Vibration Attenuation and Frequency Band Gaps in Layered Periodic Foundation: Theory and Experiment

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Summary:
Guided by the recent advances in solid-state research in periodic materials, a new type of layered periodic foundation consisting of concrete and rubber layers is proposed. The distinct feature of this new foundation is its frequency bands; as a result, it cannot transmit motions falling in the frequency band gap, so the foundation itself is a seismic isolator. Using the theory of elastodynamics and the Bloch-Floquet theorem, the mechanism of band gaps in periodic composites is explained and a finite element model is built to show the isolation characteristic of a finite dimensional periodic foundation. Based on these analytical results, moreover, a scaled model frame and a periodic foundation were fabricated and shake table tests of the frame on the periodic foundation were performed. Strong vibration attenuation is found when the exciting frequency falls into the band gaps.

Keywords: Base Isolation; Frequency Band Gaps; Periodic Materials

1.INTRODUCTION

The design of buildings and other structures capable of withstanding earthquake events has been the research focus by engineers for many decades. A commonly accepted method for the design of earthquake resistant buildings and structures, however, has not been developed up to the present time. Fortunately, the traditional design methods, based only on the static structural strength with impact factors to account for dynamic loads, have been reviewed and gradually replaced by newer methodologies over the last three decades. Concepts of structural element ductility and the importance of shear resistance have contributed to the ability to effectively design structural elements and systems that are resistant to dynamic loadings associated with earthquakes. The use of passive and active systems has been proposed and implemented in an attempt to maximize the ability of the structure to resist and survive an earthquake event. Recent design methods have also been proposed, in which seismic base isolation is utilized as a method to resist seismic loadings. One strategy implemented to date has been the addition of a base isolation system (usually a layer with low horizontal stiffness or sliding elements) between the structure and the base (foundation) of the structure. This system attempts to modify the fundamental frequency of the structure, thereby decreasing its acceleration response. The strategy of adding an isolation system between the structure and the foundation will typically result in a structure with a much lower fundamental frequency than the original un-isolated, fixed-base frequency of the structure (Naeim and Kelly 1999; Zhou *et al.* 2006; Sayani and Ryan 2009). According to the acceleration design spectrum, a change in the fundamental frequency of a structure may reduce the acceleration response significantly, thereby enhancing the overall ability of the structure to withstand and survive the earthquake event. One significant drawback of a traditional base isolation system, however, is that the structure will usually have very large residual horizontal displacements relative to the foundation after the earthquake event. To reduce these residual displacements, supplementary dampers are often prescribed.

Recently, investigations in the field of solid-state physics have shown that certain crystal arrangements may be utilized to manipulate the energy or patterns of acoustic (mechanical) wave energy (Liu *et al.* 2000; Kittel 2005; Thomas *et al.* 2006; Xiao *et al.* 2008). These materials, termed phononic crystals, can be designed to produce specific gaps in the frequency response of the material. These gaps are termed “frequency band gaps”. (The term “phononic crystal” will be replaced in this paper by the term “periodic material” for the purpose of clarity). When the frequency of a wave falls within the range of the frequency band gap of a periodic material, the wave, and hence its energy, cannot propagate through the periodic material. Fig. 1 illustrates wave propagation in both periodic and non-periodic material from a single source (Torres and Montero de Espinosa 2004). The source generated wave (and energy) cannot propagate in the periodic material on the left side of the figure when the frequencies of the wave fall within the frequency band gap of the material. The source generated wave (and energy) can, however, propagate in the homogeneous, non-periodic material on the right side of the figure since the homogeneous material possess no frequency band gap.

