Prediction of Liquefaction Induced Lateral Displacements Using Plynomial Neural Networks and Genetic Algorithms

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SUMMARY:
Liquefaction can cause ground subsidence, flow failure and lateral spreading among other effects. Estimation of the hazard of lateral spreading requires characterization of subsurface conditions. In this paper, the relation between liquefaction induced lateral displacements and both geotechnical and earthquake soil parameters is investigated. In order to assess the merits of the proposed approach, database containing 526 data points of liquefaction-induced lateral ground spreading case histories from eighteen different earthquakes are used from renowned references. This study addresses the question of whether Group method of data handling (GMDH) type neural networks optimized using genetic algorithms (GAs) could be used to estimate lateral displacement based on specified variables. At the end the results of this paper models are compared with those of a commonly used and the advantages of the proposed GMDH model over the conventional method are highlighted.

Keywords: Liquefaction, Lateral spread, GMDH, GA.

1. INTRODUCTION

Liquefaction occurs in saturated sand deposit due to increase in excess pore water pressure during earthquake induced cyclic shear stresses. It can cause destruction or serious damage to structures. In order to investigate this phenomenon and mitigate its associated damages, study of liquefaction mechanism is significant. Liquefaction mechanism contains ground subsidence, flow failure, lateral spreading among other effects. Among liquefaction mechanism, lateral spreading can be more hazardous (Youd *et al.,* 2002). Lateral spreading involves the movement of relatively intact soil blocks on a layer of liquefied soil toward a free face or incised channel. Lateral spreading can induce different forms of ground deformations and in the vicinity of natural and cut slopes can be very destructive. A number of approaches have been proposed for prediction of the magnitude of lateral ground displacements under different conditions. All of them can be categorized into Figure 1.

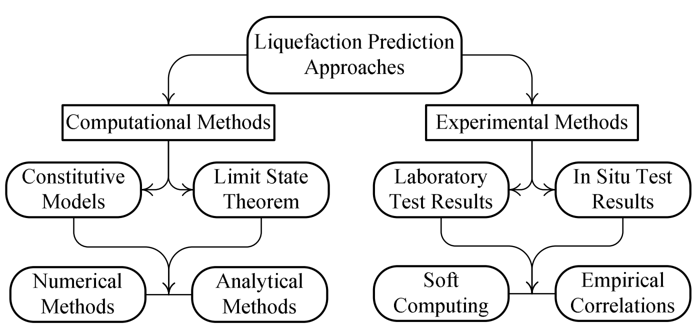

Figure 1. Classification of the approaches of lateral spreading predictions

However, all predictions based on any of the above-mentioned approaches require determination of input parameters, which are prone to uncertainties and inaccuracies. The effect of any inaccuracies of input data in the numerical and analytical approach may be studied by a sensitivity analysis of the predictions on various input data. However, due to versatility, empirical and semi-empirical correlations remain at the centre of practice (Albawab 2005).

The interdependency of factors involved in such problems prevents the use of regression analysis and demands a more extensive and sophisticated method. The Group Method of Data Handling (GMDH) type neural networks optimized by Genetic Algorithms (GAs) can be used to model complex systems, where unknown relationships exist between variables, without having specific knowledge of processes. In recent years, the use of such self-organizing networks has led to successful application of the GMDH-type algorithm in geotechnical sciences (e.g. Ardalan *et al.,* 2009; Kalantary *et al.,* 2009; Molaabasi *et al.,* 2012). This treatment aims to develop a GMDH-type NN for the prediction of Lateral Displacement, based on various soil conditions. To this end the paper first reviews previous efforts in predicting of lateral displacement, then a brief explanation of the case histories under consideration, and the phenomena of modeling with GMDH are presented. Finally the developed GMDH model is described and its accuracy is assessed through previous effort.

