version 2007 has just one modeling option (apparently linear tapering of I beams), this was the modeling used and is identified as "STAAD". The T section was approximated by specifying a very small thickness and a width equal to the web width for the bottom flange while modeling rigorously the top flange. This modeling yields in obtaining the same moment of inertia and almost the same shear area that a rigorously modeled T-section.

In DRAIN-2DX software (Prakash *et al.* 1992), the user is allowed to specify particular stiffness coefficients to model tapered elements and haunched beams. Therefore, for the traditional beam theory where shear deformations are included, the stiffness coefficients of the complete haunched beam were computed as presented by Tena-Colunga (1996) and then including them into the DRAIN-2DX input files. It is worth noting that the stiffness coefficients for the linearly varying haunched beams of T cross sections under study could also be retrieved using the tables developed by Tena and Zaldo, which are already published in the literature (Tena-Colunga 2007). This modeling option is identified as "THEORY" when discussing the results.

4.1 Lateral Displacement Profiles

The lateral displacement profiles for the 96 studied frames are compared and reported in detail in Martínez-Becerril (2011) and some of the most interesting results are depicted in Figs. 4.2 to 4.8 and briefly discussed in following paragraphs. To ease the comparisons, in each graph all lateral displacements are normalized with respect to the roof displacement of the studied frame obtained under the traditional beam theory (“Δ_{THEORY}”). The following short notation is used in Figs. 4.2 to 4.8 to identify the frame characteristics: *jj*N*i*B, where *jj* identifies the number of stories (5, 10, 15) and *i* the number of bays (2 or 3). It is worth noting that the following short notation is also used in Figs. 4.2 to 4.8, following the common notation originally proposed in PCA´s handbook of frame constants for nonprismatic members (“Handbook” 1958) and in Tena-Colunga (1996). The normalized left (α) and right (β) haunched lengths are defined as:

$$\alpha = \beta = \frac{L_h}{L} \tag{4.1}$$

whereas the relative haunched depth (γ) is defined as:

$$\gamma = \frac{h_{\max} - h_0}{h_0} \tag{4.2}$$

The results obtained for the three-bay frames are depicted in Figs. 4.2 to 4.8, as they are very similar (almost identical) to those obtained for the two-bay frames (not shown), as it can be deducted by comparing the obtained curves depicted in Fig. 4.7 (three-bay frames) and Fig. 4.8 (two-bay frames) for the fifteen story models with bay lengths L=10m and L=12m.

The following general observations can be done by comparing Figs. 4.2 to 4.8:

a) Excellent approximations (between 98% and almost 100%) are obtained under the STAAD-Pro modeling (STAAD) with respect to the traditional beam theory with shear deformations (THEORY) for all the models under study. The number of stories and the bay length (L) has a negligible impact on the approximations. However, it seems that the relative haunch depth (γ) has a reduced impact on the approximation, as better approximations are obtained for γ=2 when compared with γ=1 (compare for example models 5N3B, 10N3B and 15N3B for α=β=1/5), that is, apparently approximations improve as γ increases. Nevertheless, it seems that the most significant variable that impacts the approximations are the normalized haunch lengths α and β. Better approximations are obtained for α=β=1/5 when compared with α=β=1/3 (compare for example models 5N3B, 10N3B and 15N3B when γ=2), that is, apparently approximations improve as β decreases.

b) Relatively poor approximations (between 62% and 80%) are obtained under the ETABS-1 modeling (linear variation of the bending stiffness) with respect to the traditional beam theory with shear deformations (THEORY) for all the models under study. The number of stories and the bay length (L) has a reduced impact on the approximations. However, the most significant variable that impacts the approximations is the relative haunch depth (γ). Better approximations are obtained for γ=1 when compared with γ=2 (compare for example models 5N3B, 10N3B and 15N3B for α=β=1/5 or α=β=1/3), that is, apparently the level of approximation decreases as γ increases. Approximations are also affected by the normalized haunch lengths α and β. Better approximations are obtained for α=β=1/5 when compared with α=β=1/3 (compare for example models 5N3B, 10N3B and 15N3B when γ=2), that is, apparently approximations worsen as β increases.

