A Method for Direct Determination of Inelastic Floor Response Spectra

V. Vukobratović
University of Novi Sad, Faculty of Technical Sciences & University of Ljubljana, Faculty of Civil and Geodetic Engineering

P. Fajfar
University of Ljubljana, Faculty of Civil and Geodetic Engineering

SUMMARY:
Floor response spectra, which are usually based on the assumption that the behaviour of the primary (building) and the secondary systems (equipment) is linear, are used for the seismic design of equipment in industrial buildings. In general, essential reductions in peak values of floor response spectra can be obtained if inelastic behaviour of the primary system is taken into account. The paper presents the most important results of an extensive parametric study of floor response spectra, taking into account the inelastic behaviour of the primary system and the linear behaviour of the secondary system. Both systems were modeled as single-degree-of-freedom systems. The influences of input ground motion, ductility, hysteretic behaviour and natural period of the primary system, as well as damping of the secondary system have been studied. A simple approximate method for direct determination of floor response spectra from the design spectrum is proposed.

Keywords: floor response spectra, inelastic behaviour, primary system, secondary system, equipment

1. INTRODUCTION

In industrial buildings like nuclear and thermal power plants, floor response spectra are usually used for seismic design and evaluation of mechanical and electrical equipment (e.g. piping systems, boilers, turbines, generators, pumps, tanks, ducts, etc.).

The floor response spectra concept is based on separate (uncoupled) analysis of the structure (primary system, PS) and equipment (secondary system, SS), which means that the dynamic interaction between them is neglected. It has been proven accurate in cases of SS whose mass is significantly smaller than the mass of PS, at least a hundred times. The floor response spectra method is rational, simple in concept and very practical. It usually yields somewhat conservative results. By using it, one can avoid numerical problems due to large differences between dynamic properties of PS and SS. Main steps of the method are:

1. Performing a response-history analysis of the PS by using a set of ground motions.
2. Determining the response of a floor in terms of absolute floor acceleration.
3. Generating floor response spectrum using the absolute acceleration time-history determined in step (2) as input.

Once a floor spectrum is determined, the SS can be analysed in the same way as the PS is analysed using a design response spectrum.

In order to avoid long numerical integrations, several researchers have proposed methods that enable generation of floor response spectra directly from the design response spectrum, using the dynamic properties of the PS. Because of their simplicity, these methods are very attractive for practical applications. A list of methods proposed by different authors is provided in Villaverde (1997).

Developments of early floor response spectra methods have been based on the assumption that both PS and SS remain in linear elastic region during earthquakes. However, even in structures of great importance such as nuclear power plants, it is usually acceptable to allow some moderate amount of inelastic behaviour during very strong earthquakes. This fact is of great importance, especially in the case of re-evaluation of existing structures. Significant reductions in peak values of floor response spectra can be obtained if inelastic behaviour of PS and/or SS is taken into account.

In this paper, some results of an extensive parametric study, taking into account inelastic behaviour of the PS and linear elastic behaviour of the SS, aimed at determining some general characteristics of floor response spectra, are presented. The results mostly confirm the findings obtained by Fajfar and Novak (1995). Based on these results, a preliminary version of a practice-oriented approximate direct method for determination of floor response spectra is proposed, which takes into account the inelastic behaviour of the PS. The method is based on the method originally proposed by Yasui *et al*. (1993) for elastic PS and on the idea for the extension of this method to inelastic PS proposed by Novak and Fajfar (1994). Some results obtained by the proposed approximate method for direct determination of floor response spectra are presented and compared with the "accurate" floor response spectra obtained by the parametric study.

2. PARAMETRIC STUDY

2.1. Description of input parameters used in the study

In the parametric study, 5760 floor response spectra were calculated by the procedure described in the Introduction. A single-degree-of-freedom model (SDOF) was used for both the inelastic PS and the elastic SS, which were treated as uncoupled. The influences of natural period, hysteretic behaviour and ductility of the PS, as well as the influence of damping of the SS have been studied. The influence of the ground motion characteristics was also investigated. Two different sets, consisting of 30 ground records each, were used in the study. The records of each set were chosen so that their average spectrum matched a target spectrum. The target spectrum was the elastic spectrum defined by Eurocode 8 (2004). Type 1 spectra for soil types B and D (each for one set of records) were used with the peak ground acceleration equal to 0.35g and to 0.39g, respectively. The characteristic periods of ground motion T_C are equal to 0.5 and 0.8 sec for soil types B and D, respectively. Selection of ground records was conducted using the software REXEL (Iervolino *et al*., 2010) in the case of soil type B, whereas the software developed by Baker Research Group (Jayaram *et al*., 2011) was used in the case of soil type D.

