Seismic assessment of isolated bridges with abutment transverse restraint

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SUMMARY:
This paper proposes an analytical model and formulation for predicting the seismic response of isolated bridges with abutment transverse restraint. These are a particular class of multi-span continuous bridges in which isolation bearings are provided only between the piers top and the deck whereas seismic stoppers restrain the transverse motion of the deck at the abutments.
The bridges are modelled by two-dimensional simply supported beams with intermediate visco-elastic restraints, whose properties are calibrated to describe the substructures behaviour. First, the application of the complex mode superposition method to the specific class of bridges analyzed is illustrated, with the aim of deriving an exact benchmark solution to the seismic problem. Then, two simplified models useful for preliminary analysis and design are proposed.
The reliability of the simplified models is assessed by comparing the approximate results of the seismic analysis of realistic PRSI bridges with the refined results involving the use of the complex mode superposition method.

Keywords: Continuous bridges, transverse seismic response, partial restraint, seismic isolation.

1. INTRODUCTION

Partially restrained seismically isolated bridges (PRSI) are a particular class of isolated bridges in which isolation bearings are posed only at the top of the piers, with seismic stoppers restraining the transverse motion of the superstructure at the abutments. The growing interest on the dynamic behaviour of this class of bridges is demonstrated by numerous recent experimental (Boroschek et al. 2003, Shen et al. 2004) and numerical (Tsai 2008, Makris et al. 2009, Tubaldi and Dall'Asta 2011a, Tubaldi et al. 2011b, Tubaldi et al. 2011c, Tubaldi and Dall'Asta 2012) studies.
An analytical model commonly employed for the analysis of the transverse behaviour of PRSI bridges consists in a continuous 2-dimensional simply supported beams resting on discrete intermediate supports (usually with visco-elastic behaviour) representing the pier-bearing systems. The damping is promoted by two different mechanisms: the isolated piers, characterized by high dissipation capacity localized in the bearings, and the deck, characterized by a lower but widespread dissipation capacity.
The drastical variation of energy dissipation among the isolated structure results in non-classical damping and this makes the seismic analysis complicated to be carried out in rigorous terms. In general, the exact solution for the seismic problem of a non-classically damped system requires resorting to the direct integration of the equations of motion or to complex modal analysis (Veletsos and Ventura 1986), due to the presence of non-classic complex vibration modes. While the first approach is conceptually simple (though computationally expansive when complex and/or large-scale system are analyzed), the second approach is difficult to implement and in general not attractive for practical engineering use. For this reason, many studies have been devoted to the definition of approximate techniques of analysis and to the assessment of their accuracy (Claret and Venancio-Filho 1991, Kim 1995, Venancio-Filho et al. 2001, Morzfeld et al. 2008). With reference to fully isolated bridges, the studies of Hwang et al. (1997), Franchin et al. (2001), Lee et al. (2011) have shown that the decoupling approximation considering the real undamped modes and the diagonal terms of the modal damping matrix usually provides a fully acceptable estimate of the seismic response. However,

the effects of this decoupling approximation on the evaluation of the seismic response of PRSI bridges have not been investigated, yet. This issue becomes of particular relevance in consequence of the numerous studies regarding the dynamic behaviour of PRSI bridges that completely disregard non-classical damping (Tsai 2008, Tubaldi and Dall'Asta 2011a, Makris et al. 2010). The work of Tubaldi and Dall'Asta (2012) addresses this issue within the context of the free-vibration response of PRSI bridges. The authors observe that non classical damping influences differently the various response parameters (i.e., the transverse displacement shape is less affected than the bending moment demand by the damping non proportionality). However, a closer examination is still required in order to ensure whether the use of proportionally damped models is adequate or not for the seismic assessment of these systems.

Another approximation often introduced in the analysis of PRSI bridges is the assumption of a pre-fixed sinusoidal vibration shape (Tsai 2008, Tubaldi and Dall'Asta 2011a, Tubaldi et al. 2011c). This assumption permits to derive analytically the properties of a generalized SDOF system equivalent to the bridge and to estimate the system response by expressing the seismic demand in terms of a response spectrum reduced to account for the system composite damping ratio. It is noteworthy that the vibration modes of a simply supported homogeneous beam are purely sinusoidal. The vibration shapes of a homogeneous beam resting on continuous homogeneous restraints are sinusoidal, too. Thus, also the PRSI bridge is expected to exhibit a sinusoidal transverse deformed shape if the following conditions are met: a) the superstructure stiffness is significantly higher than the pier stiffness, b) the variations of mass and stiffness of the deck and of the supports are not significant, c) the span number is high, and d) the displacement field is dominated by the first vibration mode. Again, further investigations are required to estimate the error committed by the simplified analysis techniques involving this approximation in estimating the seismic response of real configurations of PRSI bridges.