c) Reasonable approximations (between 78% and 88%) are obtained under the ETABS-3 modeling (cubic variation of the bending stiffness) with respect to the traditional beam theory with shear deformations (THEORY) for all the models under study. The number of stories and the bay length (L) has a reduced impact on the approximations. In coincidence with ETABS-1 modeling, the most significant variable that impacts the approximations is the relative haunch depth (γ). Better approximations are obtained for γ=1 when compared with γ=2 (compare for example models 5N3B, 10N3B and 15N3B for α=β=1/5 or α=β=1/3), that is, apparently the level of approximation decreases as γ increases. Likewise, approximations are also affected by the normalized haunch lengths α and β. Better approximations are obtained for α=β=1/3 when compared with α=β=1/5 (compare for example models 5N3B, 10N3B and 15N3B when γ=1), that is, apparently approximations improve as β increases.

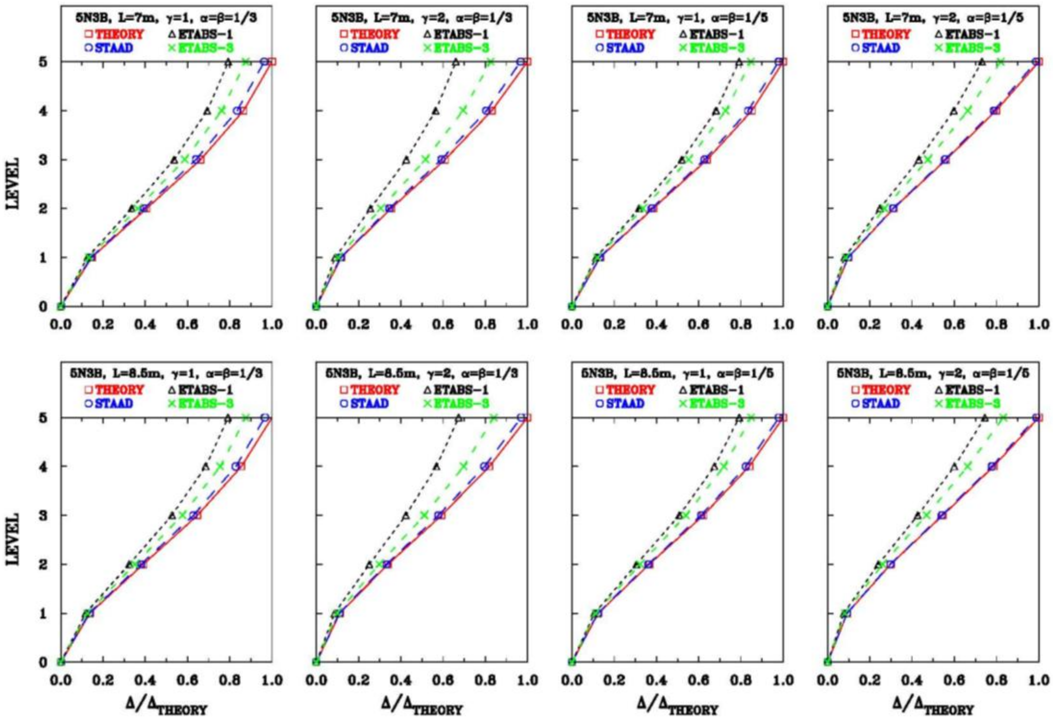

Figure 4.2. Normalized lateral displacements for five-story, three-bay frames with bay widths L=7m and L=8.5m

