The non homogeneous soil profile effect on the dynamic response of structures

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SUMMARY:
Site effect is one of the phenomena of the spatial variability of the seismic motion, which should be taken into account in the aseismic code design of structures. Generally, the geotechnical characteristics, particularly the shear modulus, increase with the depth. During the last decade an analytical formulation of the transfer function, considering a continuous variation of the soil shear modulus has been established for the zero and non-zero shear wave velocity at the free surface. In the present work, a statistical analysis of the pseudo acceleration spectra, PSA, and the shear forces of the structures is investigated under the non homogeneous soil effect. These parameters are carried out by using Monte Carlo simulation with Shinozuka spectral representation. The structures are modeled as an oscillator with concentrated mass, two supports having the same stiffness under translational and unidirectional seismic excitations which take into account the non homogeneous site effect. The Kanai-Tajimi model filtered with the Clough and Penzien model is used for the seismic input at the bedrock. Firstly, the same soil parameters are considered under the two supports (fully correlated excitations), secondly, the case of different soil parameters (partially correlated excitations) is analyzed. Results show that PSA is amplified with increasing the non homogeneous parameter and less influenced by the geological conditions variability effect in the two supports. While, the shear forces are generally more important in the case of non uniform ground motion, where the pseudo static effect is predominant. The contribution rate of the pseudo static component depends essentially on the non homogeneous parameter '*p*' and on the ratio between the soil bedrock interface shear wave velocity and the height layer, 'V_0/H'.

Keywords: Site effect-non homogeneous-Simulation-PSA-Shear forces

1. INTRODUCTION

As part of the seismic design of buildings, it is very common to assume that the whole basis of the structure is subjected to a uniform ground motion. In other words, the supports of the structure are assumed excited in the same way and synchronously by the seismic motion. So, the assumption of uniform ground motion is not appropriate in the case of extended structures. Several studies were carried out on the effects of spatial variability of seismic motions on the response of extended structures (Harichandran and Wang, 1988; Zerva 1991, Berrah and Kausel, 1992, 1993; Der Kiureghian and Neuenhofer, 1992; Zavoni-Heredia and Vanmarcke, 1994; Slimani and Berrah, 1996, 1997; Zembaty and Rutenberg, 2002; Wang et al., 2009; Mwafi et al., 2011).

All the investigations showed that during an earthquake, a structure is not only subject to inertial effects (due to dynamic loading assumed to be uniform), but also those caused by the ground motion variability in the space (generated by the differential movement of supports). The latter is the result of several phenomena that can be grouped into the following three points: wave passage effect which is due to differences between the recorded arrival times of seismic waves at various recording stations; incoherency effect defined as the loss of coherence of seismic waves during the path, and site effect defining the local geological conditions which vary from one station to another in the same site.

Unlike the first two effects that shown their influence on the structural response which increases with distance between supports, the site effects can be very important even for short distances. The study of the structural response under this effect showed that the internal forces generated vary greatly from one soil type to another (Somerville et al., 1991; Zerva and Harada, 1994; Rasem et al. 1996; Der Kiureghian et al., 1997, Zembaty and Rutenberg, 2002; Hadid and Afra, 2000).

In the present paper, we evaluate the inhomogeneity effect on the maximum acceleration spectrum (PSA) and shear forces of a one story shear building. A statistical analysis is performed on PSA and shear forces by simulating the seismic excitations at the structure supports.

2. THE INHOMOGENEOUS SOIL PROFILE EFFECT ON THE AMPLIFICATION FUNCTION

2.1. Mathematical Formulation of Model

The frequencies of vibration and the amplification function of a soil profile depend essentially on its geometrical and mechanical characteristics. The mechanical characteristics and, in particular, the shear modulus of the soil depends upon the depth, from the free surface. Experiences have shown that this variation is a power of the depth (Richart et al., 1970 Dobry et al., 1971). For example (Gazetas, 1984), for a uniform site of normally consolidated soft clays, the soil shear modulus varies linearly with depth Z_s ; for cohesionless materials, it varies square root of Z_s and for a site on stiff overconsolidated clay deposits, it is constant. Knowing the frequencies of vibration and the amplification function of a site is very important to study the variation effect in local geological conditions on the seismic design of structures. In the general case, we assume that the variation of the shear modulus of the soil layer, $\boldsymbol{G_s}$, is of the form (Idriss and Seed, 1968; Pecker, 1995; Afra, H., 1995) (see Fig. 1).

$$G_s(Z_s) = G_0 \left(\frac{Z_s}{H}\right)^p \tag{1}$$

where G_0 is the shear modulus at depth H, p is the inhomogeneous parameter which varies $0 \leq p \leq 1$ and H is the thickness of the soil profile.