The aim of this study is to define an exact analytical technique for the seismic assessment of PRSI bridges, modelled as non-classically damped continuous systems, and to measure the error in the estimate of the response deriving by the introduction of the previously described approximations. In order to accomplish this, the complex mode superposition (CMS) method (Foss 1958, Veletsos and Ventura 1986, Oliveto et al. 1997, Gurgoze and Erol 2006) is specialized and applied to the particular structural system considered. The application of the CMS method permits to test the accuracy of two simplified alternative techniques of analysis proposed in this paper. The first one employs classic modes of vibration (corresponding to the vibration modes obtained by neglecting the damping of the intermediate restraints) instead of the exact complex modes for describing the motion whereas the second technique employs the Fourier sine-only series terms (corresponding to the vibration modes obtained neglecting completely the intermediate restraints). The introduction of these approximations results in a coupling of the equation of motions projected in the space of the approximating functions which is neglected in the solution in order to simplify the response assessment. The extent of the coupling is quantified by means of two appropriately defined indexes.

Finally, a realistic case study is analyzed and the seismic response according to the CMS method is compared with the response obtained applying the proposed simplified techniques, in order to test their accuracy and establish a relation between the coupling coefficients and the error committed due to the approximations introduced.

2. DYNAMIC BEHAVIOR OF PRSI BRIDGES

In order to keep the problem as simple as possible, the PRSI bridges are modelled (Figure 1) as 2-dimensional beams resting on discrete visco-elastic supports representing the pier-bearing systems and pinned at the abutments.

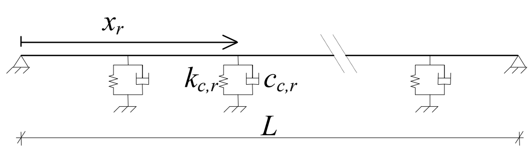

Figure 1. Analytical 2-dimensional model for PRSI bridges.

Let $V=\{v(x)\in H^2[0,L]:v(0)=v(L)=0\}$ be the space of displacement functions defined along the bridge length L and satisfying the kinematic (essential) boundary conditions, and $u(x;t)\in U=C^2(V;[t_0,t_1])$ be the motion, defined in the time interval considered [t_0, t_1] and known at the initial instant together with its time derivative (initial conditions).
The differential dynamic problem can be derived from the D'Alembert principle (Truesdell and Toupin 1960) and posed in the following form:

$$\begin{aligned}&\int_0^L m(x)\ddot{u}(x,t)\eta(x)dx+\int_0^L c_d(x)\dot{u}(x,t)\eta(x)dx+\sum_{r=1}^{N}c_{c,r}\dot{u}(x_r,t)\eta(x_r)+\int_0^L b(x)u''(x,t)\eta''(x)dx+\\&+\sum_{r=1}^{N}k_{c,r}u(x_r,t)\eta(x_r)=-\int_0^L m(x)\ddot{u}_g(t)\eta(x)dx \qquad \forall\eta\in V;\forall t\in[t_0,t_1]\end{aligned} \tag{2.1}$$

The functions $m(x)$, $b(x)$ and $c_d(x)$ are piecewise continuous and denote the mass per unit length, the transverse stiffness per unit length and the deck distributed damping constant. The constants $k_{c,r}$ and $c_{c,r}$ are the stiffness and damping constant of the visco-elastic support located at the r-th position $x=x_r$ while $\ddot{u}_g(t)$ denotes the ground motion input. N denotes the total number of intermediate supports.
In order to simplify the analytical solution of the problem, $m(x)$, $b(x)$ and $c_d(x)$ are assumed as constant and equal respectively to m_d, EI_d, c_d. The local form of the problem is obtained by integrating by parts Eqn. (2.1) and can be formally written as:

$$\begin{aligned}&M\ddot{u}(x,t)+C\dot{u}(x,t)+Ku(x,t)=-M\ddot{u}_g(t)\\&u'''(x,t)\eta\big|_0^1=0\ ,\ u''(x,t)\eta'\big|_0^1=0\end{aligned} \tag{2.2}$$

where M, C, K denote respectively the mass, damping and stiffness operator. They are expressed as (δ is the Dirac's delta function):

$$\begin{aligned}&M=m_d\\&K=EI_d\frac{\partial^4}{\partial x^4}+\sum_{r=1}^{N_c}k_{c,r}\delta(x-x_r)\\&C=c_d+\sum_{r=1}^{N_c}c_{c,r}\delta(x-x_r)\end{aligned} \tag{2.3}$$