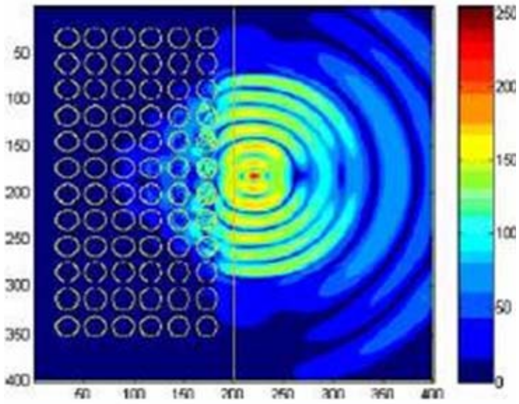

Figure 1. Wave Propagation in Periodic and Non-periodic materials (Torres and Montero de Espinosa 2004)

Guided by the recent advances in solid-state research and the concept of frequency band gaps in periodic materials, Shi and his co-workers utilize this periodic material as a new and innovative means in the seismic base isolator to mitigate the potential damage to advanced structures (Xiang and Shi 2009; Jia and Shi 2010; Xiang *et al.* 2010; Bao *et al.* 2012; Huang and Shi 2012). With this periodic material, the pattern of the earthquake event energy will be completely obstructed or changed when it reaches the periodic foundation of the structural system. This will result in the total isolation of the foundation from the earthquake wave energy because no energy will be passing through it. This total isolation will be of special significance to structures that house equipment that are highly sensitive to vibration such as research laboratories, medical facilities with sensitive imaging equipment, or manufacturing facilities specializing in the fabrication of electronic components. Further, the full isolation of emergency-critical structures such as bridges, facilities housing emergency response units or equipment, and power generation or distribution structures will result in better earthquake emergency response; consequently, there will be fewer compromises to the entire emergency response system.

Three types of periodic foundation systems are presented in Fig. 2. The diagram on the right in the figure presents a view of a section of the idealized foundation, including the overall constituents that impart the periodic nature to the foundation. Specifically, shown in Fig. 2*b* is the one-dimensional (1D) period foundation system fabricated periodically with two different materials, such as rubber and reinforced concrete. Shown in Fig. 2*c* is the two-dimensional (2D) periodic foundation consisting of a concrete matrix with multi-unit composite cylinders embedded periodically. In the case of three-dimensional (3D) periodic foundation system, as shown in Fig. 2*d*, a concrete matrix is loaded with composite balls arranged periodically in three directions.

The mechanism of band gaps in 2D or 3D periodic foundations is similar to that of 1D case. In this work, we will only take the one dimensional layered periodic foundation as an example to demonstrate how it works. Firstly, based on the theory of elastodynamics, the band gaps in the periodic foundations were analyzed by employing the Bloch-Floquet theorem (Kittel 2005). Subsequently, a parametric study was conducted to achieve a frequency band gap below 20Hz. Finally, we specialize the above concepts and results to a steel frame on the periodic foundation. The frame was subjected to seismic loading on a shaking table to simulate earthquake motion in three axes. The dynamic response of the frame shows that vibration can be attenuated significantly. If this idea is proved to be practical for civil structures, the impact on the economic savings and safety is enormous.

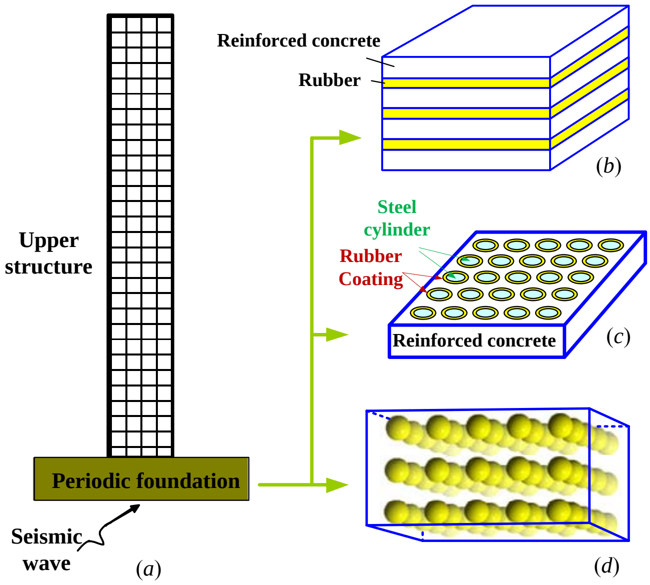

Figure 2. Schematic diagram of periodic foundations: (*a*) Periodic foundation with upper structure; (*b*) 1D layered periodic foundation; (c) 2D periodic foundation; (*d*) 3D periodic foundation.