2. REVIEW OF THE AVAILABLE METHODS

Following the concept presented in figure 1, two basic approaches are described here; computational based and experimental based approaches. In the computational methods, basic parameters are input into analytical or numerical models to predict the extend of the effect, whereas in the latter approach laboratory and/or field test results are used in conjunction with case histories to develop empirical correlations. In recent years new identification techniques have further enhanced the latter approach by providing fast and efficient codes for development of empirical models. A brief review of each approach is provided here:

2.1. Computational Based Methods

Numerical and analytical methods have widely been used in geomechanics to simulate patterns of kinematic behaviour under various loadings. The success of such methods is highly dependent on the constitutive model or the simplified geometry used. The finite element or finite difference method are perhaps the most widely used numerical methods. However these procedures are highly dependent on material parameters that are usually difficult to estimate and as a result, limited success has been achieved in producing results that are comparable to field observations (Javadi et al., 2006). Numerical methods can also been utilized in conjunction with soft computing techniques to enhance or produce databases. Analytical models have also contributed to the development of knowledge in this field.

2.2. Experimental Based Methods

Due to complexities of the phenomenon, the aforementioned constitutive models as well as simplified analytical methods have failed to capture the full effect and thus empirical models based on case histories have remained as a popular method in the past decades. Hamada *et al.,* 1986, Youd and Perkins1987, Bardet *et al.,* 1999 and Youd *et al.,* 2002 introduced empirical correlations and multi-linear regression (MLR) models for the assessment of liquefaction-induced lateral spreading. Al Bawwab 2005 used SPSS 2004 software for statistical analysis of new sets of databases and arrived at a number of correlations for determination of lateral displacement. In order to enhance the accuracy of the models, a maximum likelihood approach was considered and the effect of data uncertainty was taken into account by a probabilistic methods. Kramer and Baska 2007 proposed a variation to the correlation presented by Youd *et al.* 2002; they based their model on a square root transformation of displacement rather than the logarithmic transformation used. On a different note, Zhang *et al.,* 2004 based their empirical correlation on a cumulative shear strain model; they introduced a "lateral displacement index (LDI)" calculated by integration of maximum shear strain over potentially liquefiable layers and then use it in a couple of simple correlations for "free-face" and "ground slope" case. Idriss and Boulanger 2008 used a different cumulative strain model to arrive at LDI. Table 1 shows some of the empirical models found in the literature. Due to different form of prediction, Zhang et al., 2004, Kramer and Baska 2007 and Idriss and Boulanger 2008 models have not been included in this table1. The difficulties posed by the fact that the phenomenon is dependent on multiple parameters have partly been alleviated by soft computing techniques such as fuzzy logic, neuron computing, probabilistic reasoning, genetic algorithm. These methods of decision making and optimization have firmly established themselves as indispensable tools for modeling natural phenomena. The artificial neural network (ANN) has been used for modeling the seismically induced displacement based on the same database used in the Multi Linear Regression model developed by Bartlet and Youd 1992. In the light of the above mentioned techniques, a new approach is proposed here which combines the benefits of empirical models, neural networks with an optimization method.

