Improved Extended Kalman Filter for Parameters Identification

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SUMMARY:
The extended Kalman filter (EKF) is a useful method for physical parameters identification in system with unknown parameters. A multipoint iteration method was proposed in EKF method, which made full use of the present and the previous time information. The numerical results showed that the precision in lumped mass system could be improved more highly by multipoint iteration. A more stable and convergent solution could be obtained by this method. In addition, the results could be influenced less by the hypothetical initial value.
Keywords: Extended Kalman Filter Parameters Identification Multipoint Iteration

1. Introduction

Under the effects of environmental loads including earthquake, wind, fire, Civil engineering structure may have damage in various degrees. As a result, great changes may take place in physical and mechanical parameters. Furthermore, buildings may collapse. Therefore, how to obtain structural parameters became a key problem, which may judge the health of structures. Among all the identification methods, extended kalman filter was an effective way, which was similar with kalman filter. The difference was that rigidity and damping was put into the state vector. Through multiple iterative, the physical parameters may converge to the true value. This way could have higher efficiency, but it depended on the selection of its initial value. If it was chosen inappropriately, the identification results may be divergent. This paper proposed an improved way that could make parameters converge and improve identification efficiency.

2.Introduction of extended kalman filter

The extended kalman filter was proposed by Andrew H.Jazwinski in 1970. This paper demonstrated the filter process systematically, and pointed that the increment of state vector x met the requirements of kalman filter.

Definition of extended state vector x[4]

$$x_k = \begin{bmatrix} u_k \\ \dot{u}_k \\ c_k \\ k_k \end{bmatrix} = \begin{bmatrix} x_{k1} \\ x_{k2} \\ x_{k3} \\ x_{k4} \end{bmatrix} \tag{2.1}$$

where $c_k = \begin{bmatrix} c_1 & c_2 & \cdots & c_k \end{bmatrix}^T$ is damping array and $k_k = \begin{bmatrix} k_1 & k_2 & \cdots & k_k \end{bmatrix}^T$ is rigidity array. From dynamic equation, derivative of x was got, which is written as

$$\dot{x}(t)=\begin{bmatrix}\dot{u}_k\\ \ddot{u}_k\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}=\begin{bmatrix}\dot{u}_k\\ M^{-1}f_k-M^{-1}C_k\dot{u}_k-K_ku_k\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}=\begin{bmatrix}x_{k2}\\ M^{-1}f_k-M^{-1}C_kx_{k2}-K_kx_1\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix} \quad (2.2)$$

How to obtain the extended kalman filter formula was demonstrated as follows.

Firstly, partial differential of $\dot{x}(t)$ was written as

$$\delta\dot{x}(t)=\frac{\partial\dot{x}(t)}{\partial x_i(t)}\delta x(t) \quad (2.3)$$

Eq. (2.1) could be written as $\delta\dot{x}(t)=A_d^{"}\delta x(t)$ (2.4)

where $A^{"}=\dfrac{\partial\dot{x}(t)}{\partial x_i(t)}$. Under discrete state, Eq. (2.3) was written as

$$\delta x_{k+1}=A_d^{"}\delta x_k=e^{A^{"}\Delta t}\delta x_k \quad (2.5)$$

where $A_d^{"}=e^{A^{"}\Delta t}=I+A^{"}\Delta t+\frac{1}{2!}A^{"2}\Delta t^2+\cdots+\frac{1}{k!}A^{"k}\Delta t^k+\cdots=\sum_{k=0}^{\infty}\frac{1}{k!}A^{"k}\Delta t^k$

δy_k was increment of observation vector, which was written as

$$\delta y_k=y_k-\hat{y}_{k-1} \quad (2.6)$$

The model of incremental equation was written as

$$\begin{cases}\delta x_{k+1}=A_d^{"}\delta x_k\\ \delta y_k=C_d\delta x_k+v_k\end{cases} \quad (2.7)$$