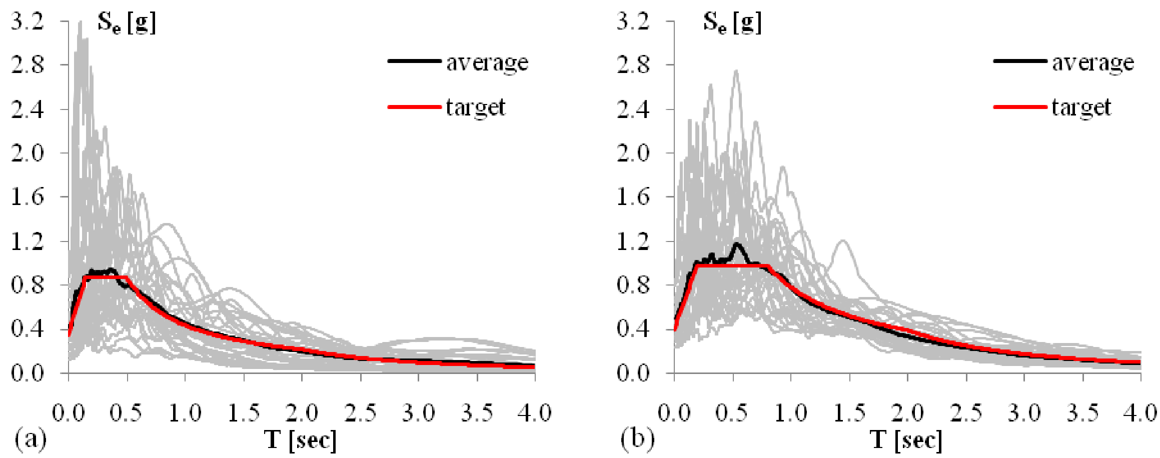


Figure 1. Elastic acceleration spectra of individual records, target and average spectrum for the case of (a) soil type B and (b) soil type D (5% damping)

Target and average spectra of the selected sets of records for both soil types are shown in Fig. 1. The natural periods of the PS amounted to 0.2, 0.3, 0.5, 0.75, 1.0 and 2.0 sec. Two different hysteretic models were assumed: elasto-plastic (EP) and stiffness degrading (Q) model with 10% hardening and unloading stiffness degradation coefficient equal to 0.5.

A constant target ductility factor μ throughout the whole period range was assumed. It amounted to 1.5, 2.0 and 4.0. "Mass-proportional" damping amounted to 5% in the case of PS and to 1% and 5% in the case of SS.

2.2. Discussion of results

The results obtained in the parametric study show some trends which can be considered as general characteristics of floor response spectra. In the following text, some representative results of the study are presented in order to provide a basis for the development of a method for the direct determination of inelastic floor response spectra. The natural periods of the PS and the SS are denoted as T_p and T_s, respectively. Floor response spectra values are denoted as A_s in the case of inelastic PS and A_{se} in the case of elastic PS. Peak acceleration of the PS is denoted as A_p. The results shown in Figs. 2-4 were obtained for the set of ground records which corresponds to the soil type B, for PS with T_p=0.3 sec. In all cases the damping of PS and SS amounted to 5%.

Floor response spectra shown in Fig. 2 represent mean values of A_s obtained for both EP and Q hysteretic models of PS.

The period range of a floor spectrum can be roughly divided into three regions, depending on the ratio T_s/T_p: short-period region ($T_s/T_p<0.8$), resonance region ($0.8<T_s/T_p<1.25$), and long-period region ($T_s/T_p>1.25$). It is obvious that in the short-period and in the resonance regions, the behaviour of the SS is strongly influenced by the behaviour of the PS. Both regions are characterized by a significant reduction of A_s due to inelastic structural behaviour. The shape of floor response spectra is influenced by the hysteretic behaviour of the PS. In the case of the EP model the peak values of A_s occur in resonance ($T_s=T_p$), whereas in the case of the Q model the peak values of A_s are shifted towards higher periods, due to increasing T_p with increasing plastic deformations. In the long-period region, floor response spectrum is controlled by the ground motion spectrum, and the inelastic structural behaviour has only a small influence on it. If $T_s>>T_p$, there is practically no reduction due to inelastic behaviour for the EP model. For the Q model, even some slight amplification can be observed. In the case of infinitely rigid SS, A_s is equal to A_p, whereas for an infinitely flexible SS the value of A_s is equal to peak ground acceleration.

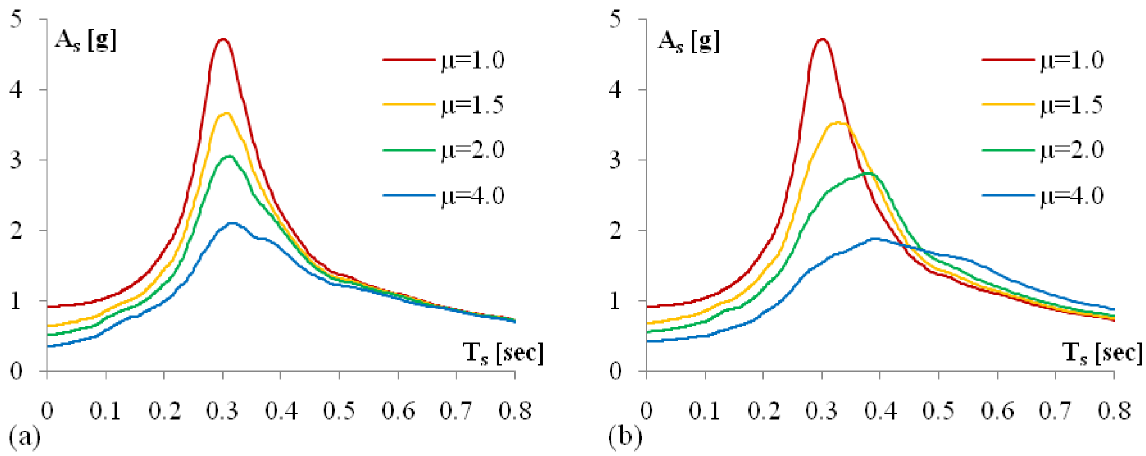


Figure 2. Mean values of floor response spectra for T_p=0.3 sec for (a) EP and (b) Q hysteretic models (soil type B, 5% damping of PS and SS)