Table 1. Empirical correlations for prediction of the lateral displacement

Method	Subset	Model	limitations
Hamada et al. (1986)		$D_H = 0.75\ H^{1/2}\ \theta^{1/3}$	Number of case histories and variables
Youd and Perkins (1987)		$\text{Log } D_H = -3.49 - 1.86 \text{ Log } R + 0.98\ M_w$	Number of case histories and specific soil profile and topography conditions
Bardet *et al.* (1999)	free-face	$\text{Log }(D_H+0.01) = -17.372 + 1.248M_w - 0.923\text{LogR} - 0.014R + 0.685\text{LogW} + 0.3\text{Log } T_{15} + 4.826\text{Log }(100\text{-}F_{15}) - 1.091D50_{15}$	Number of case histories and mistakes in databases that correct in youd models.
	Slopping ground	$\text{Log }(D_H+0.01) = -14.152+0.988M_w-1.049\text{Log } R-0.011R+0.318\text{Log } S +0.619\text{Log}T_{15}+4.287\text{Log }(100\text{-}F_{15})-0.705D50_{15}$	
Youd *et al.* (2002)	free-face	$\text{Log } D_H = -16.713+1.532M_w-1.406\text{Log } R^*-0.012R+0.592\text{LogW} +0.540\text{Log}T_{15} +3.413\text{Log }(100 - F_{15})-0.795\text{Log }(D50_{15}+0.1 \text{ mm})$	$5 \leq W \leq 20\%$ $6 \leq MW \leq 8$, $0.1 \leq S \leq 6\%$, $1 \leq T15 \leq 15$ m, gravelly and/or very silty soils,critical depth up to 10 m
	Slopping ground	$\text{Log } D_H = -16.213+1.532M_w-1.406\text{Log } R^*-0.012R+0.338\text{Log } S+0.540\text{Log}T_{15} +3.413\text{Log }(100 - F_{15})-0.795\text{Log }(D50_{15}+0.1 \text{ mm})$	
Kanibir (2003)	free-face	$\text{Log } D_H = -20.71+25.32\text{Log } M_w-1.39\text{Log } R^*-0.009R+1.15\text{Log } W+0.19T_{15}\ 0.5 - 0.02F_{15}-0.84\text{Log }(D50_{15}+0.1 \text{ mm})$	Uncertanity not assumed
	Slopping ground	$\text{Log } D_H = -7.52+8.44\text{Log } M_w+0.001R^*-0.23R+0.11S+0.6\text{Log } T_{15}-0.22F_{15} - 0.89\text{Log } D50_{15}$	
Al Bawwab (2005)	Model 1	$\text{Log } D_H = b_1 \cdot LSI+b_2 \cdot a_y/a_{max}+b_3 \cdot \tan\beta/\tan\varphi'_{eqv,liq}+b_4 \cdot z_{cr}+b_5 \cdot M_w +b_6 \cdot W+b_7$	Probabilistic analysis included
	Model 2	$\text{Log}D_H = b_1 \cdot LSI+b_2 \cdot a_y/a_{max}+b_3 \cdot \tan\beta/\tan\varphi'_{eqv,liq}+b_4 \cdot z_{cr}+b_5 \cdot M_w +b_6 \cdot \text{LogS}+b_7 \cdot \text{LogW}+b_8$	
	Model 3	$\text{Log}D_H=b_1 \cdot LSI+b_2 \cdot a_y/a_{max}+b_3 \cdot \tan\beta/\tan\varphi'_{eqv,liq}+b_4 \cdot \text{Log}z_{cr}+b_5 \cdot \text{LogM}_w +b_6 \cdot a_{max}+b_7 \cdot \text{LogS}+b_8 \cdot \text{LogW}+b9$	
	Model 4	$\text{Log}D_H= [(\theta_1 LSI+\theta_2)a_y/a_{max} +(\theta_3 LSI+\theta_4)\tan\beta/\tan\varphi'_{eqv,liq} + (\theta_5 LSI+\theta_6)\text{Log } z_{cr} +(\theta_7 LSI+\theta_8)\text{Log } M_w +(\theta_9 LSI+\theta_{10})a_{max} + (\theta_{11} LSI+\theta_{12})\text{LogS} +(\theta_{13} LSI+\theta_{14})\text{Log } W +(\theta_{15} LSI+\theta_{16})+ \varepsilon]$	

3. THE PROPOSED MODEL

Following the trend proposed by Al Bawwab 2005, a_y/a_{max}, $\tan\beta/\tan_{\varphi'eqv,liq}$, and z_{cr} variables are used instead of T_{15}, F_{15}, and D_{5015} which were used in some of the earlier models. This can be considered as a step toward reaching to a more descriptive group of variables and consequently, a more powerful representative correlation. The descriptive variables are fully explained in Table 2. Where ay is the yield acceleration (g) equal to $\tan(\varphi'_{eqv,liq}-\beta)$ with finite slope assumption, and $\varphi'_{eqv,liq}$ is the equivalent mobilized angle of internal friction of liquefied or potentially liquefiable soils. Among the descriptive variables, there are two topological parameters (W and S) which refer to sloping sites without a free face (i.e. W=0) and level sites with a free face (i.e. S=0) as in Fig 2.

