Effect of Column Stiffness on Infill Plate Yielding Distribution in Steel Plate Shear Walls

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SUMMARY:

The lateral force resistance and hysteretic energy dissipation capacity of Steel Plate Shear Walls (SPSWs) are primarily provided through yielding of the infill steel plates attached to the boundary frame of the system. Hence, it is desirable to achieve uniform infill plate yielding in an SPSW during earthquakes. This paper investigates the effects of column stiffness on infill plate yielding distributions in representative two-story SPSWs. Specially considered are the uncertainties existing in infill plate strength and lateral force distributions along the vertical direction. Three probabilistic methods: the Monte Carlo method, the Latin Hypercube Sampling method, and the Rosenblueth's 2K+1 Point Estimate method, are considered. It is shown that all methods are stable and effective; however, the Rosenblueth's 2K+1 method requires the least computational effort. The results show that increasing column stiffness and story ductility capacity can be effective to achieve infill plate yielding at each story of the SPSWs.

Keywords: Steel plate shear wall, Column stiffness, Infill Plate Yielding, Probabilistic evaluation.

1. INTRODUCTION

A typical steel plate shear wall (SPSW) consists of unstiffened thin infill steel plates attached to boundary frame members as shown in Fig. 1.1 (a). The infill plates are allowed to buckle in shear and subsequently form diagonal tension field actions during an earthquake event. Hysteretic energy dissipation and lateral force resistance of the system are primarily achieved through yielding of the infill steel plates. Previous research efforts, both analytical and experimental, have shown this system can behave in a ductile manner and have a high hysteretic energy dissipation capacity compared with conventional braced frames and concrete shear walls, making it an attractive candidate in seismic design community (Qu and Bruneau 2008).



Figure 1.1. An example two-story SPSW (a) actual system; (b) strip model with continuous column.



Seismic design guidelines for SPSWs are available in the American Institute of Steel Construction (AISC) Seismic Design Provisions for Structural Steel Buildings (ANSI/AISC 341) (AISC 2005; AISC 2010) and the Canadian Standards Association (CSA) S16-09, Design of Steel Structures(CSA 2009), which provide design methodologies and procedures to promote the implementation of SPSWs in regions with high seismicity. Similar to the design requirements for other ductile steel seismic force resisting systems, the AISC and CSA documents require capacity design for SPSW columns. Such a design strategy ensures that the SPSW columns remain elastic when the infill plates are fully yielded during an earthquake event with exception of plastic hinges at the column bases when the columns are fully fixed to ground. Additionally, ANSI/AISC 341-05 (AISC 2005) and CSA S16-09 (CSA 2009) require a minimum moment of inertia, i.e., stiffness, for SPSW columns to prevent them from excessive in-plane flexibility and buckling failures observed from prior experimental research (Lubell et al. 2000). While ANSI/AISC 341-10 (AISC 2010) excludes the SPSW column stiffness requirement based on the recent work by Qu and Bruneau (2010), which suggested that the column stiffness limit specified by the codes is uncorrelated to satisfactory SPSW column performance, it is unclear whether the code-specified column stiffness limit brings other advantages to the seismic performance of SPSWs. As such, Section F5 of the Commentary of ANSI/AISC 341-10 (AISC 2010) specifies that opportunity exists for future research to confirm or improve the applicability of the column stiffness requirement. Moreover, CSA S16-09 (CSA 2009) retains the minimum column stiffness requirement for SPSWs

One potential advantage of using relatively stiff columns in a multi-story SPSW is that it may help ensure a more uniform infill plate yielding distribution and therefore reduce drift concentration along the height of a multi-story SPSW during an earthquake event. Conceptually, if the SPSW columns pinned to ground are ideally rigid, the infill plates at different stories will yield simultaneously as a result of the earthquake loading. If the SPSW columns become relatively flexible, infill plate yielding may first occur at a certain story and then progressively spread into the other stories, possibly resulting in a premature failure due to the excessive inelastic deformation at the initially yielded story before infill plate yielding occurs at all the other stories. The yielding sequence of infill plates in a multi-story SPSW depends on its column stiffness and other factors such as infill plate strength distribution and lateral seismic force distribution along the height of the structure.