The same results are presented in a different form in Fig. 3, which shows the ratio of floor response spectra corresponding to inelastic and elastic PS.

Fig. 4 presents the floor response spectra normalized to the peak acceleration of the PS (A_s/A_p). This ratio is primarily influenced by the damping value of SS. It can be observed that in the short-period and the resonance regions, the ratio A_s/A_p slightly increases with increasing ductility in the case of the EP model, whereas in the case of the Q model in the resonance region, A_s/A_p decreases with increasing ductility.

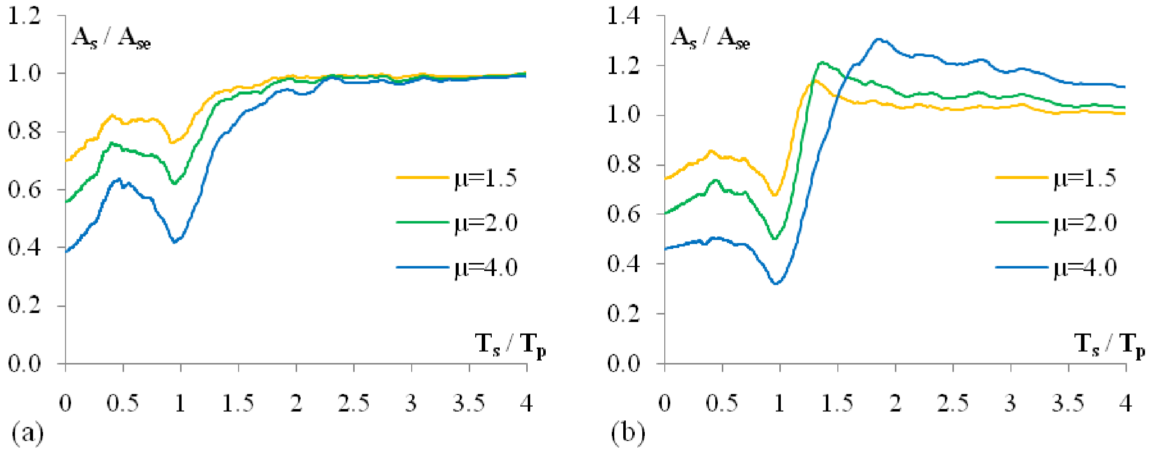


Figure 3. The ratio of floor response spectra corresponding to inelastic and elastic PS (T_p=0.3 sec) for (a) EP and (b) Q models (soil type B, 5% damping of PS and SS)

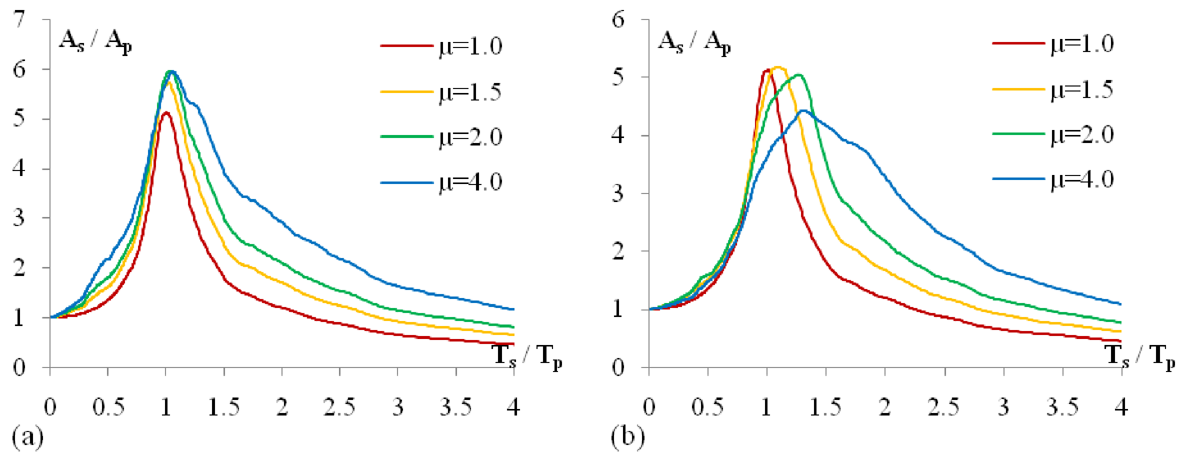


Figure 4. Floor response spectra normalized to the peak acceleration of the PS (T_p=0.3 sec) for (a) EP and (b) Q models (soil type B, 5% damping of PS and SS)

Figs. 5 and 6 present maximum values of the ratio A_s/A_p, which will be hereinafter referred to as amplification factor, AMP, for two damping values of SS and for two sets of ground motions. Results obtained for both sets of ground motions indicate that the shape of the response spectrum characterized by the characteristic period of ground motion T_C has only a small influence on the amplification factor, provided that the ratio T_p/T_C is plotted on the x-axis instead of T_p.