From the above model, it could be seen that the increment of extended state vector met requirements of kalman filter. Because of this point, it could be obtained by kalman filter that was written as

$$\delta\hat{x}_{k+1}=A_d^{"}\delta\hat{x}_k+K_{k+1}\left(\delta y_{k+1}-C_dA_d^{"}\delta\hat{x}_k\right) \quad (2.8)$$

where

$$\tilde{P}_{k|k-1}=A_d^{"}\tilde{P}_{k-1}A_d^T+Q_{k-1} \quad (2.9)$$

$$\tilde{P}_{k-1}=\left(I-K_kC_d\right)\tilde{P}_{k|k-1}\left(I-K_kC_d\right)^T+K_kR_kK_k^T \quad (2.10)$$

$$K_k=\tilde{P}_{k|k-1}C_d^T\left(C_d\tilde{P}_{k|k-1}C_d^T+R_k\right)^{-1} \quad (2.11)$$

Increment of k+1 could be written as

$$\delta\hat{x}_{k+1|k} = A_d^{''}\delta\hat{x}_k \quad (2.12)$$

which was one-step estimation. This was optimum estimation with observed values, which was the last kalman estimation.

Meanwhile $\delta\hat{x}_{k+1|k}$ could also be gotten by the way as follows:

$$\delta\hat{x}_{k+1|k} = \hat{x}_{k+1} - \hat{x}_k = \int_{k\Delta t}^{(k+1)\Delta t}\dot{\hat{x}}_k dt \approx \dot{\hat{x}}_k\Delta t \quad (2.13)$$

Through incremental equation, $\hat{x}_{k+1}$ could be written as

$$\hat{x}_{k+1} = \hat{x}_k + A_d^{''}\delta\hat{x}_k + K_{k+1}\left(\delta y_{k+1} - C_d A_d^{''}\delta\hat{x}_k\right) \quad (2.14)$$

where

$$\delta y_{k+1} = y_{k+1} - \hat{y}_k = y_{k+1} - C_d\hat{x}_k \quad (2.15)$$

then

$$\begin{aligned}
\hat{x}_{k+1} &= \hat{x}_k + A_d^{''}\delta\hat{x}_k + K_{k+1}\left(y_{k+1} - C_d\hat{x}_k - C_d A_d^{''}\delta\hat{x}_k\right) \\
&= \hat{x}_k + A_d^{''}\delta\hat{x}_k + K_{k+1}\left[y_{k+1} - C_d\left(\hat{x}_k + A_d^{''}\delta\hat{x}_k\right)\right] \\
&= \hat{x}_k + \delta\hat{x}_{k+1|k} + K_{k+1}\left[y_{k+1} - C_d\left(\hat{x}_k + \delta\hat{x}_{k+1|k}\right)\right] \\
&= \hat{x}_k + \dot{\hat{x}}_k\Delta t + K_{k+1}\left[y_{k+1} - C_d\left(\hat{x}_k + \dot{\hat{x}}_k\Delta t\right)\right] \quad (2.16)
\end{aligned}$$

Eq.(2.16) was the last extended kalman filter recursion formula, through which $\hat{x}_{k+1}$ could be gotten by $\hat{x}_k$. Before recursion, x_0 and P_0 needed to be supplied, where P_0 was variance of x_0 and supposed that $\hat{x}_0 = x_0$ .In order to accelerate rate of convergence, the value of x_0 was more accurate, the identification results were better.

Extended kalman filter was ideal in theory, but in fact how to choose x_0 was a key problem. If x_0 was chosen inappropriately, then it may increase the amount of calculation, and even worse it may result in divergence. Furthermore, the matrix $\left(CP_{k|k-1}C^T + R_k\right)$ may be singular. In order to solve this problem, this paper proposed a multipoint iteration method. Its principle is distinct. At first, x_0 was given at random. Through observation data and extended kalman filter x_{k+1} could be obtained. Because rigidity k and damping c were unchangeable in linear time-invariant systems, k and c of x_{k+1} were the optimum estimation in this case. Furthermore, k and c of x_{k+1} that chosen as the initial value of x_0 were applied into extended kalman filter again. Through multiple iteration, error may be controlled in a tolerance zone.If the identification results were divergent, the initial value of x_0 needed to be modified and then repeated the multipoint iteration method.