The purpose of this research is to evaluate the effect of column stiffness on infill plate yielding distribution in SPSWs. Specifically, analyses taking into account the uncertain infill plate strength and lateral seismic force distributions along the height of the structure were conducted to evaluate the probability of achieving infill plate yielding in each story of the SPSWs designed according to the capacity design principle and having the code-specified minimum column stiffness. This paper summarizes the results of probabilistic evaluations of representative two-story SPSWs together with discussions of the implication of the code-specified minimum column stiffness from the perspective of infill plate yielding distribution in the system. Also addressed in the paper are the stability, effectiveness, and efficiency of three different probabilistic simulation methods for evaluation of SPSWs.

2. ANALYTICAL MODEL AND CLOSED-FORM RESULTS

To understand the effect of column stiffness on seismic performance of SPSWs, consider the twostory SPSW building as shown in Fig.1.1(a). For simplicity, assumptions made in the system include: 1) infill plates at both stories have the same yield strength and exhibit elastic-perfectly-plastic hysteretic behaviour; 2) consistent with the current codes, boundary frame members of the wall are designed according to the capacity design approach, i.e., only the infill plates are allowed to develop inelastic behaviour in the system under earthquake loading; 3) story heights and column cross-section properties are constant over both stories; 4) the beam-to-column and column-to-ground connections are pinned; 5) the infill plate tension fields orientation angles are the same at both stories and equal to 45° from the vertical. It is recognized that orientation angle of the infill plate tension field action depends on infill plate height, length, and thickness, and properties of the boundary frame members; the assumed value, 45°, is typically chosen for SPSW design and analysis (Driver et al. 1997).

Results from prior investigations (Thorburn *et al.* 1983; Ji *et al.* 2009, MacRae *et al.* 2004) indicate that performance of the SPSW shown in Fig. 1.1(a) can be captured by a model shown in Fig. 1.1(b), which includes a continuous column representing the contribution of the two columns in the original SPSW and connected by rigid links to a two-story frame with pinned member-to-member connections and infill plates modelled as a series of discrete pin-ended strips inclined with the same orientation as the infill tension field action. When the system shown in Fig. 1.1(b) is under the lateral earthquake forces, infill plate yielding may initially occur in one story and then spread into the other story.



Figure 2.1. Yield progressions of the two-story system (a) wall deflection associated with Yield Progression I; (b) wall deflection associated with Yield Progression II; (c) development of infill strip stresses associated with Yield Progression II Yield Progression II; (d) development of infill strip stresses associated with Yield Progression II

Figs. 2.1(a) and (b) illustrate the two yield progressions expected in the system, i.e., Yield Progressions I and II, which represent the cases in which infill plate yielding initially occurs at the first and second stories, respectively. Infill plate yielding sequence and distribution along the height of the structure depends on the following three key parameters in the system: the relative stiffness of continuous column to infill plate, the lateral seismic force distribution, and the infill plate strength distribution. To quantify the relative stiffness of continuous column to infill plate, stiffness of continuous column to infill plate.

$$\alpha = \frac{EI_{\rm c}}{k_1 h^3} \tag{2.1}$$

where *E* represents the modulus of elasticity of steel, *h* represents the story height; I_c represents the moment of inertia of the continuous column; and k_1 represents the stiffness of the first-story infill strips that can be determined according to Thorburn *et al.* (1983) as:

$$k_1 = \frac{1}{4} E t_1 \left(\frac{L}{h}\right) \tag{2.2}$$

where L and t_1 represent length and thickness of the first-story infill plate, respectively.

As indicated in Eqn. 2.1, a smaller value of α corresponds to a more flexible continuous column and vice versa. In addition to the stiffness ratio, define the following two parameters:

$$\lambda = \frac{F_2}{F_1} \tag{2.3}$$

$$\gamma = \frac{t_2}{t_1} \tag{2.4}$$

where F_2 and F_1 are the lateral forces applied at the second and the first stories, respectively; and t_2 and t_1 are the infill plate thicknesses at the second and first stories, respectively. It is recognized that λ and γ describe the lateral seismic force and infill plate thickness distributions in the system. According to Berman and Bruneau (2004), the shear strength of each infill plate is proportional to its thickness; therefore, γ is also an index of infill plate strength distribution in the system.