Table 2. Deprive variables for predicting the lateral displacement

Descriptive variables of a particular soil sub-layer.		
Seismological	M_W Duration of shaking	Moment magnitude scale of the earthquake
	a_{max} Intensity of shaking	Maximum Horizontal Ground Acceleration (*g*)
Topographical	*W Soil profile slope*	Free-face ratio = *H/L* (%)
	S Ground conditions	Ground Surface Slope (%)
	β Ground conditions	Ground surface slope angle (degrees) = $\tan^{-1}(S/100)$
Geotechnical	tan□'eqv,liq/tan*β* Gravity force	FS Against Gravitational Forces
	LSI Distribution of liquefaction potential through the depth	Liquefaction Severity Index
	a_y/a_{max} Sliding force	FS Against sliding
	z_c Effective potentially liquefiable depth	Critical Depth

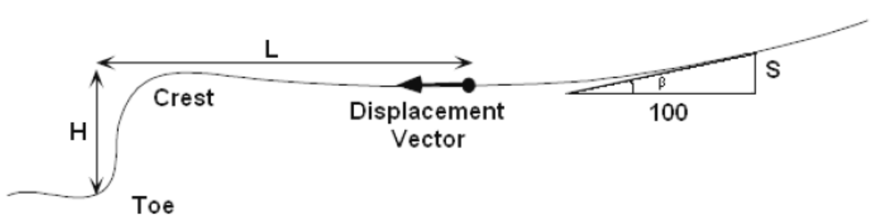

Figure 2. Topography-related descriptive variables.

With these definitions the case histories can be divided into two subsets of sloping sites without a free face and non-sloping sites with a steep face. In order to involve a model, a database is required. The database used in this paper consists of 526 case histories compiled by Youd *et al.*2002 including 1906 San Francisco–USA, 1964 Prince William Sound–Alaska, 1964 Niigata–Japan, 1971 San Fernando–USA, 1979 Imperial Valley–USA, 1983 Borah Peak–USA, 1983 Nihonkai-Chubu–Japan, 1987 Superstition Hills–USA, 1989 Loma Prieta–USA, and 1995 Hyogoken-Nanbu–Japan and 91 case histories from 7 different earthquakes added by Al Bawwab 2005, including the 1976 Guatemala, 1977 San Juan-Argentina, 1990 Luzon-Philippines, 1994 Northridge-USA, 1995 Hyogoken-Nanbu-Japan, 1999 Kocaeli (Izmit)-Turkey, 1999 Chi Chi- Taiwan, 2003 San Simeon-USA and 2003 Tokachi-Oki-Japan earthquakes.

4. PRINCIPLES OF MODELING USING GMDH TYPE NEURAL NETWORK

The GMDH algorithm is a self-organizing approach by which gradually complicated models are generated based on the evaluation of their performances on a set of multi-input single-output data pairs (xi, yi) (i=1, 2,…, m). The GMDH was first developed by Ivakhnenko 1971 as a multivariate analysis method for complex system modeling and identification. The main idea of GMDH is to build an analytical function in a feed forward network based on a quadratic node transfer function whose coefficients are obtained using regression technique. By means of the GMDH algorithm, a model can be represented as a set of neurons in which different pairs of them in each layer are connected through a quadratic polynomial, and thus, produce new neurons in the next layer. Such representation can be used in modeling to map inputs to outputs. The formal definition of the identification problem is to find a function $\hat{f}$ that can be approximately used instead of the observed one, f in order to predict output $\hat{y}$ for a given input vector $X = (x_1, x_2, x_3, \dots, x_n)$ as close as possible to its observed output y. Therefore, given M observations of multi-input, single output data pairs so that

$$Y_i = f(x_{i1}, x_{i2}, x_{i3}, \dots, x_{in}) \qquad (i = 1,2,3,\dots,M) \tag{1}$$

It is now possible to train a GMDH type neural network to predict the output values $\hat{y}_i$ for any given input vector $X = (x_{i1}, x_{i2}, x_{i3}, \dots, x_{in})$, that is

$$\hat{y}_i = \hat{f}(x_{i1}, x_{i2}, x_{i3}, \dots, x_{in}) \qquad (i = 1,2,3,\dots,M) \tag{2}$$