3. EIGENVALUE PROBLEM FOR PRSI BRIDGES

The free vibrations problem of the beam is obtained by posing $\ddot{u}_g=0$ in Eqn.(2.2). The corresponding differential boundary problem is then reduced to an eigenvalue problem solvable by expressing the transverse displacement $u(x,t)$ as the product of a spatial function $\psi(x)$ and a time-dependent function $Z(t)=Z_0e^{\lambda t}$:

$$u(x,t)=\psi(x)Z(t) \tag{3.1}$$

After substituting Eqn.(3.1) into Eqn.(2.2) for $\ddot{u}_g=0$, the following transcendental equation is obtained:

$$\left[\lambda^2m_d+c_d\lambda\right]\psi(x)+EI_d\psi^{IV}(x)+\sum_{r=1}^{N_c}(k_{c,r}+c_{c,r}\lambda)\delta(x-x_r)\psi(x)=0 \tag{3.2}$$

Eqn.(3.2) is satisfied by an infinite number of eigenvalues and eigenvectors that occur in complex conjugate pairs (Prater and Sing 1990). The i-th eigenvalue λ_i contains information about the system vibration frequency and damping, while the i-th eigenvector $\psi_i(x)$ is the i-th vibration shape. The solution of the eigenvalue problem is described in Tubaldi and Dall'Asta (2012). In the same paper, the following orthogonality conditions are also derived and reported:

$$\left(\lambda_i+\lambda_j\right)m_d\int_0^L \psi_i(x)\psi_j(x)\,dx + c_d\int_0^L \psi_i(x)\psi_j(x)\,dx + \sum_{r=1}^{N_c} c_{c,r}\psi_i(x_r)\psi_j(x_r)=0 \tag{3.3}$$

$$\begin{aligned}&EI_d\int_0^L \psi_i^{ii}(x)\psi_j^{ii}(x)\,dx + \frac{\lambda_i\lambda_j}{\left(\lambda_i+\lambda_j\right)}\left[\sum_{r=1}^{N_c} c_{c,r}\psi_i(x_r)\psi_j(x_r) + c_d\int_0^L \psi_i(x)\psi_j(x)\,dx\right]\\ &+\sum_{r=1}^{N_c} k_{c,r}\psi_i(x_r)\psi_j(x_r)=0\end{aligned} \tag{3.4}$$

4. SEISMIC RESPONSE OF PRSI BRIDGES

4.1. CMS method for seismic response assessment

In the complex modes superposition method, the displacement of the beam is expanded as the linear combination of the complex vibration modes as:

$$u(x,t)=\sum_{i=1}^{N_m}\psi_i(x)q_i(t) \tag{4.1}$$

where $\psi_i(x)$ = i-th complex modal shape, $q_i(t)$ = i-th complex generalized coordinate, and N_m = total number of modes considered.
For $\ddot{u}_g(t)=\delta(t)$, one obtains the expression of the complex modal impulse response function $h_i^c(x,t)$ corresponding to the i-th mode (Oliveto et al. 1997).

$$h_i^c(x,t)=B_i\psi_i(x)e^{\lambda_i t}=-\frac{m_d\int_0^L \psi_i(x)\,dx}{2\lambda_i m_d\int_0^L \psi_i^2(x)\,dx + c_d\int_0^L \psi_i^2(x)\,dx + \sum_{r=1}^{N} c_{c,r}\psi_i^2(x_r)}\psi_i(x)e^{\lambda_i t} \tag{4.2}$$