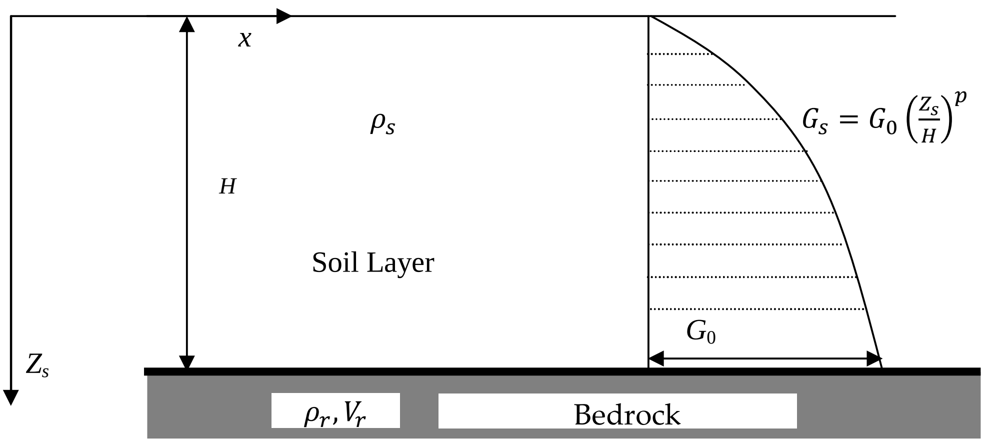

Figure 1. Inhomogeneous soil profile and distribution of shear modulus with depth

After some algebra, an analytical formulation of the transfer function of soil profile overlying bedrock is established for a zero shear wave velocity at the free surface (Pecker and Afra, 1995; Hadid and Afra, 2000).

$$H^*(\tilde{\omega}) = \frac{1}{\Gamma\left(\frac{1}{2-p}\right)} \frac{\left(\frac{1}{2-p}\tilde{\omega}\right)^{\nu}}{J_{\nu}\left(\frac{2}{2-p}\tilde{\omega}\right) + iqJ_{\nu+1}\left(\frac{2}{2-p}\tilde{\omega}\right)} \tag{2}$$

Γ is the gamma function and * indicate the conjugate.
J_{ν} is the Bessel function of the first kind of order $\nu = \frac{p-1}{2-p}$.
$\tilde{\omega} = \frac{\omega H}{V_0}$ is the dimensionless frequency, and $q = \frac{\rho_s V_0}{\rho_r V_r}$ is the impedance ratio where ρ_s, V_0 and ρ_r, V_r are the densities and shear wave velocities at depth H (for soil) and rock, respectively.

For a damped case with a damping ratio ζ, we replace G_0 in the preceding equations by $G_0(1 + 2i\zeta)$.

2.2. Parametric Study

The parametric study is based on the parameters p and q (Fig. 2), assuming a damping ratio of 5% and analysis of results highlighted the following conclusions (Afra and Hadid, 2000):

- The variation of the shear modulus in depth defined by the inhomogeneity parameter p affects both the frequency content and the amplitude of the amplification function. The amplification is larger for loose soils than for cohesive soils and rigid.
- The bedrock flexibility defined by the impedance parameter q affects only the amplitude of the amplification function. Radiative dissipation increases with increasing this parameter.

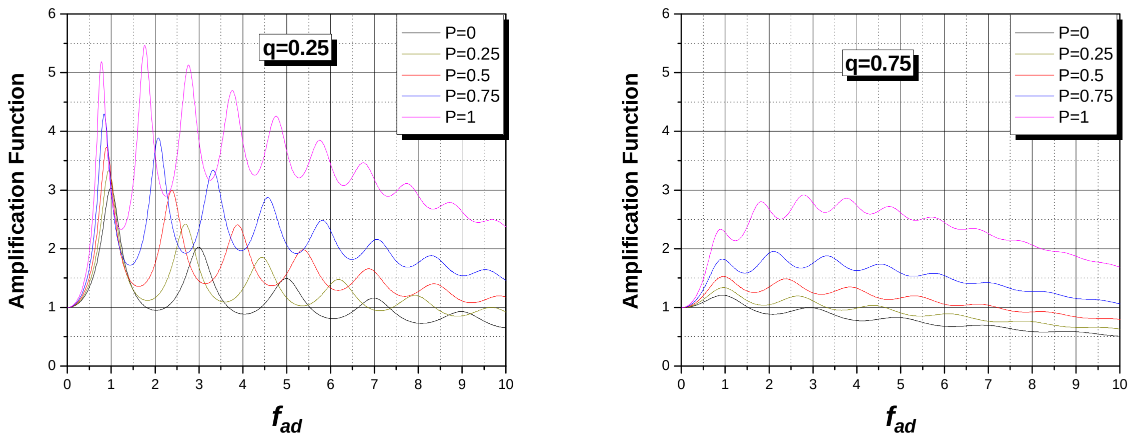

Figure 2. Influence of the inhomogeneity parameter 'p' on the transfer function ($f_{ad} = (2\omega H)/(\pi V_0)$)