For both the EP and the Q models, the main parameter that influences the amplitude of AMP is the damping of SS. AMP reaches its peak value in the region $T_p/T_C \leq 1$, and it decreases with increasing ratio T_p/T_C if the ratio is larger than 1. As stated above, AMP generally slightly increases with increasing ductility in the case of the EP model, whereas for the Q model the opposite trend is more pronounced.

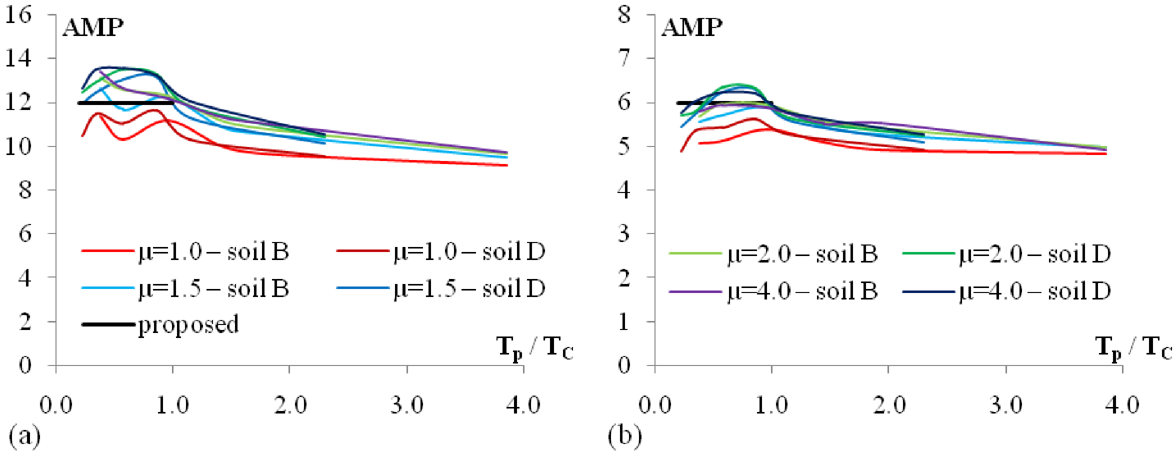


Figure 5. Computed and proposed amplification factors AMP in the case of EP model for (a) 1% damping and (b) 5% damping of SS

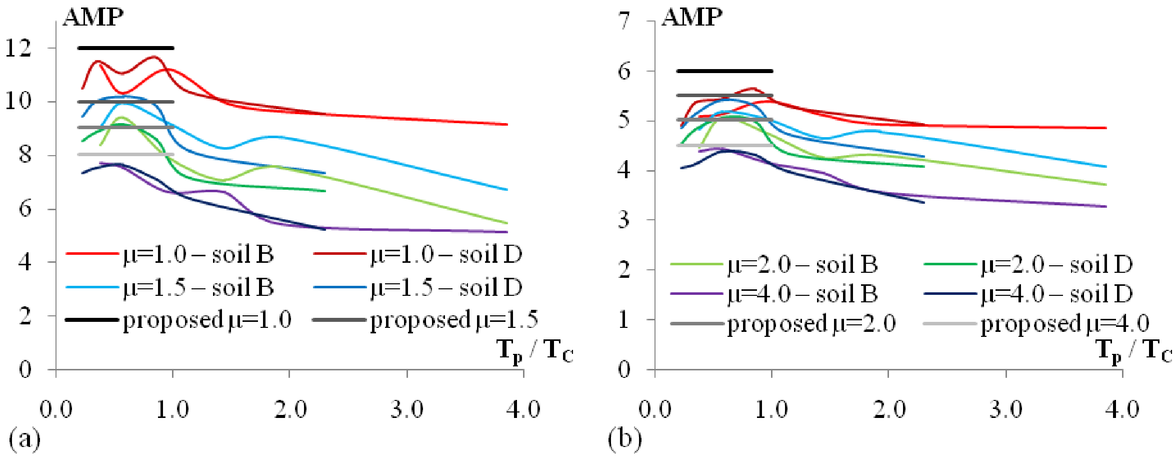


Figure 6. Computed and proposed amplification factors AMP in the case of Q model for (a) 1% damping and (b) 5% damping of SS

