Theoretical Assessment of Wire Rope Bracing System With Soft Central Cylinder

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SUMMARY:
Previous sever earthquakes have urged the necessity of seismic rehabilitation of existing structures. One rehabilitation method is to use only tension bracing systems for Moment Resistant Frames (MRFs). Wire ropes (cables), having high strength and stiffness, but low ductility, cannot be proper choice for X-braced type systems. Wire-rope bracing system with a central cylinder is a modern bracing system. As a couple of wire ropes are bundled with a cylindrical member at their intersection points, their bracing members are longer than the diagonal length of the frame. Despite low ductility of wire rope, the bracing system exhibits ductile behavior which can be used in strengthening steel and concrete MRFs. In this paper the ideal behavior of wire rope-soft cylinder bracing system and the effects of some important parameters have been studied theoretically. These parameters are dimensions and situation (horizontal or vertical) of cylinder. Optimal dimensions of cylinder have also been evaluated.

Keywords: wire rope; soft cylinder; hysteresis diagram

1. INTRODUCTION

Structural damages occurred due to the large recent earthquakes have emphasized the rehabilitation necessity of existing structures to withstand seismic loads. The flexibility of Moment Resistant Frame (MRF) systems may result in large drift and structural and nonstructural damage requesting costly post-earthquake retrofit. While MRF system has a large ductility capacity, MRF requires large columns for keeping its drifts in the allowable limit defined by seismic codes (Bruneau, Uang & Whittaker, 1998).

Different rehabilitation methods are introduced so far by the researchers, some of whom have focused on the application of braces in retrofitting. Bracing system can increase the stiffness and strength of story; however, the compression member might be buckled during the earthquake. For solving this problem, Buckling Restrained Bracing (BRB) has been introduced and improved the seismic behavior significantly (Xie, 2005; Tamai & Takamatsu, 2005; Renzi et al. 2007).

Wire rope bracing system with a central cylinder is a modern bracing system. In this system a couple of wire-ropes are bundled with a cylindrical member at their intersection point. Bracing members do not act for small and medium vibration amplitudes but they prevent large vibration amplitudes which cause large story drifts (Hou & Tagawa, 2009), shown in Fig. 1.1.

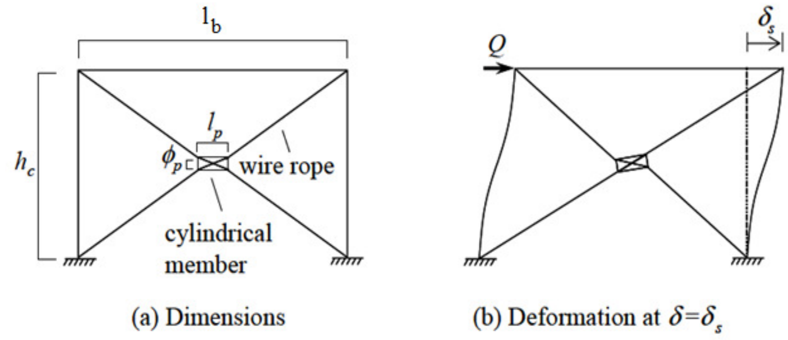

Figure 1.1. Wire rope bracing system with central cylinder

Story drift (δ_s) at which the bracing member starts acting is controlled by cylindrical member size and expressed as follows:

$$\delta_s = \sqrt{\left(2l_b + d_p\right)^2 - h_c^{\,2}} - h_b \tag{1.1}$$

$$l_b = \sqrt{\left(\frac{h_p - l_p}{2}\right)^2 + \left(\frac{h_c - \varphi_p}{2}\right)^2} \tag{1.2}$$

$$d_p = \sqrt{l_p^{\,2} + \left(\varphi_p - \varphi_b\right)^2} \tag{1.3}$$

Where, h_c is column length; l_b is beam length; l_p is length of cylindrical member; φ_p is inner diameter of cylindrical member; φ_b is wire rope diameter. Here, the ideal behavior of wire rope-cylinder bracing system is investigated. In this system the cylinder is sufficiently soft relative to wire rope members (such plastic pipe). The effects of some important parameters, the dimension and situation (horizontal or vertical) of cylinder, are also studied theoretically.