As analyzed in Qu *et al.* (2012), when $\gamma > [\lambda/(1+\lambda)]$, overstrength exists in the second story and infill plate yielding will initially occur at the first story (i.e., Yield Progression I controls); when $\gamma < [\lambda/(1+\lambda)]$, overstrength exists in the first story and infill plate yielding will initially occur at the second story (i.e., Yield Progression II controls); and when $\gamma = [\lambda/(1+\lambda)]$, the strength and demand have the same distribution along the height of the structure and both stories will yield simultaneously. While $\gamma = [\lambda/(1+\lambda)]$ is the most desirable case in which uniform plate yielding is expected, it is unlikely to occur in practice due to the facts that 1) the required plate thickness determined from the story shear force may not be always available from steel producers and thicker plates typically have to be used for design; and 2) the actual seismic forces acting on the system may be different from those assumed in design due to redistribution of live loads and progressive development of nonlinear behavior of the system.

Figs. 2.1 (c) and (d) illustrate the development of infill plate stresses as the lateral deformation increases in the system. The critical cases identified in the figures correspond to the points at which infill plate yielding just develops at both stories. Define the story ductility demand, μ_s , as ratio of the drift of the initially yielded story associated with the critical cases to the yielding drift of the story. If story ductility capacity of the wall, μ_{so} , which can be identified from experimental investigations, is greater than or equal to μ_s , infill plate yielding can be achieved in both stories of the system; otherwise, infill plate yielding will be limited to the initially yielded story when the failure due to excessive inter-story deformation occurs.

As derived and validated in Qu *et al.* (2012), μ_s can be calculated as:

$$\mu_{\rm S} = \begin{cases} \frac{\mu_{\rm tcI}}{2} \frac{\text{DCF}_{\rm CI}}{\frac{1}{[\chi](1+3\alpha)} + \frac{3\alpha}{2(1+3\alpha)}} & \text{when } \gamma > \frac{\lambda}{1+\lambda} \\ \frac{\mu_{\rm tcII}}{2} \frac{\text{DCF}_{\rm CII}}{1 - \left(\frac{1}{[\chi](1+3\alpha)} + \frac{3\alpha}{2(1+3\alpha)}\right)} & \text{when } \gamma < \frac{\lambda}{1+\lambda} \end{cases}$$
(2.5)

where

$$DCF_{CI} = \frac{2 + \frac{3\alpha\lambda}{\gamma(1+\lambda)} + \frac{3\alpha}{\gamma} - \frac{2\lambda}{\gamma(1+\lambda)} \frac{\left\{\frac{1}{|\chi|} + \frac{3\alpha}{2}\right\}}{1+3\alpha} \frac{1}{\mu_{tcI}}}{1+3\alpha}$$

$$(2.6)$$

$$DCF_{CII} = \frac{\frac{\lambda}{1+\lambda} \frac{1}{\gamma} (3\alpha+2) + \frac{3\alpha}{\gamma} - \frac{1}{3\alpha+1} \left[2 + 3\alpha - \frac{2}{[\chi]} \right] \frac{1}{\mu_{tcII}}}{\frac{\lambda}{1+\lambda} \frac{1}{\gamma} (3\alpha+1) + \frac{3\alpha}{\gamma}}$$
(2.7)

$$\mu_{tcI} = \frac{2 + 3\alpha[\chi]}{(1 + 3\alpha)[\chi]} \left[\frac{\gamma + \gamma \lambda - \lambda}{3\alpha(2\lambda + 1)} + 1 \right]$$
(2.8)

$$\mu_{\rm wII} = \frac{\left[\frac{\lambda}{1+\lambda}\frac{1}{\gamma}(3\alpha+1) + \frac{3\alpha}{\gamma}\right]\frac{(2+3\alpha)[\chi]-2}{2(1+3\alpha)[\chi]} - \frac{1}{3\alpha+1}\left[1 + \frac{3\alpha}{2} - \frac{1}{[\chi]}\right]}{\frac{3\alpha}{2\gamma}\frac{\lambda}{1+\lambda} + \frac{3\alpha}{2\gamma}}$$
(2.9)