The sum of the contribution to the complex modal impulse response function of the i-th mode and of its complex conjugate yields a real function $h_i(x,t)$, which may be expressed as (Oliveto et al. 1997)

$$h_i(x,t)=B_i\psi_i(x)e^{\lambda_i t}+\bar{B}_i\bar{\psi}_i(x)e^{\bar{\lambda}_i t}=\alpha_i(x)\left|\lambda_i\right|h_i(t)+\beta_i(x)\dot{h}_i(t) \tag{4.3}$$

where $\alpha_i(x)=\xi_i\beta_i(x)-\sqrt{1-\xi_i^2}\gamma_i(x)$, $\beta_i(x)=2\,\mathrm{Re}\left[B_i\psi_i\right]$, $\gamma_i(x)=2\,\mathrm{Im}\left[B_i\psi_i\right]$, and where $h_i(t)$ denotes the impulse response function of a SDOF system with natural frequency $\omega_{0i}=\left|\lambda_i\right|$, damping ratio ξ_i and damped frequency ω_i, whose expression is:

$$h_i(t)=\frac{1}{\omega_i}e^{-\xi_i\omega_{0i}t}\sin\left(\omega_{di}t\right) \tag{4.4}$$

In order to take advantage of the closed form expression of the impulse response function, the generic seismic input is expressed as a sum of Delta dirac functions as follows:

$$\ddot{u}_g(t)=\int_0^t \ddot{u}_g(\tau)\delta(t-\tau)d\tau \tag{4.5}$$

The seismic response in terms of transverse displacement is then expressed as:

$$u(x,t)=\sum_{i=1}^{N_m}\int_0^t \ddot{u}_g(\tau)h_i(x,t-\tau)d\tau=\sum_{i=1}^{N_m}\alpha_i(x)|\lambda_i|D_i(t)+\beta_i(x)\dot{D}_i(t) \tag{4.6}$$

where $D_i(t)$ and $\dot{D}_i(t)$ denote the response of the oscillator with natural frequency $\omega_{0i}=|\lambda_i|$, damping ratio ξ_i and damped frequency ω_{di}, subjected to the seismic input $\ddot{u}_g(t)$.

In the case of $c_{c,r}=0$, one obtains $\lambda_i=-\xi_i\omega_{0i}+i\omega_{0i}\sqrt{1-\xi_i^2}$, $\xi_i=c_m/(2\omega_{0i}m_d)$ $B_i=-\rho_i i/\left(2\omega_{0i}\sqrt{1-\xi_i^2}\right)$, $\alpha_i=\rho_i/\omega_{0i}$, $\beta_i=0$, $\gamma_i=-\rho_i/\left(\omega_{0i}\sqrt{1-\xi_i^2}\right)$, where $\rho_i=$ i-th real mode participation factor. In the case of $c_{c,r}=c_d=0$, one obtains $\lambda_i=i\omega_{0i}$, $B_i=-\rho_i i/2\omega_{0i}$, $\alpha_i=\rho_i/\omega_{0i}$, $\beta_i=0$, $\gamma_i=-\rho_i/\omega_{0i}$. Thus, in both cases, Eqn.(4.6) reduces to the well known expression:

$$u(x,t)=\sum_{i=1}^{N_m}\rho_i D_i(t) \tag{4.7}$$

Other quantities that are of interest for describing the seismic response of PRSI bridges are the abutment reactions $R_{ab}(t)=-EI_d u'''(0,t)$ and the transverse bending moments $M(x,t)=-EI_d u''(x,t)$, which depend respectively on the second $u''(x,t)$ and third order derivatives $u'''(x,t)$ of the transverse displacement. The computation of these quantities involves calculating the spatial derivatives of the modal shapes, which are exactly known.

4.2. Simplified methods for seismic response assessment

The assessment of the exact seismic response of the system entails performing a cumbersome numerical procedure for solving the transcendental equation corresponding to the eigenvalue problem. In this paragraph, two simplified approaches are investigated in order to avoid this complexity and reduce the computational cost of the seismic analysis. These approaches are based on the assumed modes method (Hamdan and Jubran 1991, Hassanpour 2010) and entail using approximate functions $y_i(x)$ instead of the exact complex vibration modes $\psi_i(x)$ for describing the displacement field.

In the first method, referred to as real modes superposition (RMS) method, the motion is expanded as the linear combination of the real modes of vibration $y_i(x)=\phi_i(x)$ of the undamped (or classically damped) structure. The calculation of the classic modes $\phi_i(x)$ involves solving an eigenvalue problem less computationally demanding than that required for computing $\psi_i(x)$, and can be easily performed within any finite element program.