2. MOVEMENT AND DEFORMATION EQUATIONS FOR A SOFT CYLINDER

Regarding very low stiffness and high ductility of very soft cylinder, in the range of $\delta \leq \delta_s$ (δ_s is the displacement at which one wire rope start working), the cylinder can be deformed so that no increase is seen in the wire ropes' lengths and their internal forces remain zero until one wire rope becomes straight. In this case, if the wire rope length is constant inside the cylinder remains, its external length is also constant for every δ less than δ_s. Regarding Fig. 2.1., the distances between the points E, F, G and H to A, B, C and D are fixed, respectively. The points E, F, G and H are on the circles with centers of A and B, C and D, respectively, and with the radius $R = \sqrt{\left(\frac{l_b - x}{2}\right)^2 + \left(\frac{h_c - y}{2}\right)^2}$. At the same time they are on the circle with center $O : \left(\frac{l_b + \delta}{2}, \frac{h_c}{2}\right)$ and diameter $S = \sqrt{x^2 + y^2}$ as well. In this case the coordinates of cylinder's corners are obtained as follows:

$$E:\begin{cases} x_E^2 + y_E^2 = R^2 \\ \left(x_E - \dfrac{l_b+\delta}{2}\right)^2 + \left(y_E - \dfrac{h_c}{2}\right)^2 = \dfrac{S^2}{4} \end{cases} \tag{2.1}$$

$$F:\begin{cases} \left(x_F - \delta\right)^2 + \left(y_F - h_c\right)^2 = R^2 \\ \left(x_F - \dfrac{l_b+\delta}{2}\right)^2 + \left(y_F - \dfrac{h_c}{2}\right)^2 = \dfrac{S^2}{4} \end{cases} \tag{2.2}$$

$$G:\begin{cases} \left(x_G - l_b - \delta\right)^2 + \left(y_F - h_c\right)^2 = R^2 \\ \left(x_G - \dfrac{l_b+\delta}{2}\right)^2 + \left(y_G - \dfrac{h_c}{2}\right)^2 = \dfrac{S^2}{4} \end{cases} \tag{2.3}$$

$$H:\begin{cases} \left(x_H - l_b\right)^2 + y_H^2 = R^2 \\ \left(x_H - \dfrac{l_b+\delta}{2}\right)^2 + \left(y_H - \dfrac{h_c}{2}\right)^2 = \dfrac{S^2}{4} \end{cases} \tag{2.4}$$

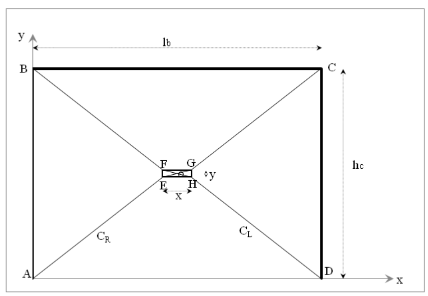

Figure 2.1. The origin of coordination in wire rope-cylinder bracing system

The cylinder is illustrated for lateral corresponding to drift of frame equal to zero and δ_s in Fig. 2.2. The length of beam and height of column are 4m and 3m, respectively, and the length of cylinder is 40 cm. The inner diameter of cylinder minus wire rope diameter is assumed as 5 cm. According to Fig 2.2, the cylinder has experienced both deformation and displacement. Therefore, if the cylinder is very soft one wire rope does not work before straightening.

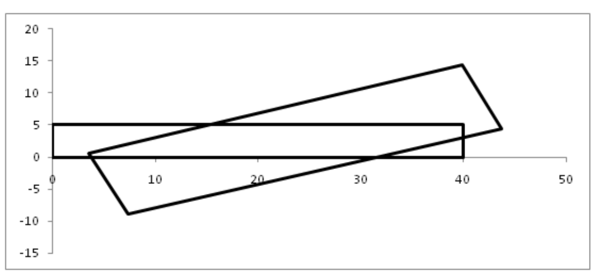

Figure 2.2. The deformation of very soft cylinder under lateral load

3. THE EFFECT OF SOFT CYLINDER'S DIMENSIONS ON WIRE-ROPE BRACING SYSTEM

Here, δ_s, a function of cylinder dimensions is determined in order to study the behavior of bracing with a very soft cylinder. Regardless the cylinder situation (horizontal or vertical) the value of δ_s increases by increasing the length of cylinder and vice versa. If the inner diameter of the cylinder decreases, the value of δ_s increases and it decreases by increasing the cylinder diameter. In other words the wire rope is deviated from its primary direction (frame's diagonal direction) and its length increases if the cylinder length increases or the inner diameter of cylinder decreases. Therefore, the value of δ_s increases, the wire rope works after receiving a larger lateral displacement and the ductility of braced frame increases.