$$[\chi] = \begin{bmatrix} \frac{\lambda}{\gamma(1+\lambda)} + \frac{1+\frac{3\alpha}{\gamma}}{1+3\alpha} \\ \frac{1+\frac{3\alpha}{2\gamma} - \frac{3\alpha}{2} \left(\frac{\frac{3\alpha}{\gamma}+1}{1+3\alpha}\right)} \end{bmatrix}$$
(2.10)

While the expressions given in Eqns. 2.5 to 2.10 are complex, the minimum story ductility required to achieve infill plate yielding in both stories, i.e., μ_s , essentially only depends on three parameters, α , γ , and λ . In practice, α can be evaluated based on the specific infill plate and column members; however, the actual values of γ and λ , may not be the same as those determined from ideal design assumptions due to the following reasons: 1) redistribution of live load and development of inelastic behavior of the system may change the lateral seismic force distribution (i.e., λ); and 2) the plate thickness determined from specific story shear force may not be available and plates thicker than required have to be used at some stories, leading to uneven infill plate overstrength distribution along the height of the structure. Therefore, it is necessary to consider γ and λ , as random variables when evaluating seismic performance of the system. These closed-form expressions for behavior of the two-story SPSW are used in the probabilistic evaluations described in the following sections.

In probabilistic performance evaluation of a system, the following function can be used to quantity the probability of failure:

$$P[Y(X_1, X_2, \dots, X_K) > y] = 1 - P[Y(X_1, X_2, \dots, X_K) \le y] = 1 - \Phi\left(\frac{y - \theta_Y}{\sigma_Y}\right)$$
(2.11)

where Y represents a selected response quantity of the system (e.g., demand) and it is function of a series of random variables describing the properties and determining the performance of the system, i.e., X_1 through X_K , y represents the performance limit, i.e., a maximum value of the response quantity allowed in the considered system without causing failures; θ_Y and σ_Y respectively represent the mean and standard deviation of Y, and Φ is the cumulative normal distribution function. It is noted that a goodness-of-fit test is required to confirm that the response quantity of interest follows the normal distribution.

For the two-story SPSWs considered here, Eqn. 2.11 can be specifically given as:

$$P\left[\mu_{\rm S}(\gamma,\lambda) > \mu_{\rm SO} \middle| \alpha = \alpha_0 \right] = 1 - P\left[\mu_{\rm S}(\gamma,\lambda) \le \mu_{\rm SO} \middle| \alpha = \alpha_0 \right] = 1 - \Phi\left(\frac{\mu_{\rm SO} - \theta_{\mu_{\rm S}}}{\sigma_{\mu_{\rm S}}}\right) \middle|_{\alpha = \alpha_0} (2.12)$$

where θ_{μ_s} and σ_{μ_s} respectively represent the mean and standard deviation of μ_s ; and α_0 represents a selected level of α of interest. Essentially, Eqn. 2.12 calculates the probability of failure to achieve infill plate yielding in both stories of the system for a given stiffness ratio.

3. PROBABILISTIC METHODS AND SELECTION OF PARAMETER RANGES

In practice, the mean and standard deviation of the response quantity of interest, which are required for calculating the probability defined in Eqns. 2.11 and 2.12, can be evaluated using a proper probabilistic analysis procedure. Three sampling and simulation methods, namely, the Monte Carlo method, the Latin Hypercube Sampling method, and the Rosenblueth's 2K+1 Point Estimate method, respectively referred to herein as M.C., L.H., and 2K+1, are considered in this investigation. Step-by-step procedures for implementing these methods in the investigation are provided in Guo (2011).