In the second method, referred to as Fourier terms superposition (FTS) method, the Fourier sine-only series terms $y_i(x)=\varphi_i(x)$ are employed. In this case, the use of the terms $\varphi_i(x)$ does not involve solving any eigenvalue problem and permits deriving an analytical solution to the dynamic problem at a very reduced computational cost, without recourse to FE analysis.

It is noteworthy that he orthogonality conditions for the terms $\phi_i(x)$ and $\varphi_i(x)$ can be derived from Eqs.(3.3) and (3.4) by posing respectively $c_{c,r}=0$ and $c_{c,r}=k_{c,r}=0$, for $r=1,2,\ldots,N$. By using the approximate functions $\phi_i(x)$ and $\varphi_i(x)$ instead of the exact complex vibration modes $\psi_i(x)$ for describing the displacement field, a system of coupled Galerkin equations is obtained in the form (Hamdan and Jubran 1991, Hassanpour 2010):

$$\mathbf{M}\mathbf{q}(t)+(\mathbf{C}_d+\mathbf{C}_c)\dot{\mathbf{q}}(t)+(\mathbf{K}_d+\mathbf{K}_c)\mathbf{q}(t)=-\mathbf{M}\mathbf{I}\ddot{u}_g(t) \tag{4.8}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{C}_d+\mathbf{C}_c$ and $\mathbf{K}_d+\mathbf{K}_c$ are respectively the damping and stiffness matrix accounting for the contribution of both the deck and the intermediate supports, and $\mathbf{q}(t)$ is the vector containing the generalized coordinates. The expressions for the matrices are:

$$\begin{aligned}
&[\mathbf{M}]_{ij}=m_d\int_0^L y_i y_j dx \\
&[\mathbf{K}_c]_{ij}=\sum_{r=1}^{N} k_{c,r} y_i(x_r) y_j(x_r),\ \ [\mathbf{K}_d]_{ij}=EI_d\int_0^L y_i'' y_j'' dx \qquad i,j=1,2,..N_m \\
&[\mathbf{C}_c]_{ij}=\sum_{r=1}^{N} c_{c,r} y_i(x_r) y_j(x_r),\ \ [\mathbf{C}_d]_{ij}=c_d\int_0^L y_i y_j dx
\end{aligned} \tag{4.9}$$

and depend on the mode shape functions $y_i(x)$ employed.

It is noteworthy that these matrices are in general non diagonal. A common approximation introduced for practical purposes (Prater and Singh, 1986, Claret and Venancio-Filho 1991) is to disregard the off-diagonal terms of the coupled matrixes to obtain a set of uncoupled equations. The resulting *i*-th equation describes the motion of a SDOF system with mass $[\mathbf{M}]_{ii}$, stiffness $([\mathbf{K}_d]_{ii}+[\mathbf{K}_c]_{ii})$ and damping constant $([\mathbf{C}_d]_{ii}+[\mathbf{C}_c]_{ii})$. Thus, traditional analysis tools available for the seismic analysis of SDOF systems can be employed to compute efficiently the generalized displacements $q_i(t)$ and the response $u(x,t)$, under the given ground motion excitation.

4.2.1. RMS method for seismic response assessment

This method corresponds to expressing the displacement $u(x,t)$ in Eqn.(4.1) in terms of the real modes of vibration $\phi_i(x)$, for $i=1,2,\ldots,N_m$. This approximation leads, upon application of the orthogonality conditions, to a set of coupled Galerkin equations due to the non-zero value terms of matrix $\mathbf{C}_c$ for $i \neq j$. It is generally accepted that if these off-diagonal terms are small compared to the diagonal terms, the errors induced by disregarding them in the response assessment are small (Prater and Singh, 1986, Claret and Venancio-Filho 1991). In order to evaluate to what extent the modal damping matrix is coupled, a coupling index is defined (Claret and Venāncio-Filho 1991) as $c.i._1=\max\alpha_{ij}$, where:

$$\alpha_{ij}=\frac{\left([\mathbf{C}_c+\mathbf{C}_d]_{ij}\right)^2}{[\mathbf{C}_c+\mathbf{C}_d]_{ii}\cdot[\mathbf{C}_c+\mathbf{C}_d]_{jj}}=\frac{\left(\sum_{r=1}^{N} c_{c,r}\psi_i(x_r)\psi_j(x_r)\right)^2}{\left(\sum_{r=1}^{N} c_{c,r}\psi_i^2(x_r)+c_d\int_0^L\psi_i^2 dx\right)\left(\sum_{r=1}^{N} c_{c,r}\psi_j^2(x_r)+c_d\int_0^L\psi_j^2 dx\right)} \tag{4.10}$$

The index assumes high values for intermediate supports with high dissipation capacity, while it assumes lower and lower values for increasing number of supports with homogeneous properties, since the behaviour tends to that of a beam on continuous visco-elastic restraints, for which $\alpha_{ij}=0$.