3. MODELING OF SPATIAL VARIABILITY OF SEISMIC GROUND MOTION

For practical considerations, seismic ground motions, exhibiting a random nature, can be modeled in a probabilistic manner by a random field. Therefore, it is reasonable to assume that each component of the seismic acceleration is a spatially-temporal homogeneous unidimensional random field. For stationary stochastic processes, the coherency function is defined as the ratio of the cross-power spectral density between two stations k and l, $S_{kl}(i\omega)$ and the square root of their corresponding auto-power spectral densities, $S_{kk}(i\omega)$ and $S_{ll}(i\omega)$. In the present study, we use the coherency model derived by Der Kiureghian (Der Kiureghian, 1996) to evaluate the site response effect on the PSA and shear forces. The coherency function accounting for only the site effect is defined by:

$$\gamma_{kl}(i\omega) = exp\left(i\theta_{kl}(i\omega)\right) \tag{3}$$

The phase difference $\theta_{kl}(\omega)$ is given by :

$$\theta_{kl}(\omega) = tan^{-1}\left(\frac{Im[H_k^*(i\omega)H_l(i\omega)]}{RE[H_k^*(i\omega)H_l(i\omega)]}\right) \quad (4)$$

where $H_k(i\omega)$ and $H_l(i\omega)$ are the soil profile transfer functions at stations *k* and *l* respectively.

4. SIMULATION OF THE SEISMIC EXCITATIONS AT THE FREE FIELD

The seismic excitations at the free field considered as stationary random processes, Gaussian and unidimensional can be simulated using the method of spectral representation proposed by Shinozuka et al. (Shinozuka et al., 1987). The expression of the simulated field model for two stations is given by:

$$\begin{cases} \ddot{u}_1(t) = 2\sum_{j=1}^{N}\sqrt{S_{11}(\omega)\Delta\omega}\cos(\omega_j t + \varphi_{1j}) \\ \ddot{u}_2(t) = 2\sum_{j=1}^{N}\sqrt{S_{22}(\omega)\Delta\omega}\,|\gamma_{12}(i\omega)|\cos(\omega_j t + \theta_{12}(\omega_j) + \varphi_{1j}) \\ \qquad +\sqrt{S_{22}(\omega)\Delta\omega}\sqrt{1-|\gamma_{12}(i\omega)|}\cos(\omega_j t + \varphi_{2j}) \end{cases} \quad (5)$$

with $S_{kk}(\omega) = S_0|F_1(i\omega)|^2|F_2(i\omega)|^2|H_k^*(i\omega)|^2, \ k = 1,2$ (6)

where

$\Delta\omega$ is the frequency increment, $\omega_j = j\Delta\omega$,
$N\,\Delta\omega$ is the maximum frequency (Nyquist frequency) and $\varphi_{1j}, \varphi_{2j}$ are independent random phase angles and uniformly distributed [0, 2π].
S_0 is a white noise amplitude and $F_1(i\omega)$, $F_2(i\omega)$ are Kanai-Tajimi (Kanai, 1957; Tajimi, 1960) and Clough and Penzien filters (Clough and Penzien, 1975), respectively. The two filters are defined as follows

$$|F_1(i\omega)|^2 = \frac{1+4\beta_g^2(\omega/\omega_g)^2}{\left[1-(\omega/\omega_g)^2\right]^2+4\beta_g^2(\omega/\omega_g)^2} \quad (7)$$

$$|F_2(i\omega)|^2 = \frac{(\omega/\omega_f)^4}{\left[1-(\omega/\omega_f)^2\right]^2+4\beta_f^2(\omega/\omega_f)^2} \quad (8)$$

in which $\omega_g = 10\pi\ rd/s$ and $\beta_g = 0.8$ for the Kanai-Tajimi filter
$\omega_f = 1.636\ rd/s$ and $\beta_f = 0.619$ are the constants proposed by Ruiz and Penzien

The simulated stationary excitations are then windowed in their beginning and ending phases to obtain more realistic seismic input. The following envelope is used:

$$a(t) = \begin{cases} (t/3)^2 & 0 \le t \le 3s & (Beginning\ phase) \\ 1 & 3 \le t \le 13s & (Strong\ phase) \\ exp[-0.26(t-13)] & t \ge 13s & (Ending\ phase) \end{cases} \quad (9)$$

The simulation and filters parameters are the same used in evaluating the inhomogeneity effect on the PGA at the free surface (Slimani et al, 2011).Time histories are simulated for a duration T=40.96sec with a 100 sps (temporal increment Δt equal to 0.01s) and Nyquist frequency value f_c = 50Hz. The simulation is carried out using the FFT technique with a frequency resolution Δf = 0.024 Hz. The S_0 value is taken equal to 11.13 10^{-5} m^2/s^3 so that the average peak acceleration (PGA) at the bedrock derived from the simulation of 1000 realizations is equal to 0.1 g.