As required in the probabilistic simulation methods, two random variables, λ and γ , which describe the lateral seismic force and infill plate thickness distributions, need to be sampled. Basically, the lateral seismic force distribution can be determined by modal analysis and is based on the reactive weight distribution and stiffness of the structure. In this investigation, λ is assumed to vary from a uniform distribution (which corresponds to $\lambda = 1.0$), to an inverted triangular distribution (which corresponds to $\lambda = 3.0$), allowing investigation of the SPSW building under a broader range of lateral seismic force distributions. Given that the shear force at the second story is always smaller than that at the first story, the infill panel at the second story should not be thicker than that at the first story, which requires

$$0 < \gamma \le 1.0 \tag{3.1}$$

It is recognized that $\gamma = 1.0$ corresponds to the case in which the infill plates at both stories have the same thickness. This may happen when the minimum thickness required at the second story is not available and the plate used in the first story is used. As discussed in Qu *et al.* (2012), Yield Progression I controls if $[\lambda/(1+\lambda)] < \gamma \le 1$, and Yield Progression II controls if $0 < \gamma < [\lambda/(1+\lambda)]$. In order to consider both Yield Progressions I and II without being biased, γ is sampled such that the probabilities of falling into each yield progression are the same. In addition, γ is assumed to follow the uniform probability distribution over the regions associated with Yield Progressions I and II.

In addition to the abovementioned two random variables defining the demand and capacity

distributions along the building height, range of the stiffness ratio, α , also must be specified. Theoretically, α varies from zero to infinity. However, this investigation will focus on the range from 0.001 to 1.0 for the following reasons: 1) when α is smaller than 0.001, the SPSW columns may be impractically flexible; and 2) when α is greater than 1.0, the difference in system performance caused by the variation of α is negligible as shown in Qu *et al.* (2012).

4. RESULT COMPARISONS AND DISCUSSIONS

Eqn. 2.12 calculates the probability of failure in the considered system based on the sample values of the random variables, γ and λ . When the sampling number approaches infinity or a sufficiently large value, ideally, the probability of failure from simulations should converge to the theoretical value. However, when the sampling number becomes more practical (which may be relatively small), it is necessary to check whether consistent results can be obtained when repeating the simulation, i.e., check stability of the considered methods. In this investigation, three analyses were conducted for each method. In the L.H. method, one hundred strata are partitioned for γ and λ , respectively, indicating that one hundred simulations are need at each considered level of α . In the M.C. method, γ and λ are sampled 5510 times at each considered level of α . Based on the values of γ and λ sampled for the M.C. method, the mean and standard deviation of γ and λ are evaluated for use in the 2K+1 method. Since the considered problem includes two random variables (γ and λ), the 2K+1 method requires only 5 simulations at each considered level of α .

Assuming the story ductility capacity, μ_{SO} , required in the probability calculation is 12, Fig. 4.1 compares the results of three analyses from each method. As shown, consistent results are observed in all the analyses of each method, indicating the stability of these methods. It is also found that all the three methods provide identical results except that the 2k+1 method slightly underestimates the probability of failure when α is smaller than 0.02. It is recognized that the 2K+1 method requires the computational efforts significantly less than the M.C. and L.H. methods and therefore the 2K+1 method is identified to be the most efficient. Beyond stability, effectiveness, and efficiency, another investigation conducted as part of this research was to evaluate whether consistent results can be observed from the considered methods when different probabilistic distributions are assumed for γ and λ . It is found that all three methods provide very similar results when the coefficient of variation of each random variable is within 0.3. Complete result comparisons of these considered probabilistic distributions are available elsewhere (Guo 2011).



Figure 4.1. Result comparisons of the considered methods.

Results generated in the investigation form a database to evaluate the effectiveness of the codespecified minimum column stiffness for achievement of infill plate yielding at both stories of the twostory SPSWs. The stiffness requirement of SPSW columns specified in both CSA S16-09 (CSA 2009) and ANSI/AISC 341-05 (AISC 2005) is intended to avoid SPSW column failures observed in Lubell *et al.* (2000). Such a stiffness limit was derived based on the plate girder flange flexibility factor, ω_h , developed in an early study of the elastic behavior of plate girders with thin metal webs subjected to transverse shear (Wagner 1931). By analogy, some aspects of SPSWs are similar to those of plate girders; therefore, the SPSW column flexibility factor at each panel is obtained from the plate girder flange flexibility factor expressed below:

$$\boldsymbol{\omega}_{\rm h} = 0.7h \left(\frac{t}{2IL}\right)^{0.25} \tag{4.1}$$

where *t* is the infill plate thickness; *I* is the moment of inertia of SPSW column, and the other variables have been defined previously.