2.THEORY AND ANALYTICAL RESULTS

Consider a periodic composite foundation of alternating layers of two isotropic materials arranged as shown in Fig. 3*a*.

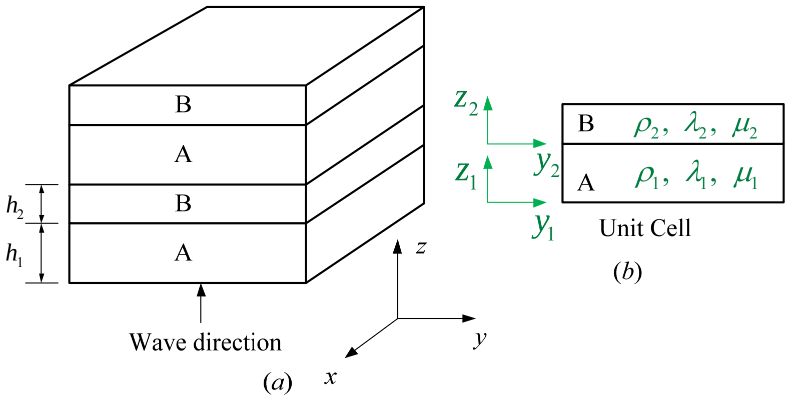

Figure 3. Configuration of a layered periodic foundation and its unit cell.

For the coordinate system specified, any two adjacent layers in the body comprise a unit cell, and this unit cell is completely invariant under a lattice translation along the z-direction. Each layer is infinitely extended in the plane. The thickness of Layer A and Layer B of a unit cell is h_1 and h_2, respectively. The periodicity of the foundation structure and displacement makes it possible to investigate the frequency band gap by studying one periodic unit (Kittel 2005), or unit cell as show in Fig. 3*b*.

2.1 Basic Equations and Solutions

Let v, w are displacements in y and z direction, respectively. Consider an elastic wave with propagation along z. The equation of motion in each layer is

$$\frac{\partial^2 u_i}{\partial t^2} = C_i^2 \frac{\partial^2 u_i}{\partial z_i^2} \tag{2.1}$$

where $u = w$ and $C = C_p = \sqrt{(\lambda + 2\mu)/\rho}$ for longitudinal wave (P wave), or $u = v$ and $C = C_t = \sqrt{\mu/\rho}$ for transverse wave (S wave). The coefficients λ and μ are Lamé's elastic constant, ρ is density. The index $i = 1, 2$ indicates layers A and B, respectively. For the free vibration analysis, a plane wave form solution to Eq. (2.1) is assumed to be

$$u_i(z_i, t) = e^{j\cdot(k\cdot z_i - \omega t)} u_i(z_i) \tag{2.2}$$

where k is the wave number, ω the angular frequency and j the imaginary unit. Substituting Eq.(2.2) into Eq.(2.1) yields

$$C_i^2 \frac{\partial^2 u_i(z_i)}{\partial z_i^2} + \omega^2 u_i(z_i) = 0 \tag{2.3}$$

The general solution of this equation is found as follows:

$$u_i(z_i) = A_i \sin\left(\omega z_i / C_i\right) + B_i \cos\left(\omega z_i / C_i\right) \tag{2.4}$$

There are four unknown constants A_1, A_2, B_1 and B_2 which are determined by boundary and continuity conditions. For the case of transverse waves, the normal stress σ_z in each layer is zero which automatically satisfies the continuous condition at the interface. The stress continuity across the interface requires that the shear stress τ is continuous. Therefore, the continuity of displacement and stress at the interface $z_2 = 0$ (or $z_1 = h_1$) are

$$u_1(h_1) = u_2(0), \tau_1(h_1) = \tau_2(0) \tag{2.5}$$

Due to the periodicity of the layered structure in the z direction, according to the Block-Floquet theorem (Kittel 2005; Xiang and Shi 2009), the displacement and stress must satisfy the following periodic boundary conditions

$$u_1(0)e^{jk\cdot h} = u_2(h_2), \tau_1(0)e^{jk\cdot h} = \tau_2(h_2) \tag{2.6}$$

where $h = h_1 + h_2$. The shear stress can be expressed as

$$\tau_i(z_i) = \mu_i \partial u_i / \partial z_i = \mu_i \omega \left[A_i \cos\left(\omega z_i / C_{ti}\right) - B_i \sin\left(\omega z_i / C_{ti}\right)\right] / C_{ti} \tag{2.7}$$