5. EQUATION OF MOTION OF THE ONE STORY SHEAR BUILDING

The equation of motion of the 1 degree of freedom linear system with natural frequency ω_0 and damping ratio β_0 can be written as follows :

$$\ddot{x}(t) + 2\beta_0\omega_0\dot{x}(t) + \omega_0^2 x(t) = -\frac{[\ddot{u}_1(t)+\ddot{u}_2(t)]}{2} = -\ddot{u}_e(t) \tag{10}$$

with $\ddot{x}(t), \dot{x}(t), x(t)$ denote oscillator acceleration, velocity and displacement respectively.
$\ddot{u}_e(t)$ is the equivalent input motion at the free surface.

- The pseudo static and total displacements

The pseudo-static and total displacements are given by the following formulas:

$$x^s(t) = u_e(t) \tag{11}$$

$$x^t(t) = x^s(t) + x(t) \tag{12}$$

- The pseudo static and dynamic Shear forces

The pseudo-static and dynamic shear forces in the kth column are given by:

$$R_k^s(t) = K\left[x^s(t) - u_j(t)\right] \tag{13}$$

$$R_k^d(t) = K\, x(t) \tag{14}$$

with *K* is the column rigidity

6. INHOMOGENEITY EFFECT ON THE PSA IN THE CASE OF IDENTICAL GEOLOGICAL CONDITIONS

Firstly, one need to analyze the effect of the soil inhomogeneity, flexibility or stiffness on the pseudo spectrum acceleration (PSA) in the case of identical geological conditions at the two supports. Figs. 3 and 4 show the effect of the inhomogeneity parameter *p*, the ratio V_0/H on the mean and standard deviation of the PSA. PSA curves are plotted versus the frequency ($f_0 = 2\pi\omega_0$) range <0-100Hz>. From these two figures, we notice that the curves exhibit peaks that coincide with the soil resonant frequencies. Generally, these peaks are not visible when the impedance ratio is low ($q \approx 1$) and the soil is flexible and especially on curves of mean PSA. In the dynamic analysis of structures, we say that a structure, subject to a given excitation is in resonance when its fundamental frequency coincides with the fundamental frequency of the ground. Now, from this analysis that takes into account higher modes of the ground, this definition is not always true. This parametric analysis of the two statistical moments of PSA shows that the spectral amplitude is also important at frequencies near those of higher modes of the soil profile and therefore the PSA is characterized by a wide band frequency range (low and high frequencies). This amplitude can be even more important, at these frequencies, compared to that corresponding to the fundamental mode (Aouali, 2008), especially when the impedance ratio is low (firm soil), soil is flexible (V_0/H is low) and the soil is greatly inhomogeneous ($p \approx 1$). In the case studied, for a soil with an impedance ratio, q=0.5, generally, the inhomogeneity of the soil leads to an amplification of the mean and standard deviation of PSA (Fig. 3). For greatly inhomogeneous profile soil, the mean and standard deviation of PSA can be amplified by more than three times on the frequency range including the resonant frequencies of the ground. One can notice that the inhomogeneity leads to PSA amplification at frequencies coinciding with natural frequencies of the soil and especially the higher modes.