4.2.2. FTS method

This method corresponds to expressing the displacement $u(x,t)$ in terms of the Fourier sine-only series terms $\varphi_i(x)=\sin(\pi x_i/L)$. The application of the orthogonality conditions leads to a set of coupled Galerkin equations due to the non-zero value terms of matrixes $\mathbf{C}_c$ and $\mathbf{K}_c$, for $i \neq j$. The terms of these equations have already been reported in Tubaldi and Dall'Asta (2011a) and have very simple closed-form expression. In order to evaluate to what extent the modal stiffness matrix is coupled, a second coupling index is defined, $c.i._2 = \max \beta_{ij}$, where:

$$\beta_{ij} = \frac{\left([\mathbf{K}_c + \mathbf{K}_d]_{ij}\right)^2}{[\mathbf{K}_c + \mathbf{K}_d]_{ii} \cdot [\mathbf{K}_c + \mathbf{K}_d]_{jj}} = \frac{\left(\sum_{r=1}^{N} k_{c,r}\psi_i(x_r)\psi_j(x_r)\right)^2}{\left(\sum_{r=1}^{N} k_{c,r}\psi_i^2(x_r) + EI_d\int_0^L (\psi_i'')^2\,dx\right)\left(\sum_{r=1}^{N} k_{c,r}\psi_j^2(x_r) + EI_d\int_0^L (\psi_j'')^2\,dx\right)} \tag{4.11}$$

It is noteworthy that $c.i._2$ assumes high values in the case of intermediate supports with a relatively high stiffness compared to the deck stiffness. Conversely, it assumes lower and lower values for increasing number of spans and homogenous support properties, since the behaviour tends to that of a beam on continuous restraints.

5. CASE STUDY

The application of the exact CMS method and of the approximate RMS and FTS techniques for the seismic analysis of PRSI bridges is illustrated by considering a realistic bridge already studied in Tubaldi and Dall'Asta (2011a). This is a four-span continuous PRSI bridge with a steel-concrete superstructure and span lengths = 40m+60m+60m+40m. Although the thickness of the steel girders varies along the deck length, a constant value of the deck transverse stiffness EI_d = 1.1E+09 kNm^2 is considered in order to simplify the problem, due to its negligible influence on the global dynamic response (Tubaldi and Dall'Asta 2012). The deck mass per unit length is equal to m_d = 16.24ton/m. The circular frequency corresponding to the first mode of vibration of the superstructure vibrating alone with no intermediate supports is ω_d = 2.03 rad/s. The deck damping constant c_d is such that the first mode damping factor of the deck vibrating alone with no intermediate supports is equal to ξ_d = 0.02. The central support has $k_{c,2}$ = 2057.61 kN/m and $c_{c,2}$ = 206.33 kN/m, while the outer supports have $k_{c1} = k_{c3}$ = 3500.62 kN/m and $c_{c,1} = c_{c,3}$ = 322.69 kN/m. It is recalled that these Kelvin models describe the stiffness and dissipation capacity of both the piers and the isolator.
The properties of the first 5 vibration modes are reported in Table 1. The eigenvalues of the system have been determined by using the command *fsolve* in Matlab (Mathworks 2011) to solve the eigenvalue problem corresponding to Eqn. (3.2). It is noteworthy that even modes of vibration are characterized by an anti-symmetric shape and a participating factor equal to zero, and thus they do not affect the seismic response of the considered configurations.

Table 5.1. Modal properties.