3. Numerical experiment

Supposed a three DOF lumped mass system shown in Fig.1 excited by earth pulsation, which was El-centro wave shown in Fig.2. The mass, rigidity and damping of each were 1000kg, 1000 $\mathrm{N/m}$ and 40 $\mathrm{N \cdot s/m}$,Sampling frequency was 50Hz and sampling point was 50.

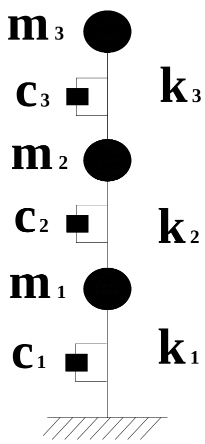

Fig.2 three DOF lumped mass system

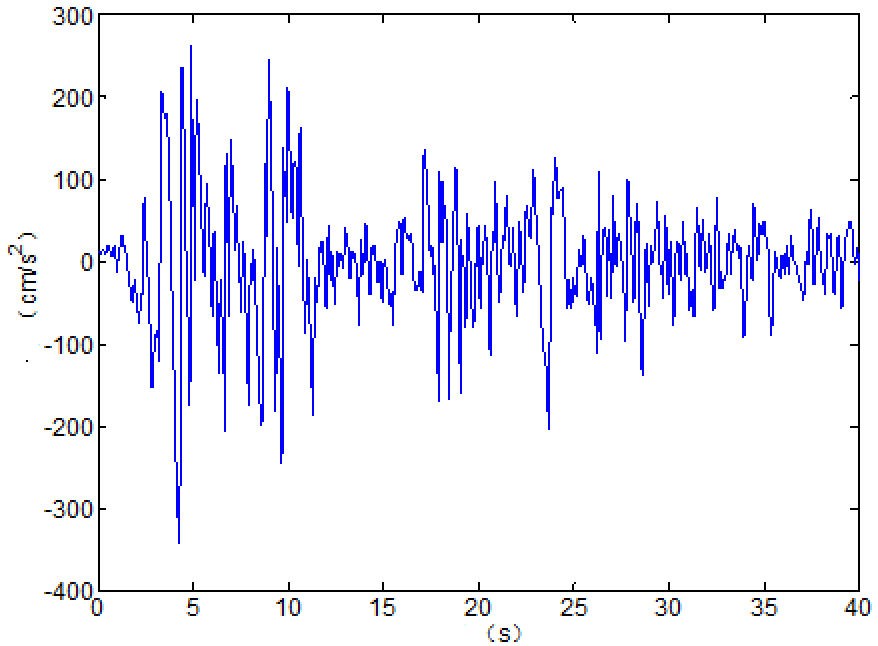

Fig.2 El-centro wave

Supposed that system was in static state, and the displacement， velocity and acceleration were obtained by central difference method. Then the calculated responses were considered as observation data, which were used to identify physical parameters of structure.

Before applying extended kalman filter, x_0 needed to be given at first. The initial value of rigidity and damping were shown in table 1.

Table 1 initial value of rigidity and damping

parameters	k_1	k_2	k_3	c_1	c_2	c_3
initial value	200	200	200	20	20	20

Because system was in static state, the displacement and velocity was zero. The variance of observed noise was $R = I \times 10^{-6}$, where I was unit matrix. The identification results were shown in table 2

Table 2 identification results with no observed noise

parameters	k_1	k_2	k_3	c_1	c_2	c_3
initial value	1000	1000	1000	40	40	40
identification results	1001.75	1001.13	1000.84	43.8	40.16	37.94
relative error	0.18%	0.11%	0.08%	9.50%	0.40%	7.65%

The error matrix of initial state vector P_0 was written as

$$P_0 = \begin{bmatrix} 10^{-6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10^{-6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10^{-6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10^{-6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10^{-6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10^{-6} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 10^{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10^{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10^{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10^{9} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10^{9} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10^{9} \end{bmatrix}_{12\times 12}$$

The responses after extended kalman filter were shown in Fig.3, Fig.4 and Fig.5. It could be learned that the filtered responses were the same as the truth-value, which indicated that r extended kalman filter could identify responses of structure.