3. PROPOSAL OF THE DIRECT METHOD

3.1. Original method

A very simple method for direct determination of floor response spectra was proposed by Yasui *et al.* (1993). The authors have derived an equation which is valid in the whole period range for the case of linear elastic behaviour of both PS and SS, which were modeled as SDOF systems. The equation was derived analytically, using the Duhamel integral for the evaluation of the responses of PS and SS. Three responses in terms of absolute acceleration were analyzed: responses of the PS and SS subjected to the ground motion and the response of the SS subjected to the absolute acceleration of the mass of the PS. The maximum values of responses were then combined with the SRSS (Square Root of Sum of Squares) combination rule in order to obtain the equation for the floor spectrum generation. The derivation was conducted separately for the non-resonant and resonant cases. Two independent equations were then combined in a single equation for floor response spectrum determination

$$A_s = \frac{1}{\sqrt{\left\{1-\left(T_p/T_s\right)^2\right\}^2 + 4\left(\xi_p+\xi_s\right)^2\left(T_p/T_s\right)^2}}\sqrt{\left\{\left(T_p/T_s\right)^2 S_e\left(T_p,\xi_p\right)\right\}^2 + S_e\left(T_s,\xi_s\right)^2} \quad (3.1)$$

where A_s is a floor spectrum value, S_e is a value from the elastic acceleration spectrum, damping values of PS and SS are denoted as ξ_p and ξ_s respectively, whereas T_p and T_s were defined above.

Input data are dynamic properties of PS and SS (damping and natural periods) and elastic acceleration spectrum representing the ground motion. Our analyses indicate that, in the non-resonance regions, floor response spectra obtained by the proposed direct method are in good agreement with more accurate floor response spectra obtained by time-history analyses. In the resonance region, a considerable inaccuracy of the direct method was observed in our studies.

3.2. Extension and modification of the method

In order to improve the accuracy of the direct method and to make the method applicable for the case of inelastic PS some changes were made.

First, elastic acceleration spectrum was replaced with the inelastic acceleration spectrum corresponding to the expected ductility demand, as proposed by Fajfar and Novak (1994). Several proposals have been made for inelastic acceleration spectra. One of them is the simplified form of spectra proposed by Vidic *et al*. (1994) which has been implemented in Eurocode 8. The inelastic spectrum, used also in the example presented in this paper, can be obtained by reducing the elastic acceleration spectrum by a reduction factor R_μ which is defined by Eqn. 3.2.

$$R_\mu = \begin{cases} \dfrac{T}{T_C}(\mu - 1) + 1, & T < T_C \\ \mu, & T \geq T_C \end{cases} \tag{3.2}$$

Second, a change in the combination rule, used for combining the amplitudes of the vibration of the PS and SS, was made. In the original method, the SRSS combination rule is used to compute the floor spectrum throughout the whole period range. Results obtained in our parametric study suggest that the SRSS combination rule gives, both in the elastic and inelastic range, adequate results only in the long-period region, whereas the results obtained in the short-period region are less accurate and unconservative. In the short-period region more accurate results can be obtained if the sum of the amplitudes of the PS and SS is used instead of the SRSS combination rule in the formula for the determination of floor response spectra.

In the resonance region, the spectral values must be limited in order to obtain realistic results. Instead of the formula provided in the original method, which in our experience provides too conservative results even in elastic region, the amplification factors which are based on AMP values obtained in the parametric study (Figs. 5 and 6) can be used for the determination of the floor response spectra in the resonance region. The research is still ongoing. In this paper some very rough preliminary proposals for amplification factors are presented in Figs. 5 and 6. For the EP model (Fig. 5), it is assumed that AMP is independent on ductility. In the region $T_p/T_C \leq 1$, values AMP=12 and AMP=6 are proposed for 1% and 5% damping of the SS, respectively. If $T_p/T_C > 1$, AMP decreases with increasing T_p/T_C. A proposal for the AMP values to be used in the simplified procedure has still to be prepared. For the Q model, the influence of ductility is considerable, therefore it is considered in our proposal. The following AMP values are proposed in the case of 1% damping of SS: 10, 9 and 8 for ductilities 1.5, 2 and 4, respectively. In the case of 5% damping of SS, the AMP values amount to 5.5, 5 and 4.5. Again, the values apply to the region $T_p/T_C \leq 1$.

In the proposed direct approach, considering the changes explained above, floor response spectra can be computed for both the EP and the Q models as follows:

1. In the short-period region, the spectral values are obtained as

$$A_s = \frac{1}{\sqrt{\left\{1-\left(T_p/T_s\right)^2\right\}^2 + 4\left(\xi_p+\xi_s\right)^2\left(T_p/T_s\right)^2}}\left\{\left(T_p/T_s\right)^2\frac{S_e\left(T_p,\xi_p\right)}{R_\mu}+S_e\left(T_s,\xi_s\right)\right\} \quad (3.3)$$

2. In the long-period region, the spectral values are obtained as

$$A_s = \frac{1}{\sqrt{\left\{1-\left(T_p/T_s\right)^2\right\}^2 + 4\left(\xi_p+\xi_s\right)^2\left(T_p/T_s\right)^2}}\sqrt{\left\{\left(T_p/T_s\right)^2\frac{S_e\left(T_p,\xi_p\right)}{R_\mu}\right\}^2+S_e\left(T_s,\xi_s\right)^2} \quad (3.4)$$

For the case of the stiffness degrading Q model, the ratio T_p/T_s in Eqn. 3.4 should be replaced with the ratio $T_{p,\mu}/T_s$, where $T_{p,\mu}$ represents the effective natural period of the PS. It depends on the inelastic deformation which is expressed in terms of ductility. It can be defined by Eqn. 3.5 proposed by Akiyama (1985).

$$T_{p,\mu} = T_p\sqrt{\frac{1+\sqrt{\mu}+\mu}{3}} \quad (3.5)$$

3. In the resonance region, the spectral values are limited to the value obtained by Eqn. 3.6. A preliminary proposal for amplification factors AMP is presented in Figs. 5 and 6.

$$A_s = AMP \times S_e\left(T_p,\xi_p\right)/R_\mu \quad (3.6)$$

Note that in Eqn. 6, as used in this paper, two rough approximations are involved. One is related to AMP (see Figs. 5 and 6) and the second one is related to approximate inelastic spectra (Eqn. 3.2 for R_μ).