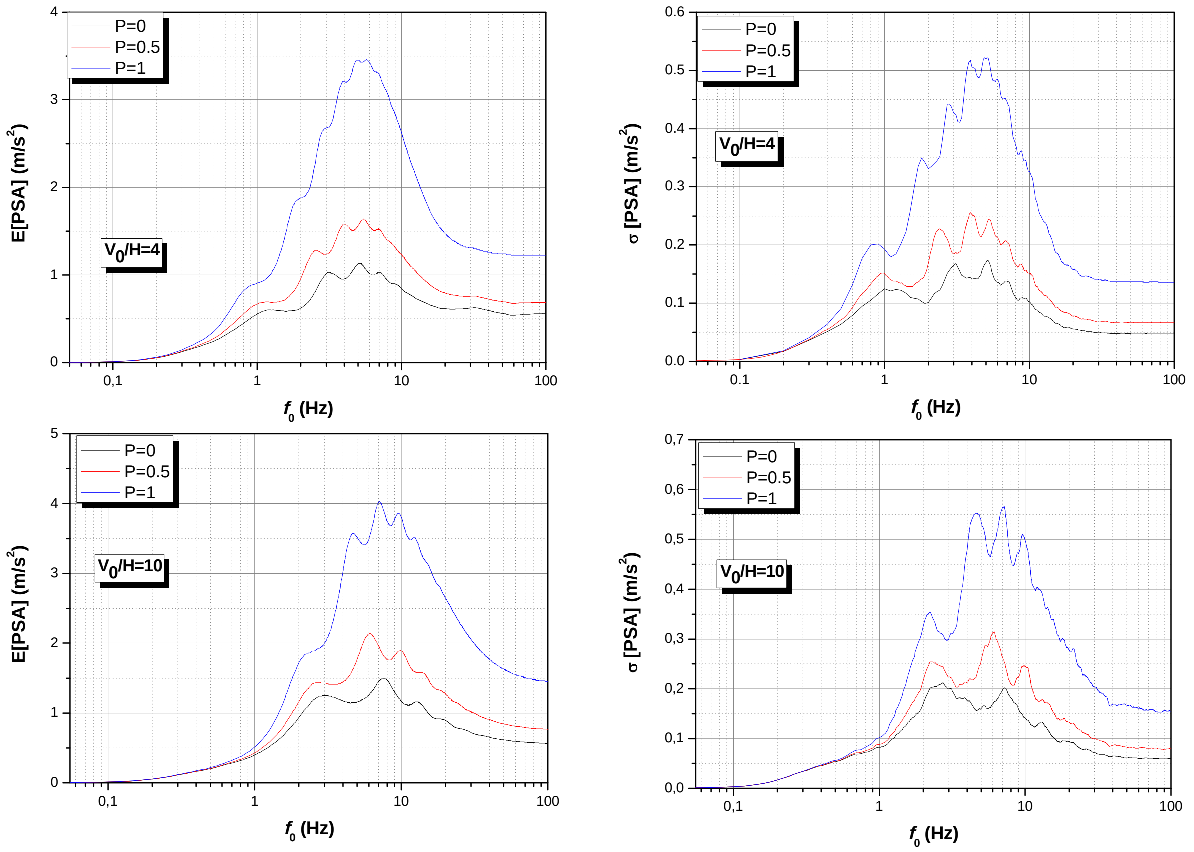

Figure 3. Inhomogeneity *p* and V_0/H parameters effects on PSA variation (q=0.5).

7. INHOMOGENEITY EFFECT ON THE PSA IN THE CASE OF DIFFERENT GEOLOGICAL CONDITIONS

7.1. Pseudo Acceleration Spectra

After analyzing the case where local site conditions are the same, it is interesting to extend the study for local site conditions variability. This latter may be due to the individual or combined effect of parameters p and V_0/H. Figure 4 shows the variation of the mean and standard deviation of the PSA as a result of the variability of local site conditions at the supports. We consider the support 1 founded at the rock and the support 2 founded at the soil (homogeneous or greatly inhomogeneous). We consider three soil types, flexible (V_0/H = 4), medium (V_0/H = 10) and rigid (V_0/H = 20).

The results show that the two statistical moments of PSA decrease when the soil becomes flexible. As was already shown for the case of identical local conditions, the mean and the standard deviation increase with the degree of inhomogeneity of the soil (Fig. 3). One can notice that if the support 1 is founded at the rock, the statistical parameters values are generally lower compared with those corresponding to identical geological conditions at the two supports and particularly for inhomogeneous soils. This allows to say that the PSA is positively influenced by the effect of the variability of geological conditions at the two supports.