Through the results of numerical experiment shown in table 2, it could be learned that extended kalman filter may identify rigidity accurately and the identification of damping was relatively bad. Generally speaking, rigidity was a common parameter that was used to judge the health of structure, while damping was almost not be used. Then the identification accuracy of damping had no effects on structural damage detection.

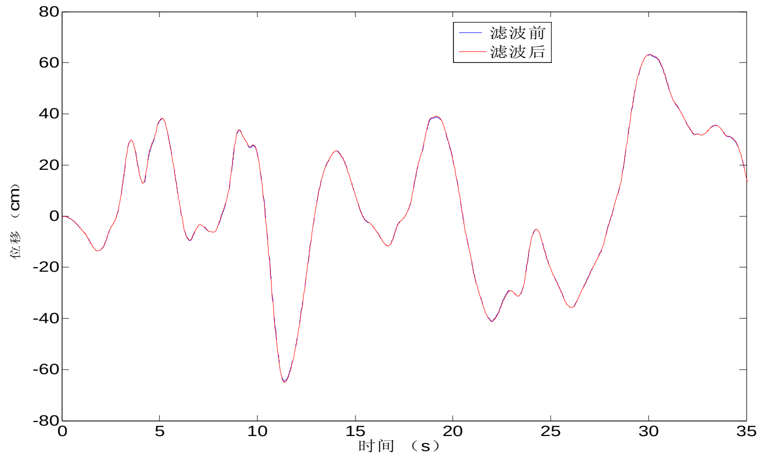

Fig.3 displacement of 1

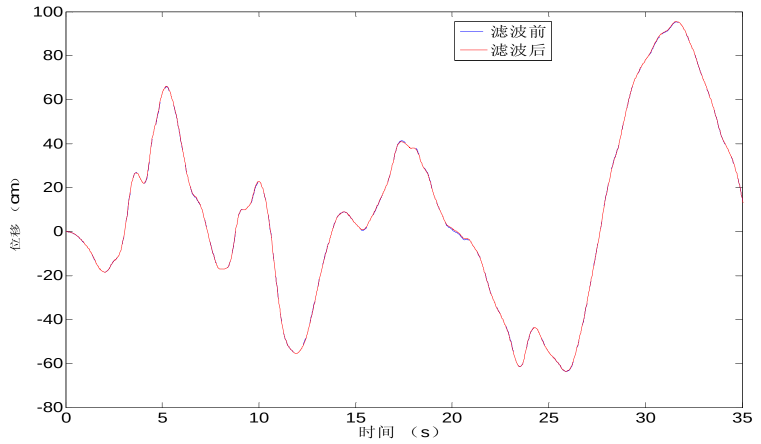

Fig.4 displacement of 2

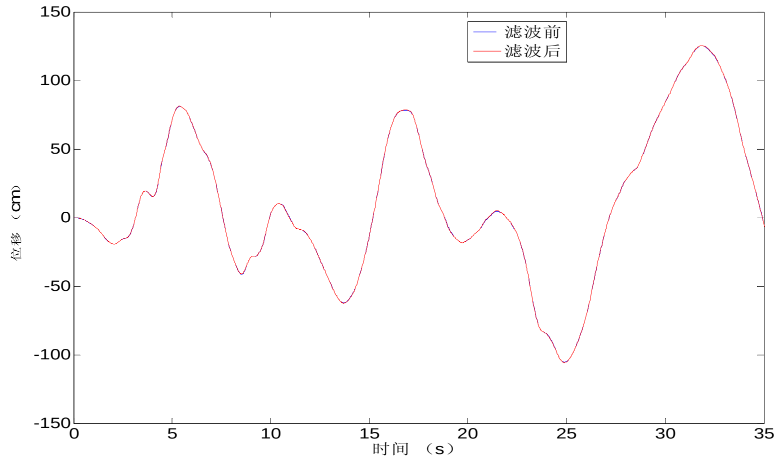

Fig.5 displacement of 3

Then if the responses had observed noises, the method could also get the physical parameters. The mean value of observed noises was zero and variance was 5,.The observed displacement were shown in Fig.6, Fig.7 and Fig.8.

Through multiple iteration proposed by this paper, the filtered responses were shown in Fig.9, Fig.10 and Fig.11.