The problem is now to determine a GMDH type neural network such that the square of differences between the observed output and predicted one is minimized, that is

$$\sum_{i=1}^{M} \left[\hat{f}(x_{i1}, x_{i2}, x_{i3}, \dots, x_i) - y_i\right]^2 \to \min \tag{3}$$

The general connection between input and output variables can be expressed by a complicated discrete form of the Volterra functional series, known as the Kolmogorov-Gabor polynomial; hence:

$$y = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_i x_j + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} a_{ijk} x_i x_j x_k + \cdots \tag{4}$$

This full form mathematical description can be represented by a system of partial quadratic polynomials consisting of only two variables (neurons) in the form of:

$$\hat{y} = G(x_i, x_j) = a_0 + a_1 x_i + a_2 x_j + a_3 x_i x_j + a_4 x_i^2 + a_5 x_j^2 \tag{5}$$

By this means, the partial quadratic description is recursively used in a network of connected neurons to build the general mathematical relation between inputs and output given in Eq. (4). The coefficients a_i in Eq. (5) are calculated using regression techniques, so that the difference between the observed output, y, and the calculated one, $\hat{y}$, for each pair of x_i, y_i as input variables is minimized. Apparently, a tree of polynomials is constructed using the quadratic form given in Eq. (5) whose coefficients are obtained in a least squares scheme. In this way, the coefficients of each quadratic function G_i are derived to fit optimally the output in the whole set of input–output data pairs, that is

$$E = \frac{\sum_{i=1}^{M}(y_i - G_i())^2}{M} \rightarrow \min \quad (6)$$

In the basic GMDH algorithm, all possibilities of two independent variables out of the total n input variables are taken in order to construct the regression polynomial in the form of Eq. (5) that best fits the dependent observations$(y_i, i = 1,2,\dots,M)$ in a least squares sense. Consequently, $\binom{n}{2} = \frac{n(n-1)}{2}$ neurons will be built up in the first hidden layer of the feed forward network from the observations $\{(y_i, x_{ip}, x_{iq});\ (i = 1,2,\dots M)\}$ for different $p, q \in \{1,2,\dots,n\}$.

In other words, it is now possible to construct M data triples $\{(y_i, x_{ip}, x_{iq});\ (i = 1,2,\dots M)\}$ from observations using $p.q \in \{1,2,\dots,n\}$ in the form of:

$$\begin{bmatrix} x_{1p} & x_{1q} & y_1 \\ x_{2p} & x_{2q} & y_2 \\ x_{Mp} & x_{Mq} & y_M \end{bmatrix}. \quad (7)$$

Using the quadratic sub-expression in the form of Eq. (5) for each row of M data triples, the following matrix equation can be readily obtained as

$$Aa = Y \quad (8)$$

$$a = \{a_0, a_1, a_2, a_3, a_4, a_5\} \quad (9)$$

$$Y = \{y_1, y_2, y_3, \dots, y_M\}^T \quad (10)$$

Where; a is the vector of unknown coefficients for the quadratic polynomial in Eq. (5), and Y is the vector of output values from observation. It can be readily seen that:

$$A = \begin{bmatrix} 1 & x_{1p} & x_{1q} & x_{1p}x_{1q} & x_{1p}^2 & x_{1q}^2 \\ 1 & x_{2p} & x_{2q} & x_{2p}x_{2q} & x_{2p}^2 & x_{2q}^2 \\ 1 & x_{Mp} & x_{Mq} & x_{Mp}x_{Mq} & x_{Mp}^2 & x_{Mq}^2 \end{bmatrix} \quad (11)$$

The least squares technique from multiple regression analysis leads to solution of the normal equations,

$$a = (A^T A)^{-1} A^T Y \quad (12)$$

This determines the vector of best coefficients of Eq. (5) for the whole set of M data triples. It should be noted that this procedure is repeated for each neuron of the next hidden layer according to the connectivity topology of the network. However, such a solution directly from normal equations is rather susceptible to round off errors and, more importantly, to the singularity of these equations. There are two main concepts involved within GMDH type neural networks design, namely, the parametric and the structural identification problems. Nariman-Zadeh *et al.,* 2005 present hybrid GA and singular value decomposition (SVD) method to optimally design such polynomial neural networks. The methodology and general description of this technique is beyond the scope of this study, and complementary information may be found in Kalantary *et al.,* 2009.