Mode	λ_i [-]	ω_{0i} [rad/s]	ξ_i [-]
1	-0.1723-2.6165i	2.62	0.0657
2	-0.2196-8.3574i	8.36	0.0263
3	-0.2834-18.4170i	18.42	0.0154
4	-0.1097-32.5180i	32.52	0.0034
5	-0.1042-50.7860i	50.79	0.0021

In general, the effect of the intermediate restraints on the dynamic behaviour of the PRSI bridge is to increase the vibration frequency and the damping factor. The fundamental vibration frequency shifts from ω_d = 2.03 rad/s to ω_{0i} = 2.62 rad/s, corresponding to a first mode vibration period of about 2.4s. The first mode damping factor increases from ξ_d = 0.02 to ξ_1 = 0.0657, whereas the damping factor of the internal and external supports for a harmonic motion at the same circular frequency are respectively 0.13 and 0.12. This is a result of the low dissipation capacity of the deck and of the dual load path behaviour of the bridge (Tubaldi et al. 2011b, Tubaldi et al. 2012). The composite damping ratio is in general very low and tends to decrease significantly with the increasing mode order.

Figure 2 shows the response of the midspan transverse displacement (Figure 2a) and of the abutment reactions (Figure 2b), for a unit impulse load acting at the supports. The analytical exact expression of the i-th mode displacement impulse response is reported in Eqn.(4.3). The different response functions plotted in Figure 2 are obtained considering 1) the contribution of the first mode only, 2) the contribution of the first and third mode, and 3) the contribution of the first, third and fifth mode. It is noteworthy that the even modes of vibration are characterized by an anti-symmetric shape and a participating factor equal to zero, and thus they do not affect the seismic response of the configurations considered (uniform support excitation is assumed). Modes higher than the 5th also have a negligible influence on the response.

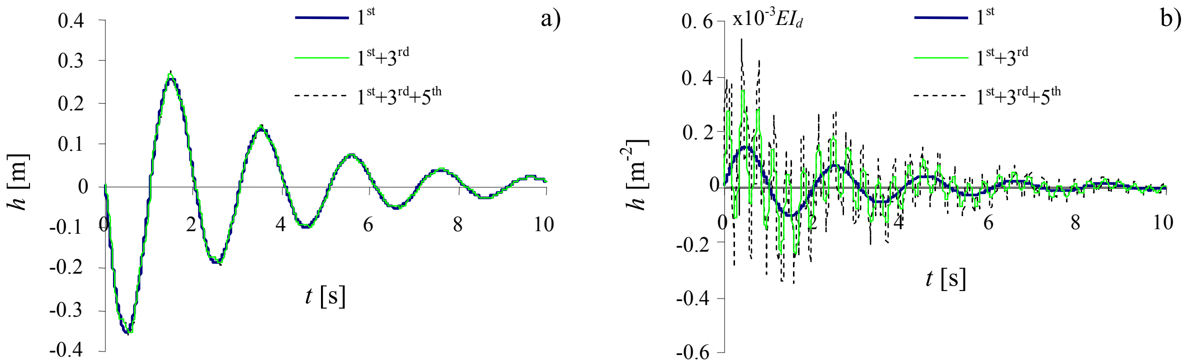

Figure 2. Response to impulsive loading in terms of a) midspan transverse displacement and b) abutment reactions.

It is observed that the midspan displacement response is dominated by the first mode only while higher modes strongly affect the abutment reactions. The impulse response in terms of transverse bending moments, not reported due to space constraint, is also influenced by the higher modes contribution, but at a less extent than the abutment reactions. Based on this results, it is expected that the approximations introduced in the response assessment will also affect differently the various response parameter considered.
The CMS method and the simplified RMS and FTS techniques are applied to the seismic analysis of the bridge under a set of 7 real ground motion records. These records are compatible with the Eurocode 8-1 (ECS 2005) design spectrum (Figure 3), corresponding to a site with a peak ground acceleration (PGA) of 0.35Sg where S is the soil factor, assumed equal to 1.15 (ground type C), and g is the gravity acceleration. They have been selected from the European strong motion Database (Ambraseys et al. 2000) and fulfil the requirements of Eurocode 8 (ECS 2005). For each record, the maximum values of the response parameter of interest attained during the seismic action are evaluated and averaged. These parameters are the midspan transverse displacement, the transverse bending moments and the abutment shear. Only the contribution of the first three anti-symmetric vibration modes are considered due to the negligible influence of the other modes.

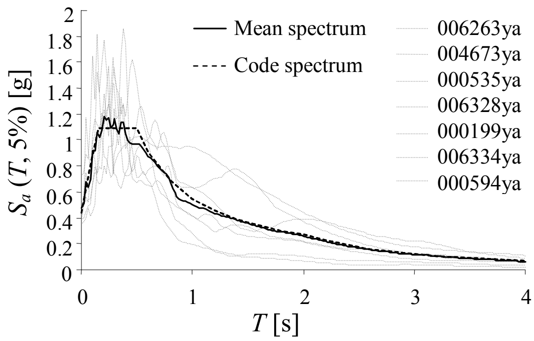

Figure 3. Input response spectrum and response spectra of natural records vs period T.