Substituting Eqs. (2.4) and (2.7) into Eqs. (2.5) and (2.6), we have

$$\begin{bmatrix} \sin\left(\omega h_1 / C_{t1}\right) & \cos\left(\omega h_1 / C_{t1}\right) & 0 & -1 \\ \mu_1 C_{t2} \cos\left(\omega h_1 / C_{t1}\right) & -\mu_1 C_{t2} \sin\left(\omega h_1 / C_{t1}\right) & -\mu_2 C_{t1} & 0 \\ 0 & e^{jk\cdot h} & -\sin\left(\omega h_2 / C_{t2}\right) & -\cos\left(\omega h_2 / C_{t2}\right) \\ \mu_1 C_{t2} \cdot e^{jk\cdot h} & 0 & -\mu_2 C_{t1} \cos\left(\omega h_2 / C_{t2}\right) & \mu_2 C_{t1} \sin\left(\omega h_2 / C_{t2}\right) \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \\ A_2 \\ B_2 \end{bmatrix} = 0 \tag{2.8}$$

A necessary and sufficient condition for the existence of a non-trivial solution to Eq. (2.8) is that the determinant of the coefficient matrix is zero. After the expanding the determinant, one obtains the dispersion relation for ω as a function of k , which is given by

$$\cos(k \cdot h) = \cos(\frac{\omega h_1}{C_{t1}})\cos(\frac{\omega h_2}{C_{t2}}) - \frac{1}{2}(\frac{\rho_1 C_{t1}}{\rho_2 C_{t2}} + \frac{\rho_2 C_{t2}}{\rho_1 C_{t1}})\sin(\frac{\omega h_1}{C_{t1}})\sin(\frac{\omega h_2}{C_{t2}}) \tag{2.9}$$

Because $\left|\cos(k \cdot h)\right| \le 1$, Eq.(2.9) is satisfied only when the value of the right-hand side is between -1 and +1. The band gaps are the values of ω and k that are the solutions to Eq. (2.9) but cos(k h) falls outside the range of -1 to 1. Following the same procedure, one can derive a similar result for the case of longitudinal waves. If materials A and B are the same, i.e. $C_{t1} = C_{t2} = C_t$ and $\rho_1 = \rho_2$, we get the dispersion relation for a homogenous material as $\cos(k \cdot h) = \cos(\omega h / C_t)$ where $\omega = kC_t$. For any value of k , we can find a frequency ω to satisfy this relation. This is the reason why there are no band gaps in a homogenous material. In general, the dispersion equation that defines the relation between ω and k is numerically solved to find values of ω and k Though the wave vector k is unrestricted, it is only necessary to consider k limited to the first Brillouin zone (Kittel 2005), i.e., $k \in [-\pi / h, \pi / h]$. In fact, if we choose a wave vector k_0 different from the original k in the first

Brillouin zone by a reciprocal lattice vector, for example $k_0 = k + 2n\pi / h$ where n is an integer, we may obtain the same set of equations because of the exponential $e^{jk_0 h} = e^{jk\cdot h}$ in Eq.(2.8). As an example, two common materials, concrete and rubber, are used to fabricate the periodic foundation. The thickness of both layers are $h_1 = h_2 =$ 0.2m. Fig. 4 presents the variations of frequencies ω for both transverse wave and longitudinal wave as a function of the reduced wave number k in the first Brillouin zone. The introduction of inhomogeneities implies the opening of a gap at the Brillouin zone boundary $k = -\pi / h$ or $k = \pi / h$. The curves are related to real wave numbers and the frequency band gaps are related to complex wave numbers (evanescent wave), which are not calculated and don't appear in Fig. 4. For transverse wave, the first four band gaps are: 6.6Hz-15.0Hz, 17.8Hz-30.0Hz, 31.6Hz-45.0Hz and 46.1Hz-60Hz. For longitudinal modes, the first band gap starts from 25.0Hz to 57.2Hz and the second band gap is 67.9Hz-114.3Hz. Notice that the rubber layers used in this design will not produce a large horizontal displacement as is the case for the rubber layers in the conventional laminated elastomeric seismic isolator. This is so because the motion is reflected from the periodic material. In the shake table test discussed below, the results show that the horizontal displacement at the rubber layer is quite small.