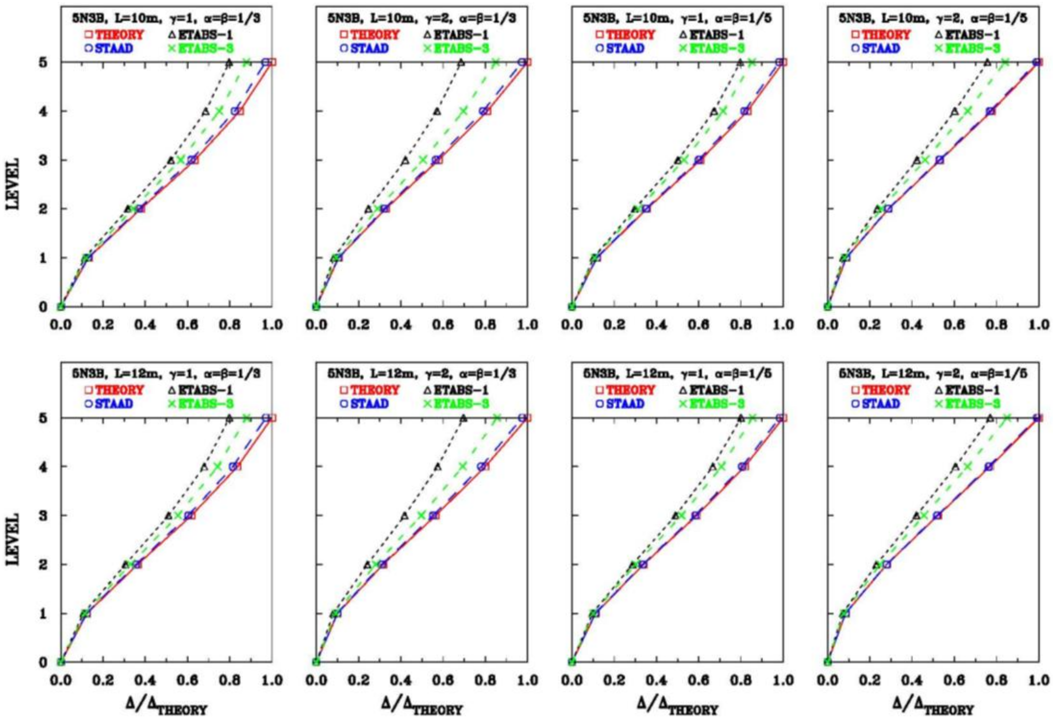

Figure 4.3 Normalized lateral displacements for five-story, three-bay frames with bay widths L=10m and L=12m