5. MODELING LATERAL DISPLACEMENT USING GMDH-TYPE NEURAL NETWORK

In order to demonstrate the prediction ability of evolved GMDH-type neural networks, experimental data have been divided into two different sets, namely, training and testing sets. The GMDH type neural networks are now used for such inputs-output data to find the polynomial model of Lateral spread displacement in respect to its effective input parameters. The structure of the evolved 2-hidden layer GMDH type neural networks for free face is shown in Figure 3.a corresponding to the genome representations of debbggah for Lateral spread displacement in which a, b, d,g and h stand for m_w, a_{max}/g, w, a_y/a_{max} and tanβ/tan φ , respectively.

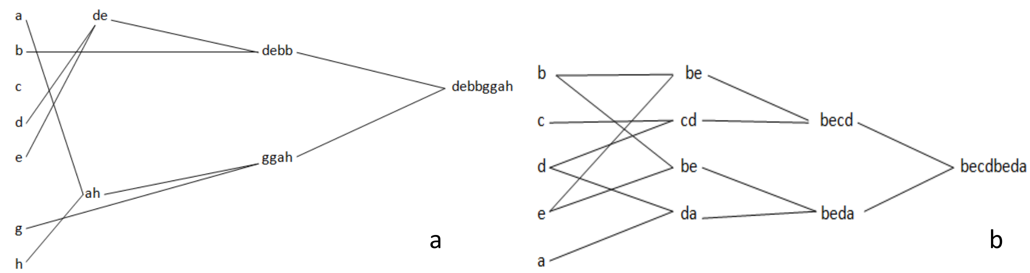

Figure 3. Evolved structure of the generalized GMDH neural network for free space condition

The structure of the evolved 2-hidden layer GMDH type neural networks for gently slope is also shown in Figure 3.b corresponding to the genome representations of becdbeda for Lateral spread displacement in which a, b, c, d, and e stand for m_w, a_{max}/g, s, w, a_y/a_{max} and LSI, respectively. The good behaviour of such GMDH-type neural network models also illustrated in Figure. 5 and 6.