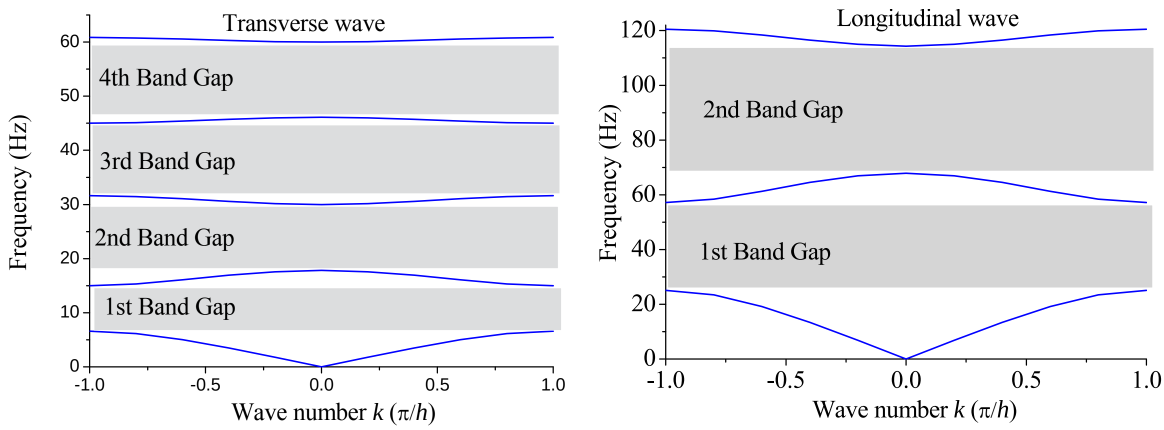

Figure 4. Dispersion curves

2.2 Numerical Harmonic Analysis

To show the isolation characteristic of the periodic structure, as shown in Figure 5a, an ANSYS finite element model is built for a three-story steel frame with the layered periodic foundation consisting of three reinforced concrete layers and two rubber layers. The system is strictly theoretical for preliminary analyses. Denote u_x, u_y and u_z as the displacement in *x*, *y* and *z* direction, respectively.

Firstly, a horizontal harmonic ground motion with amplitude δ_i in *x* direction is applied to the bottom of the foundation. The other DOFs of the bottom are fixed, i.e., u_y=0 and u_z=0. Fig. 5*b* gives a comparison between the transmitting Frequency Response Functions (FRFs) of the system with the periodic foundation and without foundation. The FRF in the vertical axis defined as $20\log(\delta_o / \delta_i)$ where δ_0 is the amplitude of displacement of the point A, as shown in Fig. 5*a*, at the top of the frame. Note that if the input and output displacements are the same then the log will be 0. Therefore, a negative number in FRF indicates a very effective isolation of the structure. As shown in Section 2.1, the first two band gaps for S-wave in the periodic foundation are 6.6Hz-15.0Hz and 17.8Hz-30.0Hz. In the band gaps, the response is significantly reduced. It is worth mentioning that the natural frequency of the frame falls into the band. As it is well known, the excitation of a building at or near the fundamental frequency of the building will result in resonance. Resonance of the structure will lead, in turn, to magnification of the overall building response and likely result in serious damage. When the excitation frequency is an integer multiple of the fundamental frequency of the building, resonance will also occur. The multiple band gaps may be thought of as the inverse of the fundamental frequency multiples and indicate that in a periodic structure the excitation input at the structure's fundamental frequency and its multiples will be blocked, avoiding resonance at both the fundamental frequency and its integer multiples.