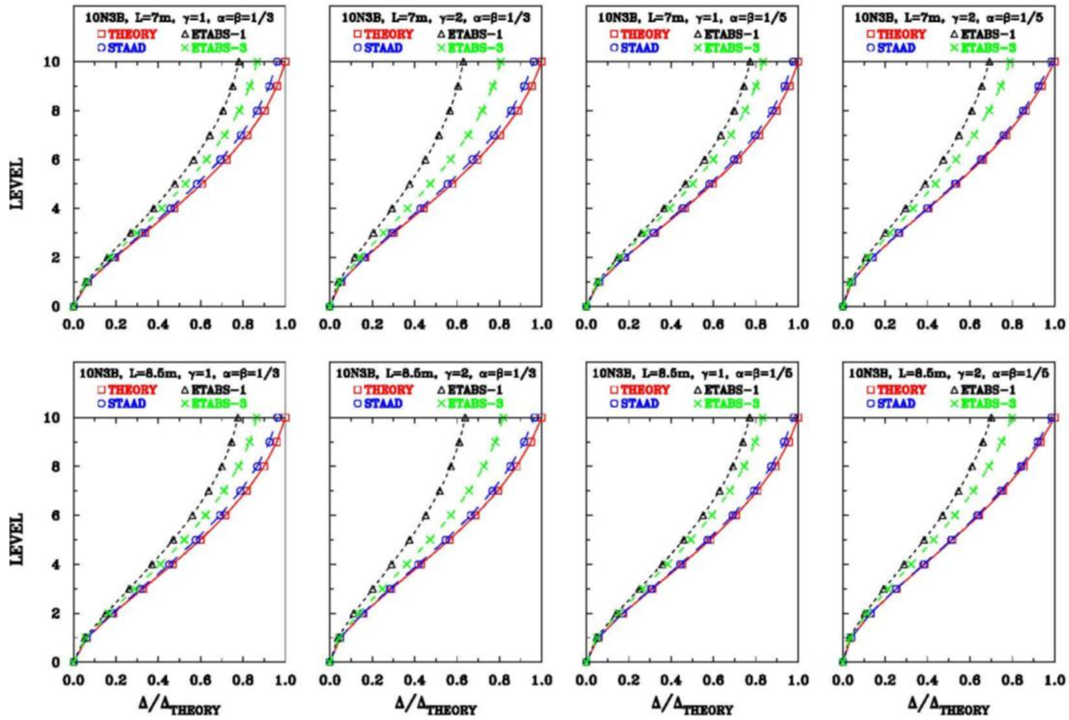

Figure 4.4. Normalized lateral displacements for ten-story, three-bay frames with bay widths *L*=7m and *L*=8.5m

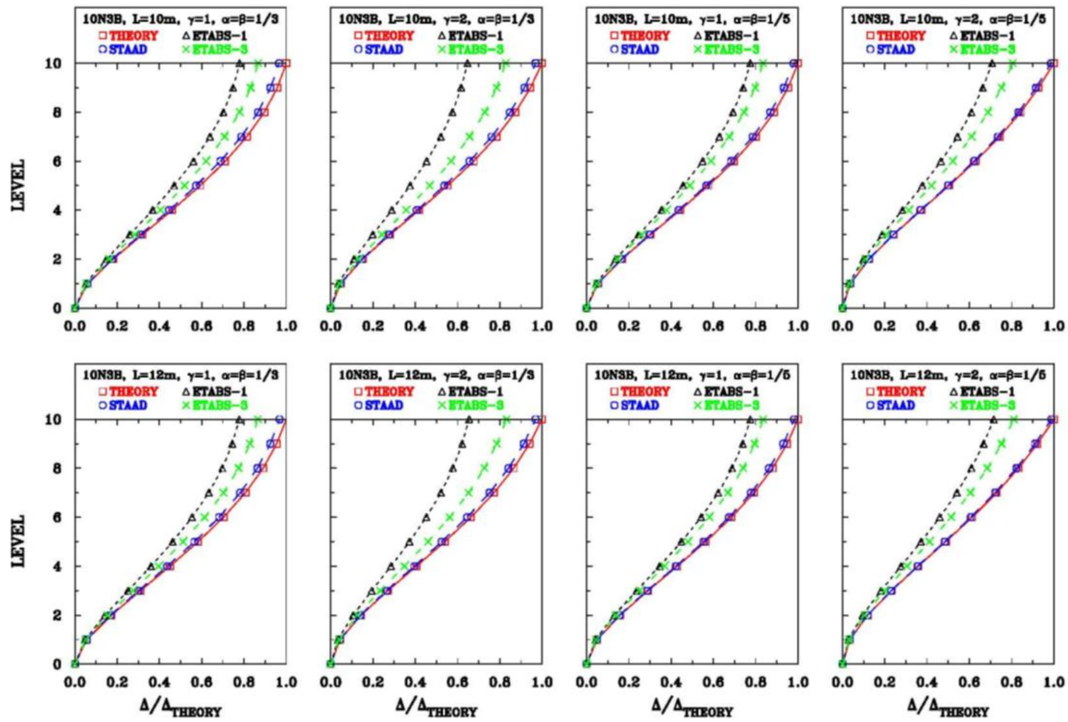

Figure 4.5. Normalized lateral displacements for ten-story, three-bay frames with bay widths *L*=10m and *L*=12m