The average of the envelopes of the transverse displacements and bending moments according to the

three analysis techniques are reported in Figure 4. The three analysis techniques provide practically the same transverse displacement envelope, which perfectly coincides with a sinusoidal shape. This result was expected, since vibration modes higher than the first have a negligible influence on the displacements, as already pointed out in Figure 2.
With reference to the transverse bending moments envelope, the shape according to RMS and FTS agree well with the exact shape, the highest difference being in correspondence of the intermediate restraints. The influence of the third mode of vibration on the bending moments shape is well modelled by both the simplified techniques.

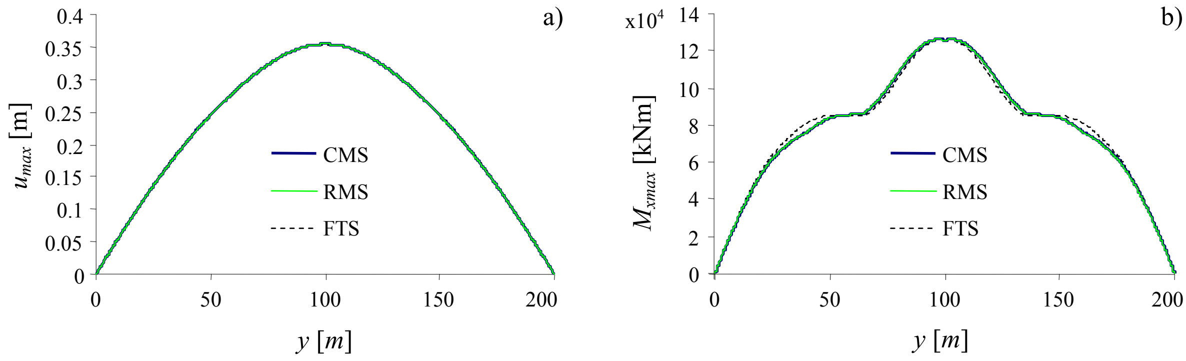

Figure 4. Transverse displacements and bending moments along the deck according to the exact and the simplified analysis techniques.

The average of the maxima of the midspan displacement $v_{\max}$, the transverse abutment reaction $R_{ab,\max}$ and the midspan transverse bending moments $M_{sag,\max}$ according to the three methods considered are shown and compared in Table 2.

Table 5.2. Comparison of results according to various analysis techniques.

Response parameter	CMS	RMS	FMS
$v_{\max}$ [m]	0.3543	0.3543	0.3537
$R_{ab,\max}$ [kN]	3051.5	3049.5	3090.9
$M_{sag,\max}$ [kNm]	126240	126230	126780

With reference to this application, it can be concluded that both the simplified analysis techniques provide very accurate estimates of the response despite their reduced computational cost, the highest relative error being less than 0.06% for RMS method and 1.4% for FTS method.
The coupling index for the damping matrix is $c.i._1 = 0.226$ for the RMS method and $c.i._1 = 0.224$ for the FTS method, whereas the coupling index for the stiffness matrix is $c.i._2 = 5.60\text{e-}4$ for the FTS method ($c.i._2 = 0$ for the RMS method). However, if only modes 1 and 3 are considered due to their higher contribution to the response with respect to mode 5, the coupling indexes reduce to $c.i._1 = 0.0454$ for the RMS method and $c.i._1 = 0.0461$ and $c.i._2 = 0.00056$ for FTS method. Thus, the extent of coupling in both the stiffness and damping terms is very limited and the approximate techniques are expected to provide accurate estimates of the seismic response.

6. CONCLUSIONS AND FURTHER STUDIES

The proposed simplified analysis techniques provide very accurate estimates of the seismic response of the realistic PRSI bridge considered in this paper, at a fraction of the computational cost and complexity required by the application of the CMS method.
Further parametric studies are currently being carried out in order to investigate the accuracy of these techniques for an extended set of bridge configurations. The additional results of these studies will permit to explicit the relationship between the coupling coefficients and the error committed in estimating the response parameter of interest.

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