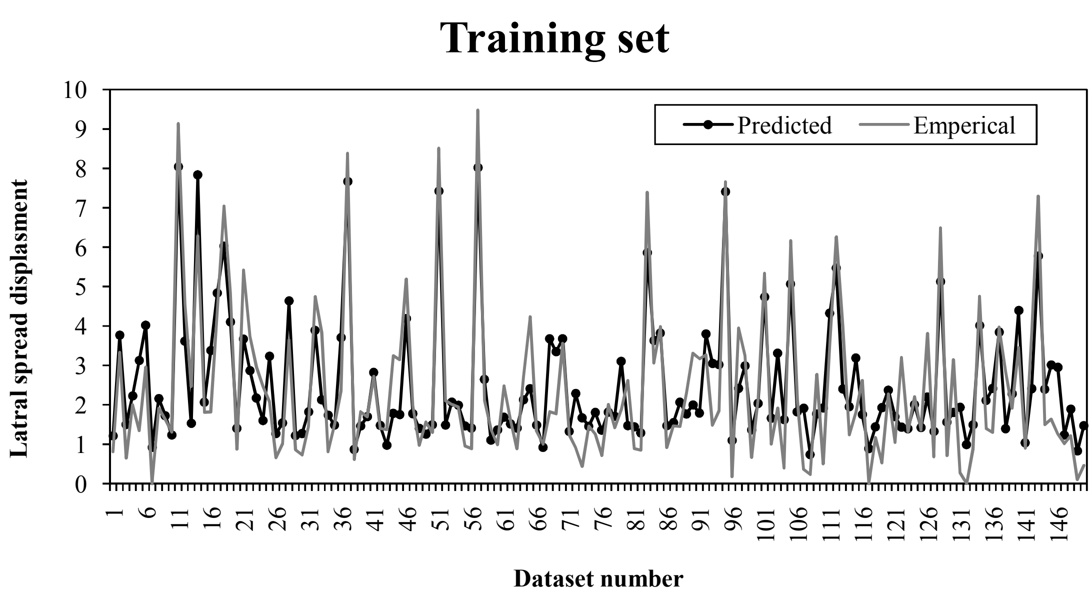

Figure 5. Neural network model predicted performance in comparison with actual data for the training set in free space model condition (150 input-output data)

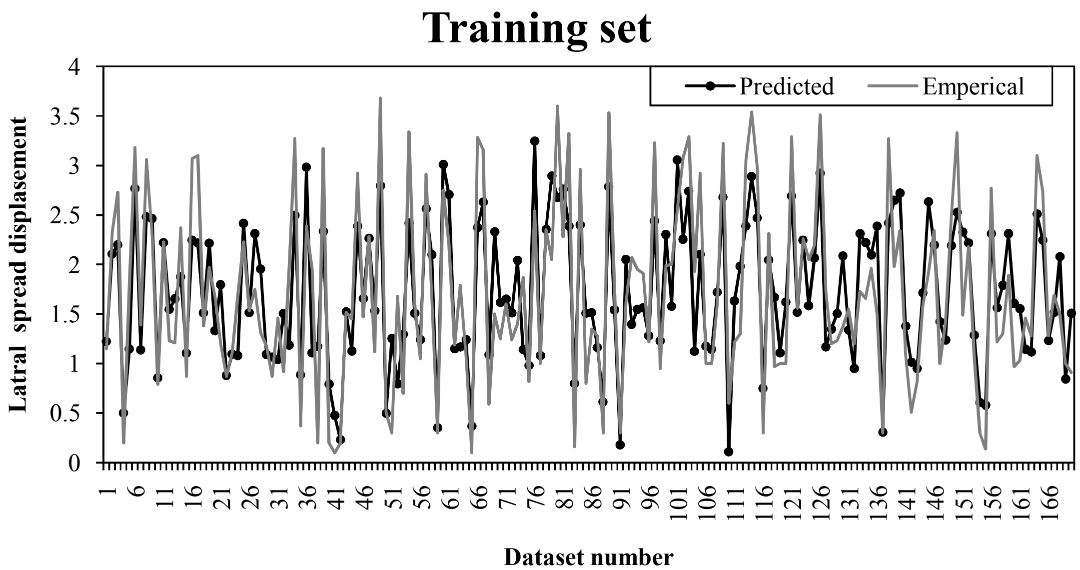

Figure 6. Neural network model predicted performance in comparison with actual data for the training set in gently slope condition (170 input-output data)

Some statistical measures given in Table. 3 are used in order to determine the accuracy of models. These statistical values are based on R^2 as absolute fraction of variance, MSE as mean squared error, and MAD as mean absolute deviation which is defined as follows:

$$R^2 = 1 - \left[\frac{\sum_{i=0}^{M}(Y_{i(Model)}-Y_{i(Actual)})^2}{\sum_{i=1}^{M}(Y_{i(Actual)})^2}\right], MSE = \frac{\sum_{i=0}^{M}(Y_{i(Model)}-Y_{i(Actual)})^2}{M}, MAD = \frac{\sum_{i=1}^{M}|Y_{i(Model)}-Y_{i(Actual)}|}{M} \quad (13)$$

Table3. Model statistics and information for the group method of data handling-type neural network model for predicting the Lateral spread displacement

Ground condition	Subset	Performance criteria		
		R^2	MSE	MAD
Free space	Training	0.91	0.86	0.77
	Testing	0.92	0.91	0.8
Gently sloping	Training	0.94	0.25	0.42
	Testing	0.94	0.21	0.39

The obtained polynomial model is now tested for unforeseen data during the training process which accordingly demonstrates the prediction ability of the model. Figure 7 shows the comparison of such behaviour with the actual values as a Sample for free face model.