4.2 Bending Moments

To complete the picture, bending moments at the (haunched) beam ends obtained with the software under study were normalized with respect to those obtained with a traditional beam theory with shear deformations. The results are reported in detail in Martínez-Becerril (2011). The most noticeable differences were observed in frames 10N2B and 10N3B; however, for space constraints, the normalized bending moments for frame 10N2B (α=β=1/3, γ=2 and *L*=7m) are only briefly discussed. Approximations for the bending moments are very good (from 0.98 to 1.02) for the STAAD-Pro modeling for all haunched beams along the height of the building, increasing the normalized bending moment from top to bottom stories. Similarly, approximations for the bending moments are good (from 0.82 to 0.99) for the ETABS-3 modeling for all haunched beams along the height of the building, having much closer approximations at the bottom stories, that is, approximations improve from top to bottom stories. In contrast, approximations are inconsistent and not good enough (from 0.73 to 1.08) under the ETABS-1 modeling.

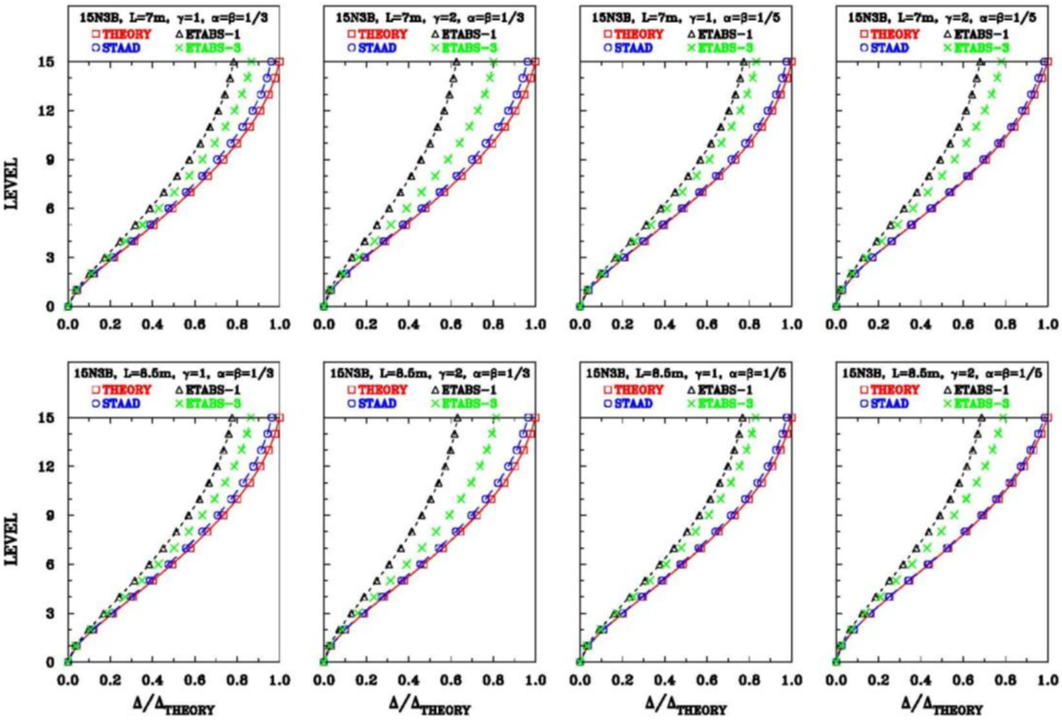

Figure 4.6. Normalized lateral displacements for fifteen-story, three-bay frames with bay widths *L*=7m and *L*=8.5m