4. EVALUATION OF THE PROPOSED DIRECT METHOD

The proposed direct method was used to compute a large number of floor response spectra which were then compared with the “accurate” spectra obtained in the parametric study, in order to evaluate the accuracy of the method. Results shown below are obtained for the sets of ground records which correspond to the soil types B and D, for PS with T_p=0.3 sec. In all cases two different values of μ are considered. Damping of PS is equal to 5%, whereas damping of SS is equal to 1% and 5%.

In practice, the natural period of the PS cannot be accurately determined due to uncertainties in input parameters, such as the material properties of the structure and soil, damping values and soil-structure interaction. In the conventional floor response spectrum method, which is described in the Introduction, these uncertainties are usually considered by broadening the peaks of floor response spectra. According to USNRC (1978), the frequency region where the spectrum should be broadened is obtained by considering a ±15% variation in the frequencies associated with the spectral peaks.

Figs. 7-14 show the mean, mean plus standard deviation (σ) and broadened mean values of “accurate” floor response spectra, as well as the approximate spectra computed by the proposed direct method, for the EP and the Q models of the PS.

It is obvious that in most cases the proposed method provides somewhat conservative results in the resonance region. Peak values of the direct spectra are mostly located between mean and mean+σ peak values of the “accurate” spectra. The exceptions that occur in the case of μ=4 are the result of a difference between the value from the “accurate” inelastic design spectrum and the approximate value obtained by using the factor R_μ according to Eqn. 3.2.

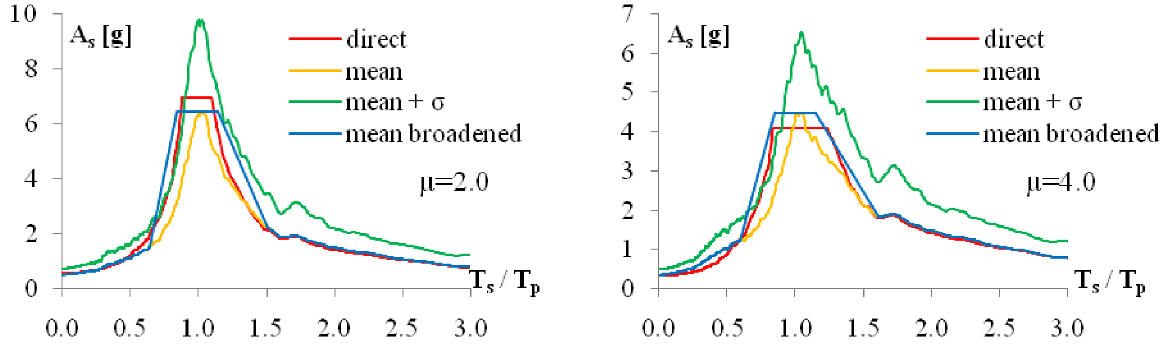


Figure 7. Floor response spectra for EP model of PS (soil type B, T_p=0.3 sec, 5% damping of PS and 1% damping of SS)

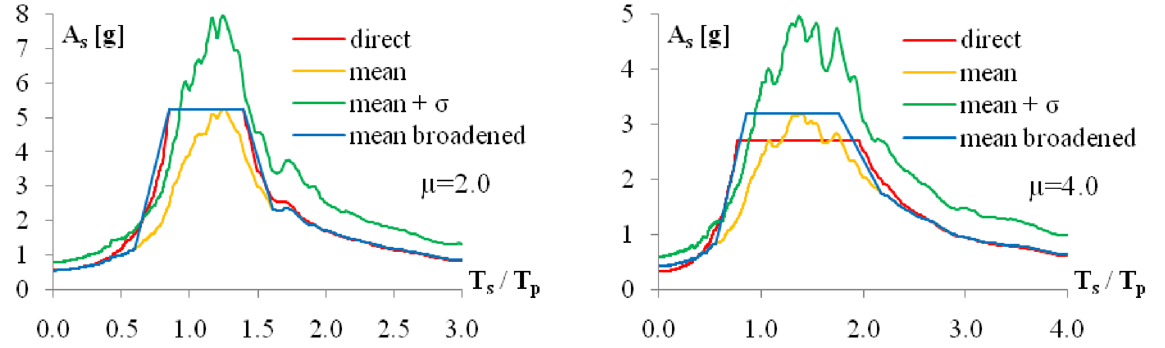


Figure 8. Floor response spectra for Q model of PS (soil type B, T_p=0.3 sec, 5% damping of PS and 1% damping of SS)