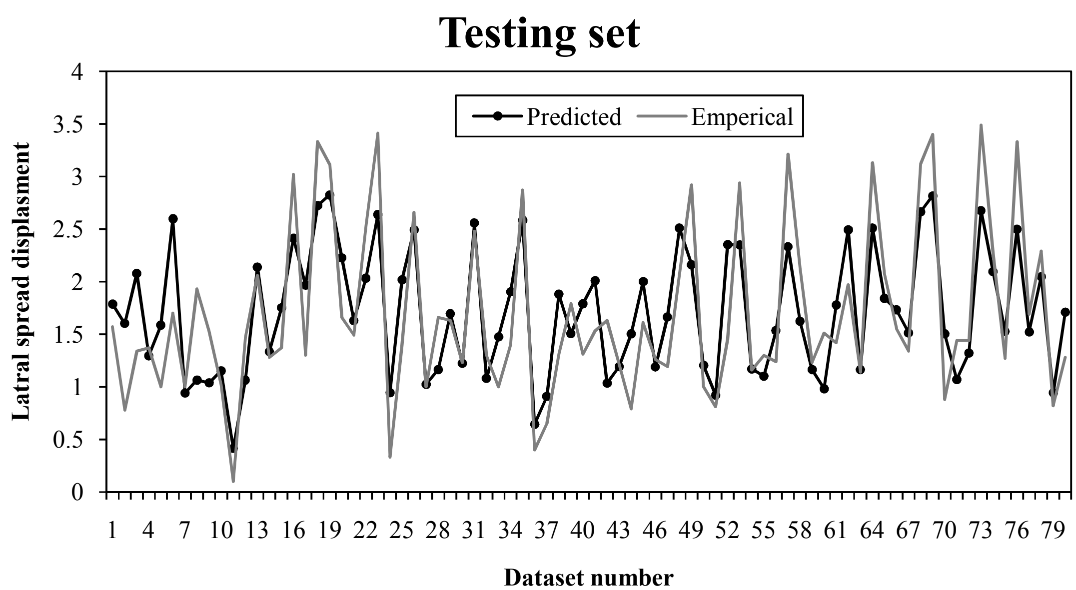

Figure 5. Neural network model predicted performance in comparison with actual data for the testing set in free space model condition (80 input-output data)

6. COMPARISON ANALYSIS

The accuracy of the proposed model, in predicting lateral displacement, is compared with correlations presented previously by Hamada *et al.,* 1987, Youd *et al.,* 2002 and Al Bawab 2005 models (cf. Table 1). The statistical comparison is performed for all the 526cases initially used for model development. Table.5 illustrates the accuracy of this study.

Table5. The accuracy of different methods

Methods		R^2
Hamada et al. (1986a)		13%
Youd et al. (2002b)		74%
Al Bawab (2005) models	Model 1	66%
	Model 2	71%
	Model 3	74%
	Model 4	85%
This study Method	Free Face Condition	92%
	Gently Slope Condition	94%

7. CONCLUSIONS

It has been attempted in this study to deploy a system identification technique to develop the lateral displacement correlation over geotechnical soils properties. The evolved GMDH type neural networks have been used to obtain a model for the prediction of lateral displacement. Databases of case histories consisting of 526 databases from 18 earthquakes were compiled. A polynomial model was developed for lateral displacement based on geotechnical and earthquake conditions. The validation and performance of the new model was assessed, and contrasted with previous statistical correlations. For all 526 case records, including lateral displacement and geotechnical soil properties, predicted and measured lateral displacement values were compared. The results manifest that predictions by the correlations of Hamada et al. (1986a), Youd et al. (2002b) and Al Bawab (2005) models, however the proposed approach predicts with high accuracy and low variance. Results obtained from this study and previous researches reveal that empirical correlations derived from a local dataset should not implemented for different sites with significantly varying features. Therefore, these proposed relationships should be used with caution in geotechnical engineering and should be checked against measured lateral displacements.

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