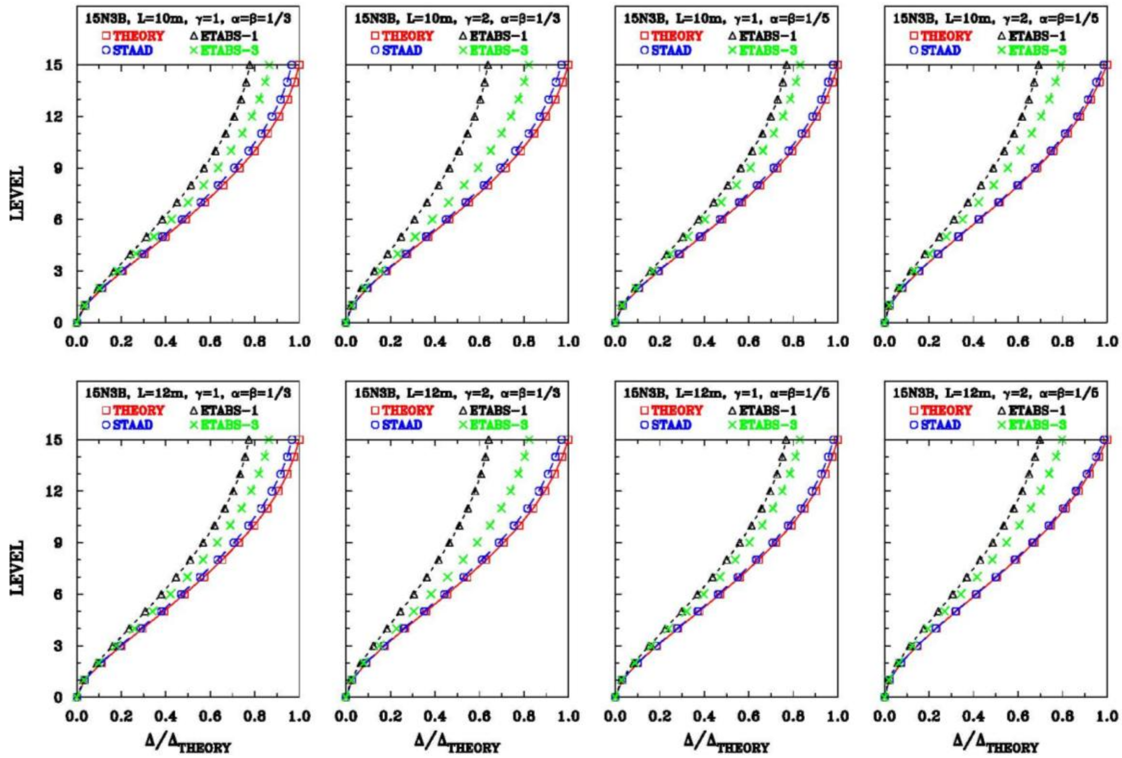

Figure 4.7. Normalized lateral displacements for fifteen-story, three-bay frames with bay widths *L*=10m and *L*=12m