The horizontal cylinder will be effective by satisfying the below equation:

$$\frac{l_p}{\varphi_p - \varphi_b} > \frac{l_b}{h_c} \tag{3.1}$$

The value of δ_s is zero when $\dfrac{l_p}{\varphi_p - \varphi_b} \le \dfrac{l_b}{h_c}$ and the behavior of wire rope is similar to that of ordinary X-cable bracing. If cylinder is vertical, it is effective when $\dfrac{l_p}{\varphi_p - \varphi_b} > \dfrac{h_c}{l_b}$ and ineffective when $\dfrac{l_p}{\varphi_p - \varphi_b} \le \dfrac{h_c}{l_b}$. If the diameter and length of cylinder increase simultaneously, δ_s increases by the length increase. However, δ_s decreases if the inner diameter of cylinder increases. Therefore, the inner diameter and length of the cylinder can increase while δ_s remains constant.
The dimensions of cylinders with the same δ_s can be obtained by the below formula:

$$\delta_s = \sqrt{\left(\sqrt{(l_b - x)^2 + (h_c - y)^2} + \sqrt{x^2 + y^2} \right)^2 - h_c^2} - l_b \tag{3.2}$$

If the cylinder is horizontal, x = cylinder length (l_p) and y = inner diameter of cylinder minus wire rope diameter ($\varphi_p - \varphi_B$); in case of vertical cylinder x = $\varphi_p - \varphi_B$ and y = l_p. The constant δ_s is assumed in Eqn. 3.2. for a portal frame with certain dimension and the x-y curve is depicted representing the dimensions of different cylinders with the same δ_s, Fig. 3.1 . The curve is plotted by MATLAB software; the frame has l_p = 4m and h_c = 3m for fixed δ_s.

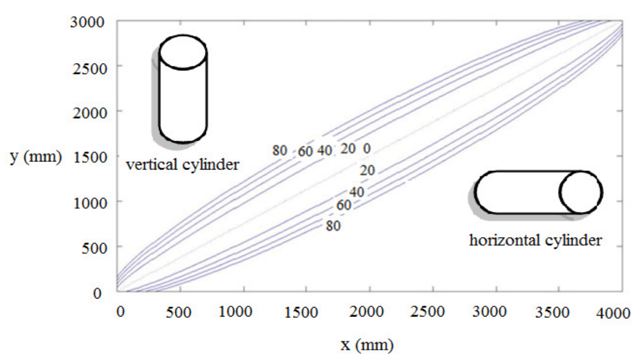

Figure 3.1. The dimensions of very soft cylinder with constant δ_s (δ_s is as per mm)

In Fig. 3.1. the straight line which is in the direction of frame diagonal shows that $\delta_s=0$. In this case the slope of wire rope in the cylinder is equal to that outside the cylinder; i.e. equal to the frame diagonal slope in the ordinary x bracing. The curves, bellow the straight line, indicate the horizontal cylinders in which the slope of wire rope in the cylinder is less than that of frame diagonal. The curves over the straight line are corresponded to the vertical cylinders in which the slope of wire rope in the cylinder is more than that of frame diagonal. Here, δ_s values are 20, 40, 60 and 80 mm which increase in accordance with the distances between curves and straight line. A considerable point in Fig. 3.1 is that by increasing δ_s, the distances between adjacent curves are reduced gradually. For example the space between the curves of δ_s=20mm and δ_s=40mm is higher comparing with δ_s=40mm and δ_s=60mm and lower in comparison with δ_s= 0mm ,δ_s=20mm. Another noticeable point in Fig. 3.1. is that the curves are symmetric to the frame diagonal line, corresponding to $\delta_s=0$. This fact is controlled by considering a random point, A (x_o,y_o), in Fig. 3.1. representing a cylinder with specified dimensions. l_b is the beam length and h_c is the column height. According to the relationships in analytic geometry, the coordinates of symmetrical point (A) with respect to the line $y=\frac{h_c}{l_b}x$ is:

$$A'=\left(x_0{}',y_0{}'\right)=2\times\frac{l_b x_0+h_c y_o}{l_b{}^2+h_c{}^2}(l_b,h_c)-(x_0,y_0) \tag{3.3}$$