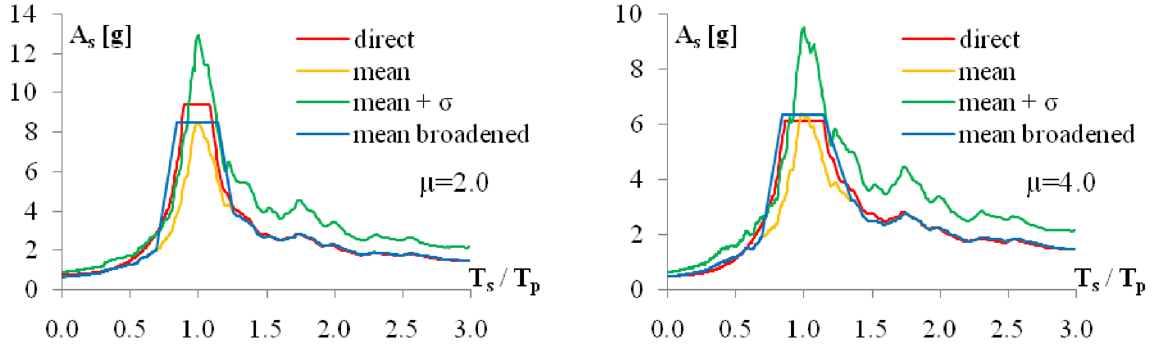


Figure 9. Floor response spectra for EP model of PS (soil type D, T_p=0.3 sec, 5% damping of PS and 1% damping of SS)

Outside of the resonance region, the results show a very good agreement between a broadened and the direct spectra.

We believe that the proposed direct method can also be applied on multi-degree-of-freedom primary systems (MDOF) which can be properly simulated by an equivalent SDOF system. This will be the subject of our future research.

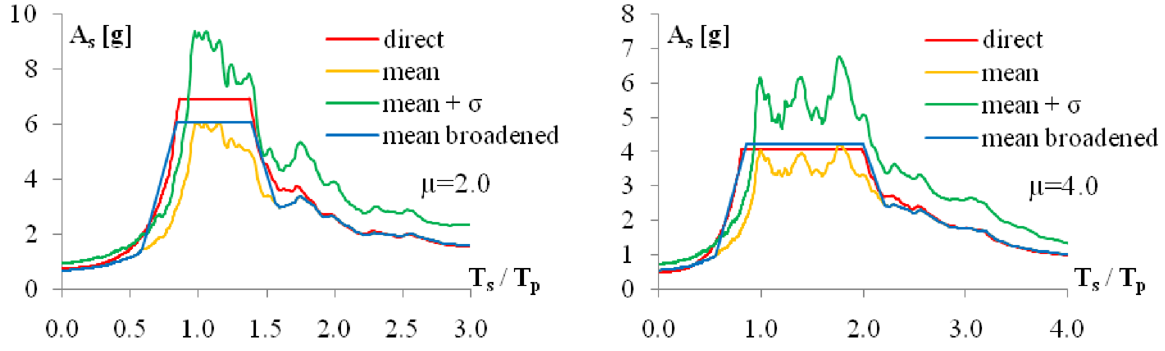


Figure 10. Floor response spectra for Q model of PS (soil type D, T_p=0.3 sec, 5% damping of PS and 1% damping of SS)

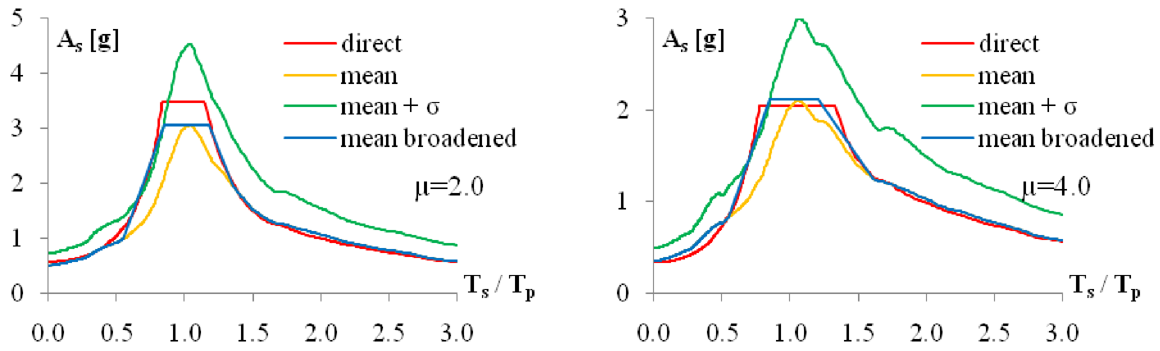


Figure 11. Floor response spectra for EP model of PS (soil type B, T_p=0.3 sec, 5% damping of PS and SS)