By contrast, it could be known that the filtered responses were almost the same as the truth-value, which indicated that extended kalman filter could also identify physical parameters even the data had ambient noises.

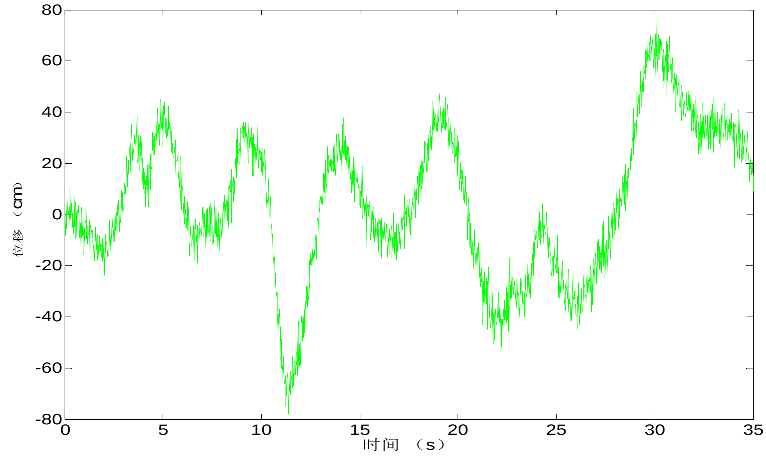

Fig.6 displacement of 1 with noise

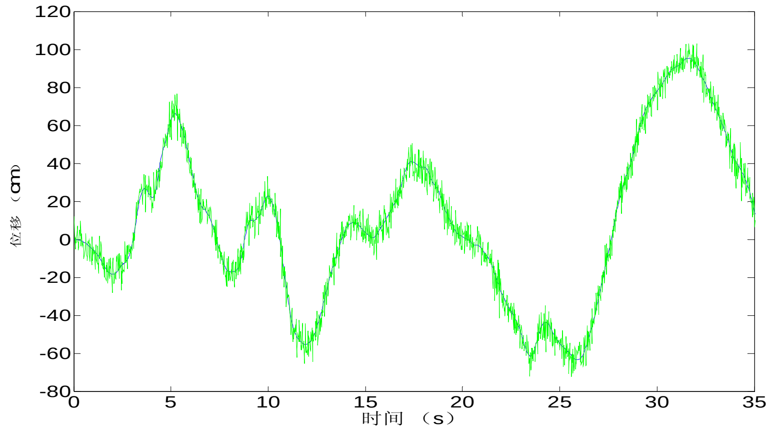

Fig.7 displacement of 2 with noise

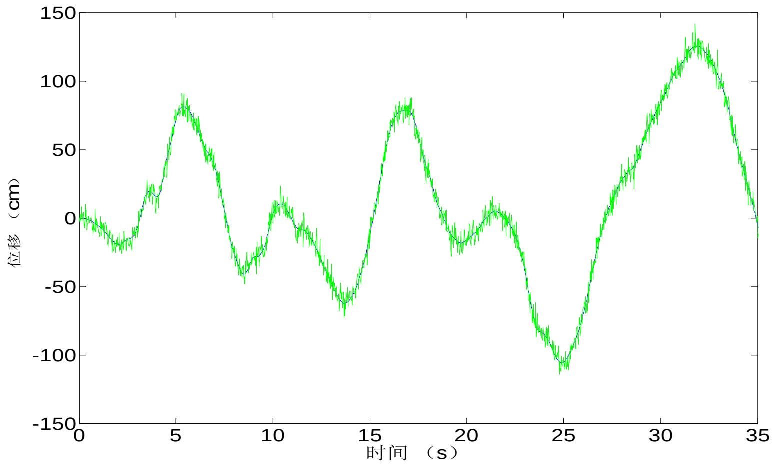

Fig.8 displacement of 3 with noise

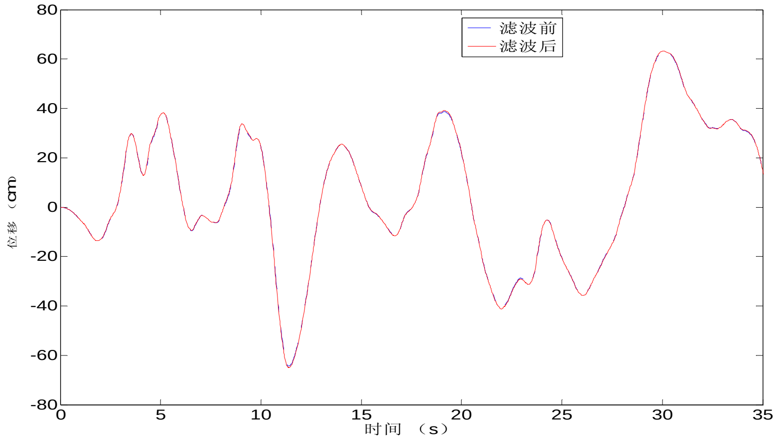

Fig.9 comparison of 1

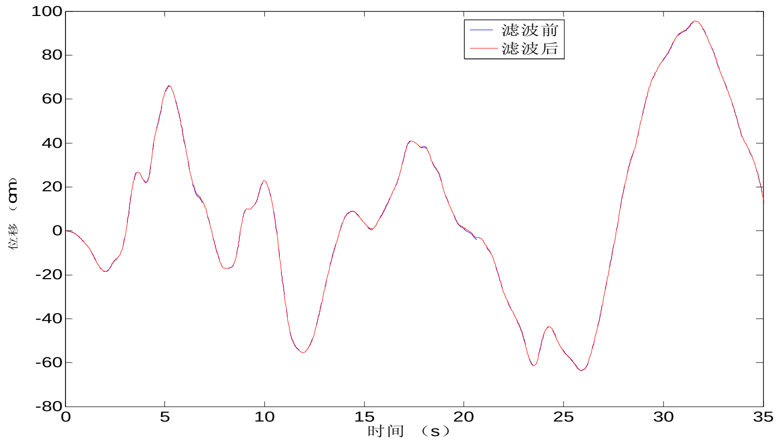

Fig.10 comparison of 2

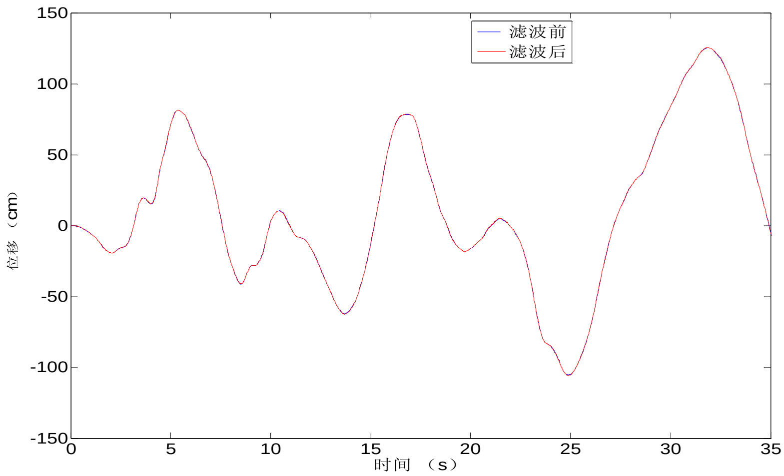

Fig.11 comparison of 3

The identification results were shown in table 3

Table 3 identification results with observed noise

parameters	k_1	k_2	k_3	c_1	c_2	c_3
initial value	1000	1000	1000	40	40	40
identification results	998.57	1002.5	1004.28	44.7	39.91	37.57
relative error	0.24%	0.25%	0.43%	11.75%	0.23%	6.01%

4.Conclusion

1. Extended kalman filter could identify structural rigidity and damping effectively even when the input of structure was unknown. The principle of it was demonstrated in this paper.

2. Numerical experiment showed that through multipoint iteration in extended kalman filter, it could reduce amount of calculation and accelerate the rapid of convergence. The numerical results also showed that the identification had high accuracy. With the identification shown in table 2 and 3, the accuracy of rigidity was far higher than damping. When measured data had observed noises, through multipoint iteration, the extended filter may also identify physical parameters accurately.

References

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