The proof is as follows:

$$\delta_{S(A)}=\delta_{S(A')}\rightarrow\sqrt{\left(\sqrt{(l_b-x_A)^2+(h_c-y_A)^2}+\sqrt{x_A^2+y_A^2}\right)^2-h_c^2}-l_b \tag{3.4}$$

$$=\sqrt{\left(\sqrt{(l_b-x_{A'})^2+(h_c-y_{A'})^2}+\sqrt{x_{A'}^2+y_{A'}^2}\right)^2-h_c^2}-l_b$$

$$\rightarrow\sqrt{(l_b-x_A)^2+(h_c-y_A)^2}+\sqrt{x_A^2+y_A^2}=\sqrt{(l_b-x_{A'})^2+(h_c-y_{A'})^2}+\sqrt{x_{A'}^2+y_{A'}^2}$$

The above equations show that wire ropes have the same total lengths in both cases. The first and second terms in the relation represent the lengths of wire rope outside and inside the cylinder, respectively. It is assumed that the lengths of inner wire ropes are identical in both cases (corresponding to the points A and A'); the lengths of outer wire ropes are also identical in both cases. The assumption is proved as follows:

$$x_A^2+y_A^2=x_{A'}^2+y_{A'}^2 \tag{3.5}$$

$$(l_b-x_A)^2+(h_c-y_A)^2=(l_b-x_{A'})^2+(h_c-y_{A'})^2 \tag{3.6}$$

To proof the Eqn. 3.5, the right side is calculated to reach the left side:

$$right\ side=x_{A'}^2+y_{A'}^2=\left(2l_b\frac{l_b x_A+h_c y_A}{l_b^2+h_c^2}-x_A\right)^2+\left(2h_c\frac{l_b x_A+h_c y_A}{l_b^2+h_c^2}-y_A\right)^2 \tag{3.7}$$

$$=4l_b^2\left(\frac{l_b x_A+h_c y_A}{l_b^2+h_c^2}\right)^2+x_A^2-4x_A l_b\left(\frac{l_b x_A+h_c y_A}{l_b^2+h_c^2}\right)+4h_c^2\left(\frac{l_b x_A+h_c y_A}{l_b^2+h_c^2}\right)^2+y_A^2-4y_A h_c\left(\frac{l_b x_A+h_c y_A}{l_b^2+h_c^2}\right)$$

$$=4\left(l_b^2+h_c^2\right)\left(\frac{l_b x_A+h_c y_A}{l_b^2+h_c^2}\right)^2+x^2+y^2-4\frac{\left(l_b x_A+h_c y_A\right)^2}{l_b^2+h_c^2}$$

$$=4\frac{\left(l_b x_A+h_c y_A\right)^2}{l_b^2+h_c^2}+x^2+y^2-4\frac{\left(l_b x_A+h_c y_A\right)^2}{l_b^2+h_c^2}=x^2+y^2=\textit{left side}$$

To proof Eqn. 3.6, the right side is calculated to reach the left side:

$$\textit{right side}=\left[l_b\left(1-2\frac{l_b x_A+h_c y_A}{l_b^2+h_c^2}\right)+x_A\right]^2+\left[h_c\left(1-2\frac{l_b x_A+h_c y_A}{l_b^2+h_c^2}\right)+y_A\right]^2 \tag{3.8}$$

$$=l_b^2\left(1-2\frac{l_b x_A+h_c y_A}{l_b^2+h_c^2}\right)^2+x_A^2+2l_b x_A\left(1-2\frac{l_b x_A+h_c y_A}{l_b^2+h_c^2}\right)$$

$$+h_c^2\left(1-2\frac{l_b x_A+h_c y_A}{l_b^2+h_c^2}\right)^2+y_A^2+2h_c y_A\left(1-2\frac{l_b x_A+h_c y_A}{l_b^2+h_c^2}\right)$$

$$=l_b^2\left[1+4\left(\frac{l_b x_A+h_c y_A}{l_b^2+h_c^2}\right)^2-4\left(\frac{l_b x_A+h_c y_A}{l_b^2+h_c^2}\right)\right]+x_A^2+2l_b x_A\left(1-2\frac{l_b x_A+h_c y_A}{l_b^2+h_c^2}\right)$$