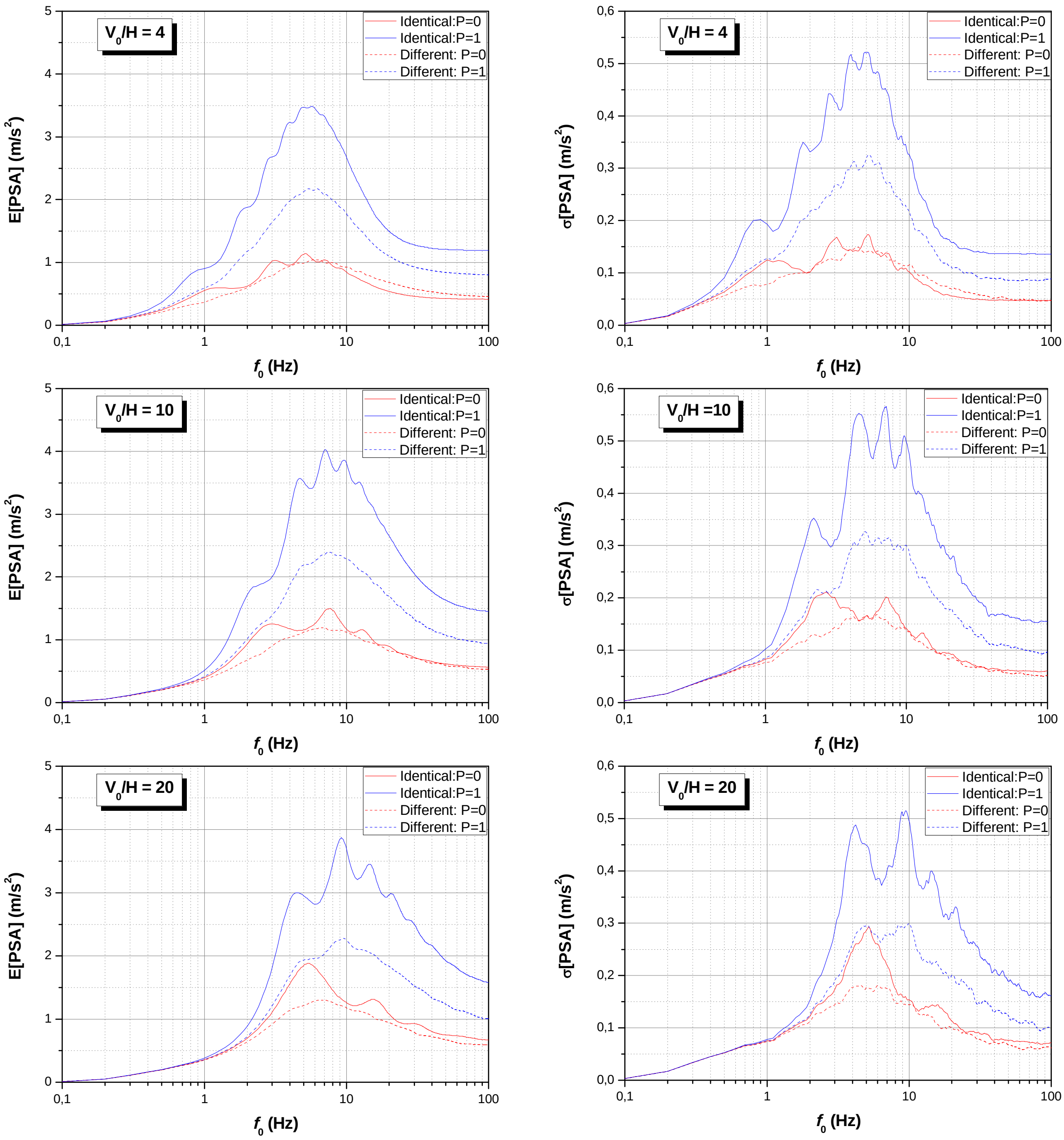

Figure 4. Inhomogeneity *p* and V_0/H parameters effects on PSA variation in the case of different local geological conditions (q=0.5).

7.2. Shear forces spectra at the shear building supports

Indeed, if the seismic input is uniform (identical site conditions), the shear force (SF) at the base or the support reaction force is proportional to the dynamic displacement (relative displacement between the oscillator mass and the supports), and has the same value at all supports. In other words, the pseudo-static shear force is zero when the seismic motion at the supports is uniform. However, when there is a ground motion variability, the contribution of the pseudo-static component exists and the total shear force is not identical at the supports (Fig. 5). In the present work, to show the influence of this variability, we assume that the support 1 is founded at the rock and the support 2 founded at the soil (homogeneous or greatly inhomogeneous). We consider four soil types, very flexible (V_0/H = 2), flexible (V_0/H = 4), medium (V_0/H = 10) and rigid (V_0/H = 20).

Figs. 5 and 6 show clearly that the mean and standard deviation of the maximum total shear force is proportional both to the soil inhomogeneity (p ↑) and flexibility (V_0/H ↓). For an infinitely stiff oscillator ($\omega_0 \rightarrow \infty$), the shear force converges to the pseudo-static component which is proportional