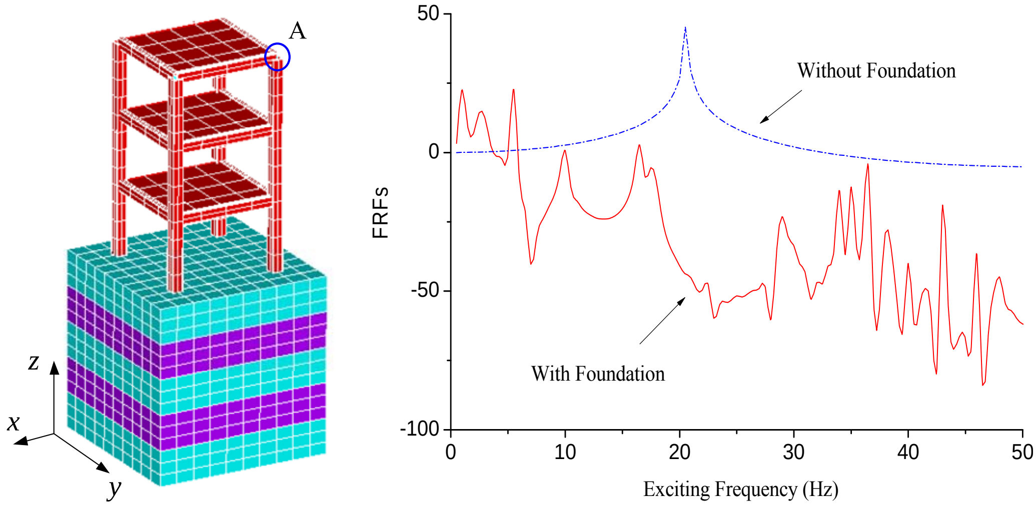

Figure 5. FEM model for a frame on a layered periodic foundation, and its FRFs subjected to a ground motion

Secondly, a vertical harmonic ground motion is applied to the bottom of the periodic foundation and the other DOFs of the bottom are fixed, i.e., u_x=0 and u_y=0. For P-wave, the first two band gaps in the foundation are: 25.0Hz-57.2Hz and 67.9Hz-114.3Hz. Again, the dynamic response is also reduced when the exciting frequency of the ground motion falls into the band gap. The results indicate that the periodic foundation can serve as a multidimensional seismically isolated foundation. When the periodic foundation is replaced by the previously mentioned 2D or 3D periodic foundation, vibration attenuation can be found in a similar way.

3.EXPERIMENTAL RESULTS

As presented above, the theoretical derivation for this concept is promising. Several challenges must be addressed in order to demonstrate the practical application of the concept. One such challenge is to achieve a good understanding of the effect of the high heterogeneity in materials at the scale of a full size base isolator. This is important because any heterogeneity is likely to hinder the desired performance of the isolation system. Another issue is concerned with the effect of debonding on the performance of the isolator. In order to begin to address these and other practical design issues and to validate the theoretical results, a scaled model and a periodic foundation were fabricated and recently tested using the shake table facility at the National Center for Research on Earthquake Engineering (NCREE) in Taiwan. As shown in Figure 6, Specimen A is a steel frame fixed on the shaking table. Specimen B is a steel frame of the same design as that of specimen A but is fixed on a 1-D layered periodic foundation. The concrete layers and rubber layers are bonded together by polyurethane (PU) glue for which the anti-pull strength is larger than 1MPa and the tear strength is larger than 6MPa. The contact area of the rubber and reinforced concrete (RC) slab is 1m^2. The resulting nominal anti-pulls and tear forces of the glue between the rubber and RC slab exceed 1000kN and 6000kN, respectively. The total mass of the test specimen with the periodic foundation is about 1.5 tons. Considering the maximum test peak ground acceleration (PGA) value as not greater than 3g for the test scheme, which is the acceleration limit of the shaking table, the maximum shear force across the rubber and RC base plate is about 45kN, which is much less than the nominal tear force of the PU glue. This assures that the PU glue prevented any loss of bond between the rubber and RC layers.