$$+h_c^2\left[1+4\left(\frac{l_b x_A+h_c y_A}{l_b^2+h_c^2}\right)^2-4\left(\frac{l_b x_A+h_c y_A}{l_b^2+h_c^2}\right)\right]+y_A^2+2h_c y_A\left(1-2\frac{l_b x_A+h_c y_A}{l_b^2+h_c^2}\right)$$

$$=\left(l_b^2+h_c^2\right)\left[1+4\left(\frac{l_b x_A+h_c y_A}{l_b^2+h_c^2}\right)^2-4\left(\frac{l_b x_A+h_c y_A}{l_b^2+h_c^2}\right)\right]+x_A^2+y_A^2+\left(2l_b x_A+2h_c y_A\right)\left(1-2\frac{l_b x_A+h_c y_A}{l_b^2+h_c^2}\right)$$

$$=l_b^2+h_c^2+4\frac{\left(l_b x_A+h_c y_A\right)^2}{l_b^2+h_c^2}-4\left(l_b x_A+h_c y_A\right)+x_A^2+y_A^2+2l_b x_A+2h_c y_A-4\frac{\left(l_b x_A+h_c y_A\right)^2}{l_b^2+h_c^2}$$

$$=l_b^2+h_c^2+x_A^2+y_A^2-2\left(l_b x_A+h_c y_A\right)=\left(l_b-x_A\right)^2+\left(h_c-y_A\right)^2=left\ side$$

According to the proved Eqns. 3.5 and 3.6, if the dimensions, length (x) and height (y), of two cylinders with the same δ_s are symmetric to the line $y=\frac{h_c}{l_b}x$, the internal and the external lengths of their wire ropes are equal.

δ_{sh} is the δ_s for horizontal position of the cylinder and δ_{sv} is the δ_s for vertical position of the cylinder. Regarding Fig. 3.1. if in a wire rope-cylinder braced frame, $l_b > h_c$, then δ_{sv} is greater than δ_{sh}. In order to obtain δ_{sv}, the symmetrical point of A= (x_o,y_o) to the line y = x, should be calculated. The new point has larger distance from the line $y=\frac{h_c}{l_b}x$ and therefore larger $\boldsymbol{\delta_s}$ in comparison with the point A. By contrast if $l_b < h_c$, then δ_s in the new point is larger for the horizontal position of cylinder comparing to its vertical position.

Regarding a constant $\boldsymbol{\delta_s}$, if the longitudinal direction of cylinder is parallel to smaller dimension of the frame, the cylinder's dimensions are smaller comparing to that of parallel to larger dimension. In order to obtain the minimum length of cylinder, its smallest possible inner diameter is chosen and its longitudinal direction is paralleled to the smaller dimension of the frame.

Based on Fig. 3.1., the cylinder's dimension, small or large, is not a factor in $\boldsymbol{\delta_s}$ determination. It is obvious that smaller dimensions of cylinders are better due their lower spaces regarding themselves and their rotations. On the other side, as the wire ropes should pass through the cylinder, its dimensions cannot be smaller than a specific value. If the diameters of the wire ropes are equal to φ_b, then the inner diameter of the cylinder should be larger than $2\varphi_b$.. Consequently, in a special design with a specified $\boldsymbol{\delta_s}$, the smallest possible inner diameter ($2\varphi_b$) is recommended and the cylinder's length is calculated by Eqn. 3.2. If $l_b > h_c$, then the smallest possible dimensions of the cylinder are found for vertical position of cylinder; however, they are calculated for a horizontal position where $l_b < h_c$.

4. IDEAL HYSTERSIS CURVES FOR THE FRAMES WITH WIRE ROPE– SOFT CYLINDER BRACING

Ideal hysteresis diagrams of an MRF braced with wire rope-soft cylinder bracings is drawn by superposing two ideal hysteresis diagrams plotted for an MRF (Fig. 4.1.) and a simple frame braced with wire rope-soft cylinder (Fig. 4.2.) separately.