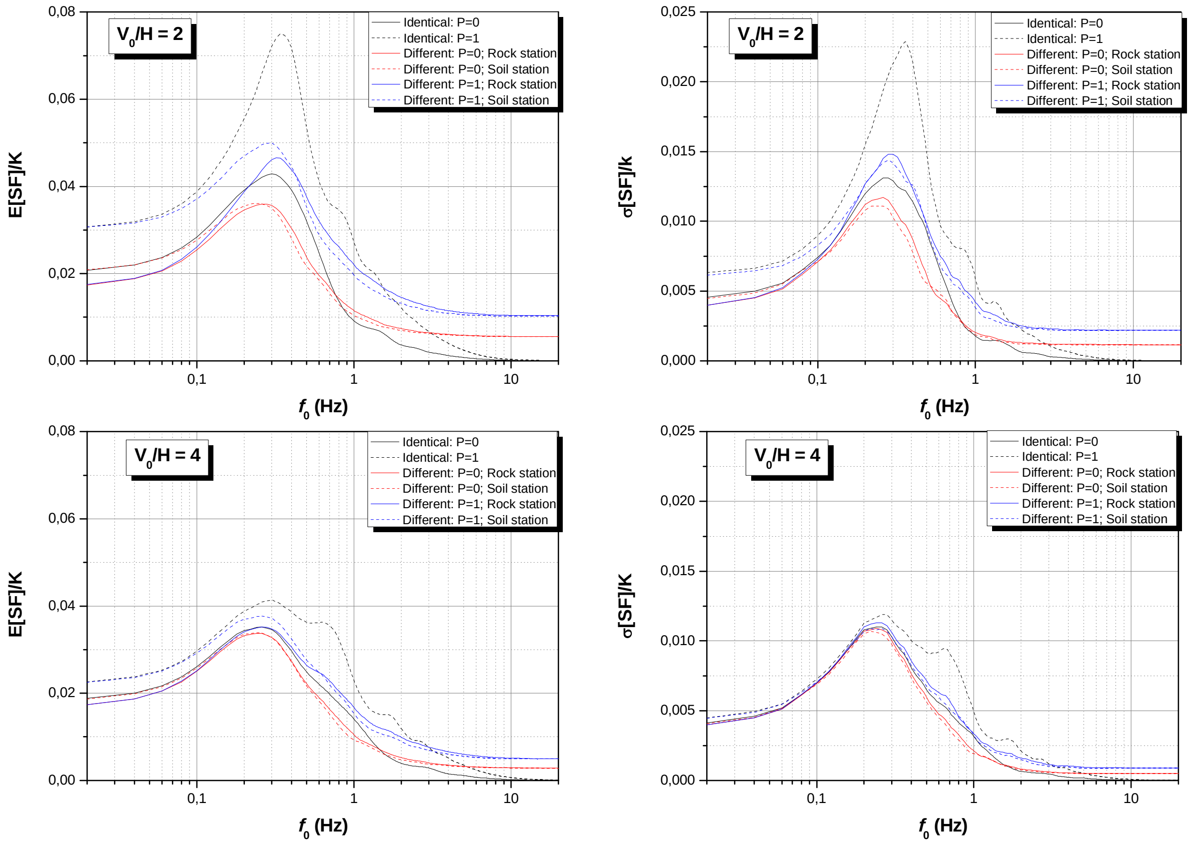

Figure 5. Inhomogeneity p and V_0/H parameters effects on shear forces variation in the case of different local geological conditions: very flexible and flexible soils (q=0.5).

to the differential displacement between the two supports. It has the same value at both supports but the cross-correlation is perfectly negative ($\rho_{R_1^s R_2^s} = -1$) between pseudo static components at the two supports. It is given by

$$R_1^s(t) = -k\, R_2^s(t) = k\left[\frac{u_2(t)-u_1(t)}{2}\right] \tag{15}$$

When the oscillator is infinitely flexible ($\omega_0 \rightarrow 0$), the total displacement is equal zero and the shear force at the support k is directly related to the corresponding displacement excitation, $R_k^t = -K\, u_k(t)$. Thus, the mean and standard deviation of the total shear forces at the support k normalized with respect to column rigidity K converge with those of PGD ($max[u_k(t)]$).

One can notice that at very low frequencies, the two statistical moments of the shear forces have larger values at the second support which is founded on a soil. The differential shear forces are important for inhomogeneous and flexible soils at the support 2. For rigid soils, the shear forces become nearly inhomogeneity independent. Moreover, the cross-correlation between pseudo-static and dynamic components is positive for the case of the second support as can be shown in the stochastic analysis, which leads to a larger value of total shear force. On the other hand, one can notice that in the intermediate frequency range, the mean and standard deviation of the total shear force is slightly significant at the first support. This is explained by the fact that the cross correlation is positive for the case of the first support. Also, when the cross correlation is zero, the support reactions have identical values.