Noting that the previously tested SPSW specimens which exhibited undesirable column failures had flexibility factors of 3.35 and that all other known tested SPSWs that behaved in a ductile manner had flexibility factors of 2.5 or less (Montgomery and Medhekar 2001), an upper bound of 2.5 on ω_h was empirically selected, which forms the current column stiffness requirement specified in CSA S16-09 (CSA 2009).

Imposing the upper bound of 2.5 on Eqn. 4.1 and solving for *I* leads to the following requirement implemented in ANSI/AISC 341-05 (AISC 2005):

$$I \ge \frac{0.00307th^4}{L}$$
(4.2)

Considering Eqns. 2.1, 2.2, and 4.2, one can obtain the following requirement

$$\alpha_{\min} = 0.02456 \left(\frac{L}{h}\right)^{-2} \tag{4.3}$$

where (L/h) represents the length-to-height aspect ratio of the infill plate. While no theoretical limits exist on the aspect ratio, ANSI/AISC 341-05 (AISC 2005) recommends the following limits on infill plate aspect ratio for desirable system performance:

$$0.8 \le \frac{L}{h} \le 2.5 \tag{4.4}$$

Based on Eqn. 4.3, the above limits indicate the following values for α_{\min} :

$$\alpha_{\min} = \begin{bmatrix} 0.00393 & 0.03838 \\ 0.003838 & 0.03838 \end{bmatrix}$$
(4.5)

The above values for α_{\min} are included in the results compared in Fig. 4.2. The story ductility capacity, μ_{SO} , are chosen to be 6, 8, 10 and 12 based on the previous experimental investigations (Berman and Bruneau 2005; Vian *et al.* 2009; Chen and Jhang 2006). The following observations can be consistently obtained from Fig. 4.2 for the walls with columns designed according to the minimum code-specified stiffness:

- 1. As indicated by the probability of failure close to 1.0, infill plate yielding in both stories is unlikely to occur in the walls with the upper bound value of the length-to-height aspect ratios (see the results at $\alpha_{\min} = 0.00393$).
- 2. For the walls with the lower bound value of the length-to-height aspect ratios, the probability of failure is typically smaller than 0.2, indicating infill plate yielding is more likely to occur in both stories (see the results at $\alpha_{\min} = 0.03838$).
- 3. Generally, the probability of achieving plate yielding in both stories is higher in the walls with larger α_{\min} values, i.e. in the walls with smaller aspect ratios as indicated in Eqn. 4.3.

4. The story ductility capacity of SPSWs plays an important role. As shown, the system with a smaller story ductility capacity has a higher probability of failure to achieve infill plate yielding in both stories.



Figure 4.2. Probability of failure to achieve infill plate yielding in both stories of the two-story SPSW.

5. CONCULUSIONS

This paper investigates the effect of column stiffness on whether infill plate yielding can occur at each story of representative two-story SPSWs. Specially considered in the investigation are the uncertainties existing in infill plate strength and lateral seismic force distributions along the vertical direction of the structures. The SPSWs were evaluated based on the simplified model consisting of a strip model connected by rigid links to a continuous column which takes into account the contribution of the SPSW columns. A total of three probabilistic simulation methods, the M.C., L.H, and 2K+1 methods, were adopted. It is found that all the probabilistic evaluation methods are stable and effective for SPSW seismic performance evaluations; however, the 2K+1 method is the most efficient and is recommended for future investigations. Additionally, the results indicate that infill plate yielding is more likely to occur in the SPSWs with a smaller infill plate length-to-height aspect ratio if the columns are designed according to the capacity design approach and have the code-specified minimum column stiffness. Moreover, it is found that increasing the column stiffness and story ductility capacity is effective for achieving a more uniform infill plate yielding distribution in SPSWs subjected to earthquake loading.

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