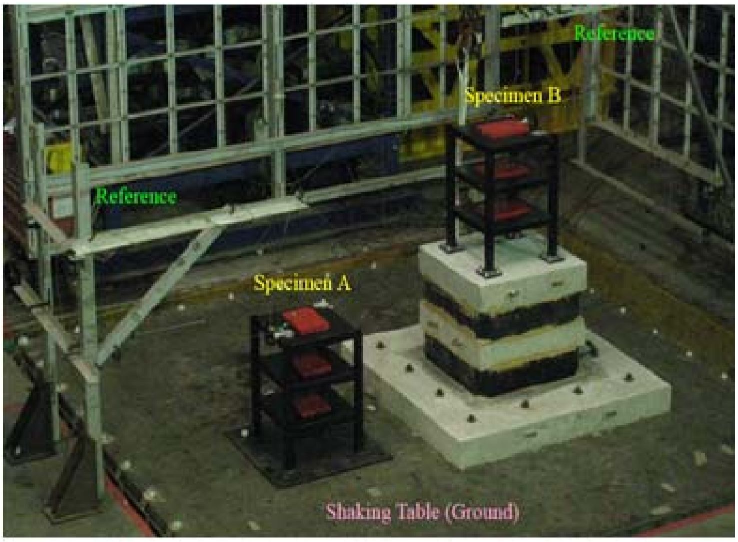

Figure 6. Test setup for specimens A (without periodic foundation) and B (with periodic foundation)

The dynamic responses induced by background vibration, i.e., ambient and engine vibration, was recorded. The main frequency of the background vibration is about 50Hz, which falls into the fourth band gap of the foundation as shown in Fig.4a. The horizontal acceleration time histories of the top of the frames are shown in Fig. 7. It is seen in Fig.7 that for the frame on a periodic foundation, the peak acceleration is reduced significantly, compared to that of the frame without periodic foundation. The result indicates that the periodic foundation can be used as a filter to isolate environmental vibration.

The shaking table used has six degree of freedom to simulate earthquake motion in three directions. Because of the symmetry of the specimen, a biaxial shaking table test was performed. The 1975 Oroville seismogram obtained from the PEER Ground Database (PEER 2011) was used as the input motion for the shaking table test. The nominal PGA is scaled to 0.418g. The main frequency of this seismogram falls into the 2nd band gaps of the periodic foundation (17.8Hz-30.0Hz). Acceleration and displacement responses of the specimens were recorded. The horizontal acceleration time histories of the top of the frames are shown in Fig. 8. It is found that for the frame on a periodic foundation, the peak horizontal acceleration is reduced by as much as 50%, compared to that of the frame without periodic foundation.

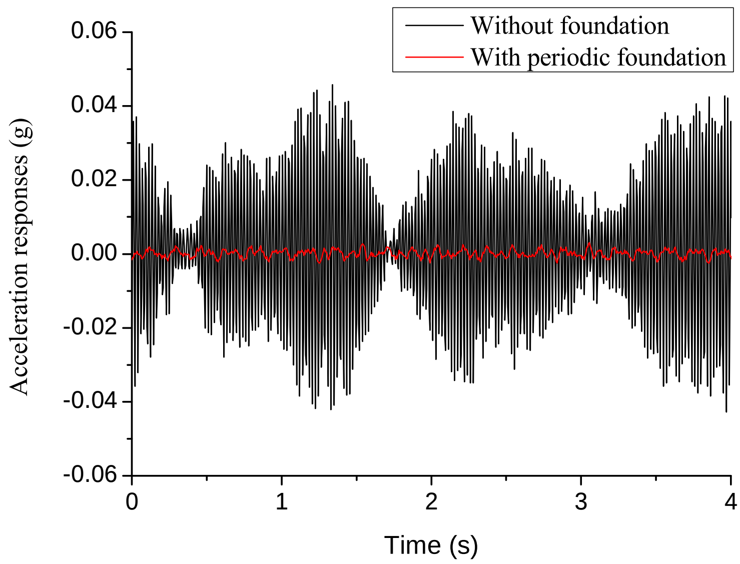

Figure 7. Dynamic responses induced by background vibration (ambient and engine vibration, about 50Hz)