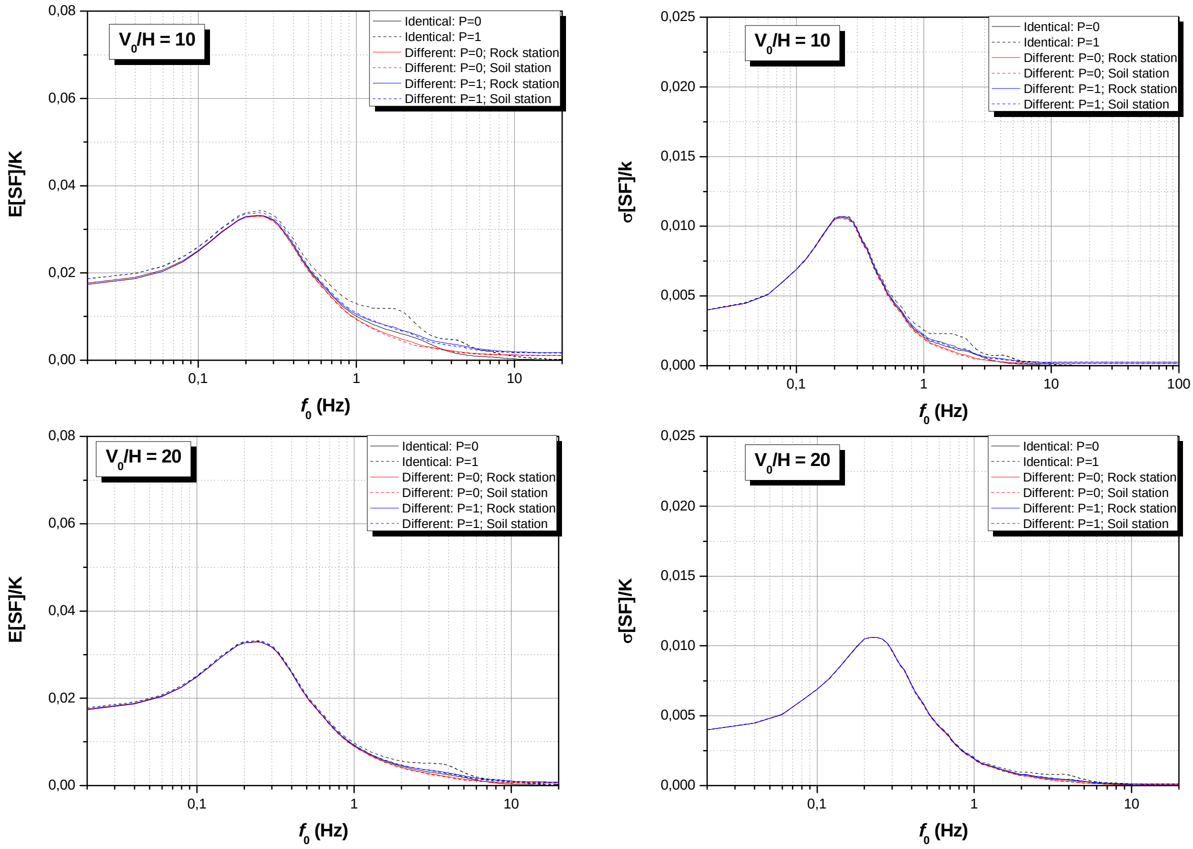

Figure 6. Inhomogeneity *p* and V_0/H parameters effects on shear forces variation in the case of different local geological conditions : medium and rigid soils (q=0.5).

8. CONCLUSION

The vibration natural frequencies and the transfer function of a soil profile depend essentially on its geometrical and mechanical properties. The mechanical characteristics and particularly the soil shear modulus varies generally with depth from the free surface. In this paper, a statistical analysis of the effects of inhomogeneity and soil fundamental frequency is performed on the pseudo acceleration spectra PSA and shear forces in a one story shear building columns using Monte Carlo simulation technique. We consider the seismic response of a linear oscillator system under both identical and different seismic input at the two supports. We note that the mean and standard deviation of PSA will be amplified with increasing soil inhomogeneity or rigidity and can reach more than three times on the frequency range including the resonant frequencies of the soil profile. One can notice that if one column of the oscillator is founded at the rock, the statistical parameters values are generally lower compared with those corresponding to identical geological conditions at the two supports and particularly for inhomogeneous soils and implies that PSA is positively influenced by the effect of the variability of geological conditions. For shear forces at columns, it is clear that the mean and the standard deviation depend only on the dynamic component in the case of uniform seismic input and the pseudo static component will be important in the case of stiff oscillator founded on different local geological conditions: one column on rock and the other on flexible soil profile. So, these parameters are important in the case of inhomogeneous or/and flexible soil profile. So, the shear forces have different values in columns for particularly flexible oscillator founded on flexible inhomogeneous soil profile. The differential shear forces can be neglected in the case of rigid soil profile.

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