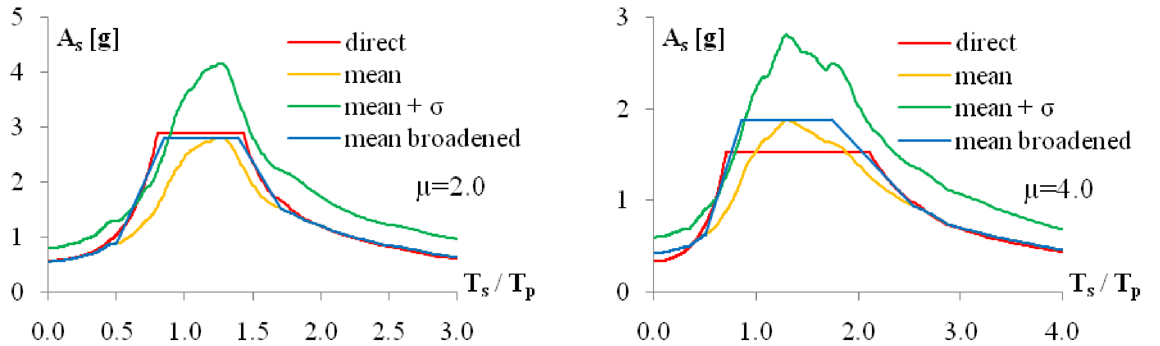


Figure 12. Floor response spectra for Q model of PS (soil type B, T_p=0.3 sec, 5% damping of PS and SS)

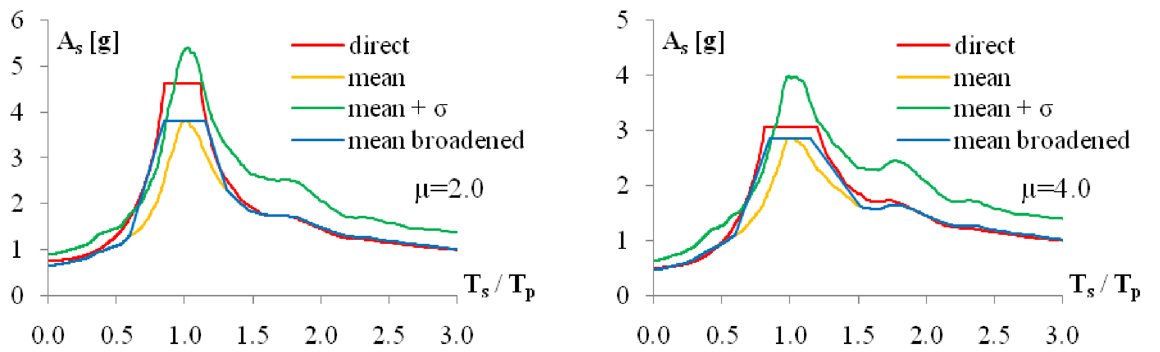
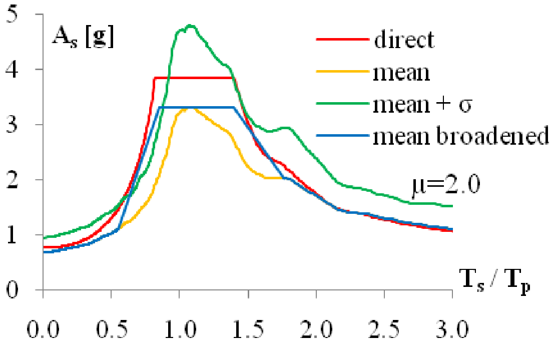


Figure 13. Floor response spectra for EP model of PS (soil type D, T_p=0.3 sec, 5% damping of PS and SS)



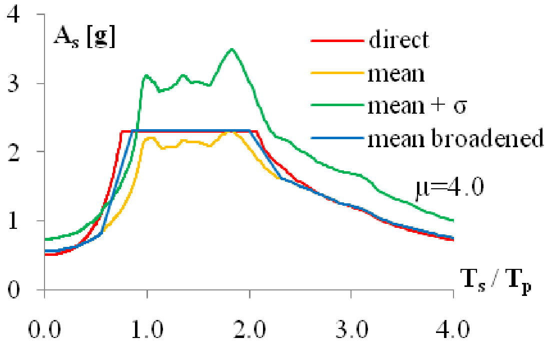


Figure 14. Floor response spectra for Q model of PS (soil type D, T_p=0.3 sec, 5% damping of PS and SS)

5. CONCLUSIONS

Inelastic behaviour of the PS significantly reduces peak values of floor response spectra. In the case of the EP model, peak values of inelastic floor response spectra occur in the resonance region, whereas they are shifted towards higher periods in the case of the Q model. For infinitely rigid equipment, the peak acceleration of equipment is equal to the peak acceleration of the structure. The ratio A_s/A_p is primarily influenced by the damping of SS. The characteristics of ground motion in terms of the characteristic period of ground motion T_C have only a small influence on the maximum values of the ratio A_s/A_p, provided that the ratio T_p/T_C is plotted on the x-axis. Based on the results obtained in the parametric study, a preliminary version of a simple approximate practice-oriented method for determination of inelastic floor response spectra directly from the design response spectrum is proposed. The method is applicable for both EP and Q models of the PS. The results obtained by the proposed method show a satisfactory agreement with the broadened "accurate" results. The proposed approximate direct method can be used for a quick determination of the seismic demand for acceleration controlled equipment.

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