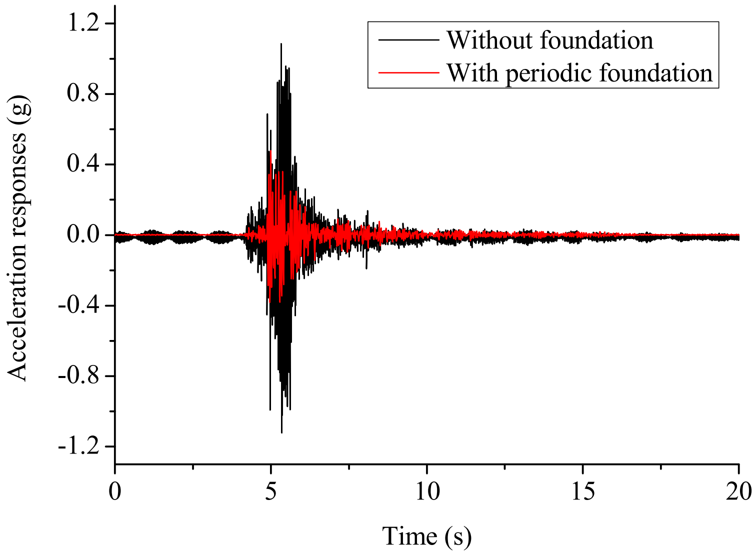

Figure 8. Acceleration responses of the top of the frames

As discussed previously, there are transverse and longitudinal band gaps in the foundation, so the vertical displacements of the specimens were recorded by Linear Variable Differential Transformers (LVDTs) to highlight that the foundation can isolate vertical vibration also. As shown in Fig. 9, the reduction of the vertical displacement in the frame on a periodic foundation is observed. These test results are promising and support that the periodic foundation can be served as a multi-dimensional base isolation.

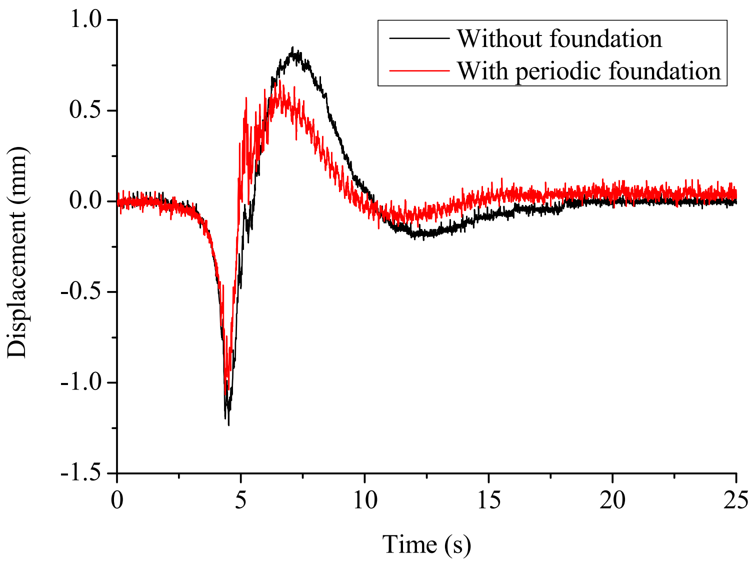

Figure 9. Vertical displacement time history of the top of the frames

4.CONCLUSIONS

A layered periodic foundation is designed to mitigate the potential seismic damages to structures. Different from traditional seismic base isolation, such as damping rubber bearings, lead-rubber bearings or friction pendulum bearings, the isolating mechanism of periodic foundation is that periodic composite can block and reflect seismic wave. The periodic foundation itself is also served as a isolator, so an additional isolator is not required. Moreover, the periodic foundation can be served as a multi-dimensional base isolation. By proper design, one can adjust the frequency band gap to match with the strong frequency range of the design earthquake, so the strong component of seismic waves will be blocked or reflected. This periodic foundation, then, can filter out the strong motion with specific frequencies that structures may be subjected to. Or, alternatively one can adjust the frequency band gap to match the fundamental frequency of the upper structure so that the motion transmitted from the periodic foundation does not contain this frequency. Theory and experimental results show that strong vibration attenuation is found when the exciting frequency falls into the band gaps.

ACKNOWLEDGEMENTS

This work is supported by the National Natural Science Foundation of China (Proj. Nos. 50808012 and 51178036), the Fundamental Research Funds for the Central Universities (2011JBM071) and US Department of Energy NEUP program (Proj. No. 3219). The opinions expressed in this study are those of the authors and do not necessarily reflect the views of the sponsors.

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