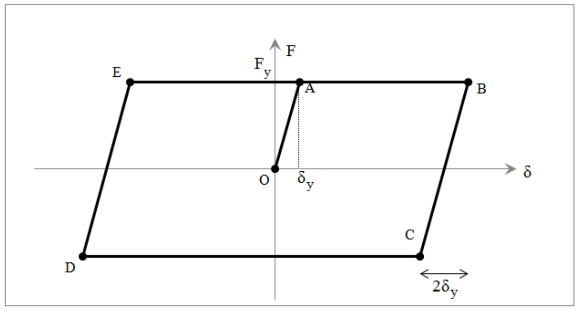

Figure 4.1. Ideal hysteresis diagram of an MRF

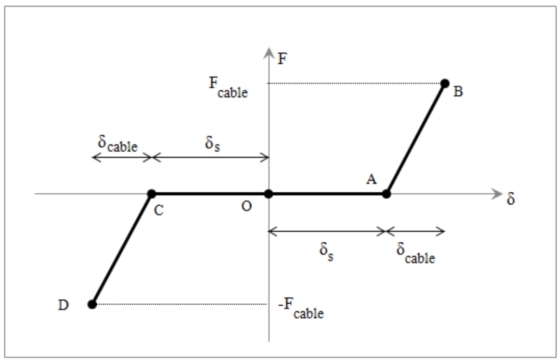

Figure 4.2. Ideal hysteresis diagram of a simple frame braced with wire rope-soft cylinder

Wire rope bracing reaches its ultimate strength at the lateral force (F_{cable}) and its ultimate strength at the drift (δ_{cable}). According to Fig. 4.2., the wire ropes are not effective until lateral displacement reaches δ_s and the system acts like without bracing. In this case the length of wire rope bracing is more than frame's diameter for the drifts less than δ_s. Therefore, as lateral load increases, the angel between the wire ropes inside and outside the cylinder is gradually reduced up to zero in $\boldsymbol{\delta_s}$ and the wire rope becomes straight and starts working. In case of existing no cylinder and no loose wire ropes, δ_s would be zero and the wire rope bracing act as X-cable bracing.

In Fig. 4.3. the hysteresis diagram of a MRF with wire rope-soft cylinder bracing is obtained by superposing the hysteresis diagrams of Figs. 4.1. and 4.2.

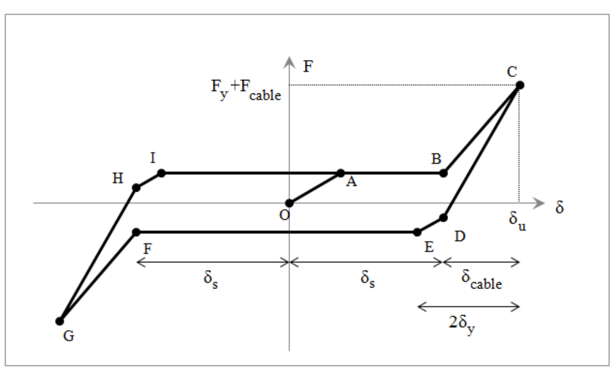

Figure 4.3. The hysteresis diagram of MRF braced with wire rope-soft cylinder bracing

For optimum design of very soft cylinder, the dimensions of the cylinder should be selected in such a way that the collapse of frame of and tear of wire rope occur simultaneously. In this case the optimized $\boldsymbol{\delta_s}$ is as follows:

$$\delta_{s_{opt}} = \delta_u - \delta_{cable} \tag{4.1}$$

If $\delta_s < \delta_{s_{opt}}$, then the wire rope is torn before the frame reaches its ultimate strength and the frame's ductility cannot be used.
If $\delta_s > \delta_{s_{opt}}$, then the frame reaches its ultimate strength before tearing the wire rope and therefore the total capacity of wire rope cannot be used. Theoretically, the best frame with wire rope-cylinder bracing is the one in which $\delta_s = \delta_{s_{opt}}$ unless drift limitation forced us using smaller δ_s.

5. CONCLUSIONS

In this study the behavior of wire rope-cylinder bracing is discussed theoretically regarding very soft cylinder. The behavior of such bracing depends on the dimensions of the cylinder. That is by increasing the length of cylinder or decreasing its inner diameter, the bracing acts in larger drifts and causes the increase of frame's ductility. If lb > hc, then δs is larger in the vertical position of cylinder comparing to that in its horizontal position. If lb < hc, then δs is larger in horizontal position of cylinder in comparison with that in its vertical position. Moreover, for obtaining the smallest dimensions of cylinder its longitudinal direction be parallel to the smaller dimension of the frame. The dimensions of the cylinder should be calculated with respect to the allowable drift of the frame. In case of being no limitation on drift, the cylinder is designed optimally based on the synchronization of frame collapse and wire rope tear. Hysteresis diagram of an MRF with wire rope-cylinder is expressed by superposing the hysteresis diagrams of MRF and simple frame braced with wire rope-cylinder bracing plotted separately.

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