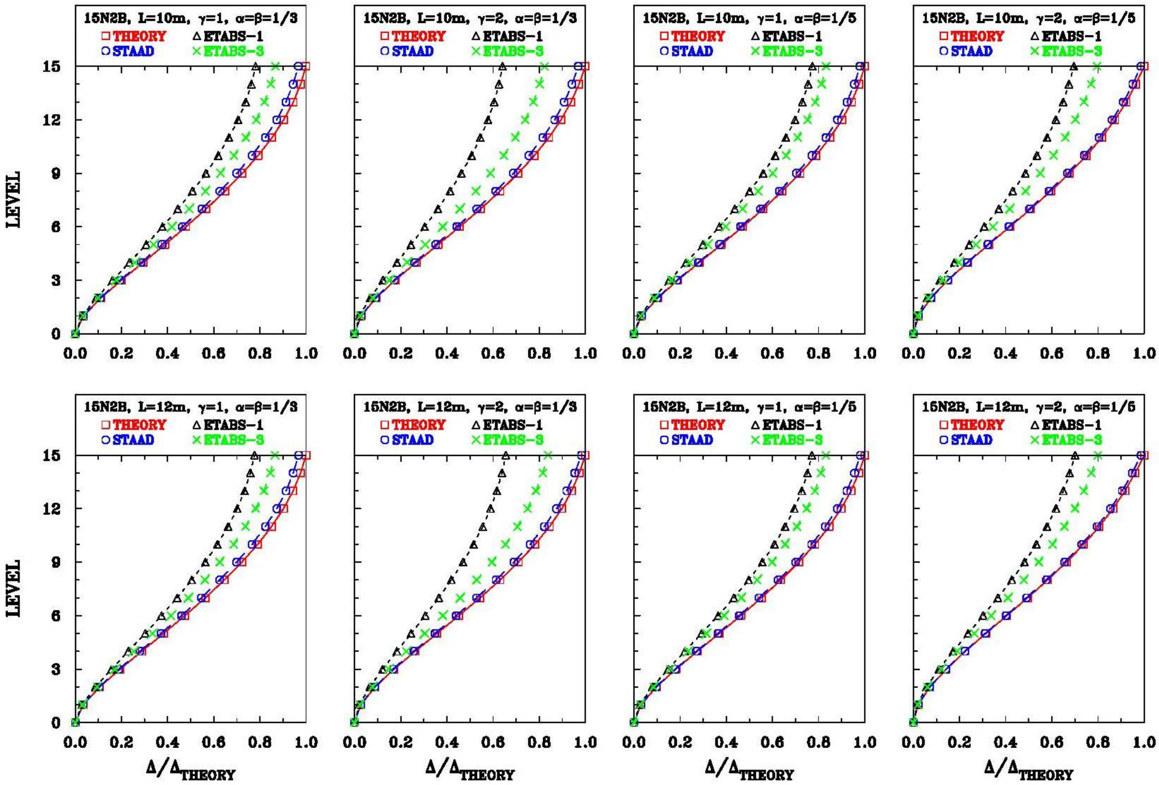

Figure 4.8. Normalized lateral displacements for fifteen-story, two-bay frames with bay widths *L*=10m and *L*=12m

5. CONCLUDING REMARKS

Many structural engineers worldwide use commercial software such as ETABS or STAAD-Pro to analyze structures with haunched beams. Therefore, it is of paramount importance to evaluate how accurate the solutions obtained with commercial software are when compared to those obtained with a recognized method already proposed in the literature, particularly for building in seismic zones, where reasonable estimates for the lateral displacements under earthquake loading are crucial for performance-based design methods.

In this paper, the approximations obtained with commercial software for a set of 96 RC-MRFs with symmetric haunched beams (linear tapering of the web depth) under lateral loading are reported when compared to those obtained with a traditional beam theory when shear deformations are included. Based upon the results of the reported parametric study, the considered building heights, number of bays and length of the bays (*L*) have a reduced impact on the approximations of the methods used in commercial software with respect to those obtained with a traditional beam theory for linearly tapered haunched beams.

However, the most significant variables that impact the approximations are the normalized haunch lengths α and β and the relative haunched depth γ, particularly for the modeling options of ETABS. For all the studied software, the level of approximation apparently decreases as γ increases. The level of approximation decreases as α and β increases for STAAD-Pro and when a linear variation of the bending stiffness is considered in ETABS. However, the level of approximation in ETABS increases as α and β increases when a cubic variation of the bending stiffness is used.

Based upon these observations and the availability of software, it can be recommended the following for the numerical modeling of RC-MRFs with symmetric haunched T-beams (linear tapering of the web depth) under lateral loading: (1) one can use with confidence STAAD-Pro for an accurate modeling of haunched T-beams of such frames (approximations between 98% and almost 100%), (2) when using ETABS, a cubic variation of the bending stiffness must be used in order to get good results for haunched beams, as reasonable approximations (between 78% and 88%) were obtained for haunched T-beams of RC-MRFs under such modeling and, (3) as relatively poor approximations (between 62% and 80%) were obtained with ETABS when a linear variation of the bending stiffness was used to model haunched beams, such modeling option should be avoided for haunched T-beams similar to those that were studied.

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