Evaluation of Fatigue and Park and Ang damage indexes in steel structures

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SUMMARY:
This paper deals with the estimation of inelastic demand parameters on simplified structural models representative of typical steel structures under constant relative strength scenarios with particular emphasis on Park and Ang as well as Fatigue Damage Indexes. Mean inelastic demands on bi-linear systems (simulating moment resisting frames) are considered as the basis for comparative purposes. Additional models representing steel structures with various levels of pinching (like partially-restrained frames) are introduced and employed to assess the influence of different force-displacement relationships on inelastic demand ratios. The studies presented in this paper illustrate that the hysteretic shape of pinching models can lead to significant differences in inelastic behaviour when compared against predictions based on bi-linear idealizations, especially in the short-period range. It is also shown that the level of pinching in the hysteretic response together with the post-elastic stiffness can significantly influence the level of cumulative damage experienced by steel structures.

Keywords: Inelastic demands, steel structures, fatigue, Park and Ang index

1. INTRODUCTION

Current earthquake performance-based design and assessment methodologies pay special attention to the reliable determination of structural response like peak deformations and energy dissipation. To this end, considerable research has already been carried out into the probabilistic distribution of peak displacements under various suites of ground-motion (Ruiz-Garcia and Miranda, 2003). Most of these studies are based primarily on elastic-plastic single-degree-of-freedom (SDOF) systems, and those studies that include pinching behaviour simultaneously incorporate severe deterioration levels typical of reinforced concrete structures (Goda et al., 2009). In contrast, steel structures exhibiting pinching (like frames employing semi-rigid connections) not always present significant strength deterioration up to considerably high levels of deformation demand (Elghazouli, et al., 2009; Málaga-Chuquitaype and Elghazouli, 2011). In light of this discussion, there is a need to improve the understanding of the structural response of SDOF systems representative of commonly used steel structures, with the aim of informing their seismic assessment, in particular with relation to cumulative structural damage measures like Fatigue and Park and Ang Damage indexes.

This paper deals with the estimation of inelastic demands in simplified structural models representative of typical steel structures under constant relative strength scenarios. It considers the relative trends in mean peak displacements, Fatigue Damage and Park and Ang indexes. Mean inelastic deformation demands on bi-linear systems (simulating moment resisting frames) are considered as the basis for comparative purposes. Additional SDOF models representing partially-restrained frames are introduced and employed to assess the influence of pinching force-displacement relationships on peak inelastic displacement ratios. The studies presented in this paper illustrate that the hysteretic shape of pinching models can lead to significant differences when compared against predictions based on bi-linear idealizations, especially in the short-period range. Additionally, it is shown that structures exhibiting different levels of post-elastic stiffness will experience proportional reductions in the mean cumulative damage as quantified by Park and Ang and Fatigue indexes.

2. STRUCTURAL SYSTEMS AND EARTHQUAKE GROUND-MOTIONS

2.1. Structural Systems Investigated

Bi-linear (i.e. elastic-perfectly plastic) systems are used in this study as benchmark models for comparison purposes. In addition, the response of SDOF models representative of partially-restrained structures is considered. The Modified Richard-Abbott model as proposed and validated by Nogueiro et al. (2007) is used here to represent the response of Partially-Restrained steel structures. The Modified Richard-Abbott model is based on the alternation between two limiting curves of the Richard-Abbott type (Richard & Abbott, 1975). As shown in Fig. 1, the boundary curves are characterized by their initial stiffness (k), post-elastic stiffness (k_p) and strength capacities (F_{sp} and F_{yp} for the lower and upper bound curves, respectively) where F_{yp} is the yield strength. Also presented in Fig. 1 is the corresponding bi-linear approximation of the Modified Richard-Abbott backbone. The pinching factor (P) is defined here as the ratio between the structural capacity during pinching intervals and the overall capacity:

$$P = \frac{F_{sp}}{F_{yp}} \tag{2.1}$$

Similarly, the strain hardening coefficient (Sh) is defined as the ratio between the post-elastic stiffness, k_p, and the initial stiffness, k, in the upper bound curve

$$Sh = \frac{k_p}{k} \tag{2.2}$$

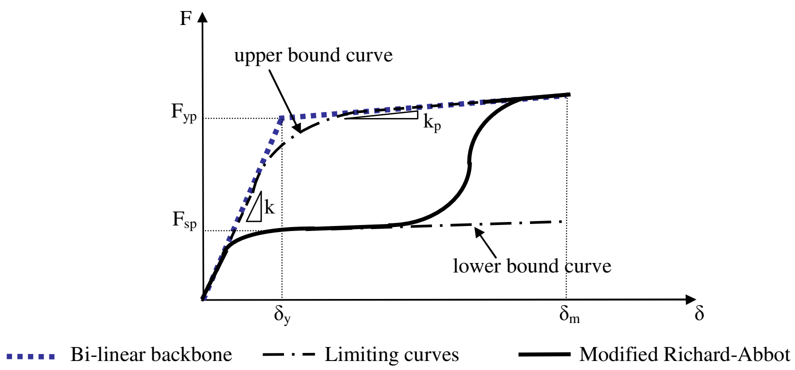


Figure 1. Bi-linear and modified Richard-Abbot force-deformation relationships

2.2. Ground Motion Dataset

A total of 100 records from 27 earthquakes with magnitudes M_w ranging from 5.65 to 7.51, and distances ranging from 6.28 to 293 Km, were used in this study. The acceleration records were obtained from the PEER-NGA database, and involve different site classes (according to the NEHRP classification) in order to address the impact of site conditions on the variability in inelastic response. Special attention was given to the lowest usable frequency in order to avoid undesired noise and filtering effects. Table 2.1 summarizes the catalogue of earthquakes used while more detailed information can be found elsewhere (Málaga-Chuquitaype, 2011).

Table 2.1. Summary of earthquake ground-motion dataset

Earthquake name		Magnitude M_w	Distance [km]		PGA [cm/s^2]		NEHRP site class
			Min.	Max.	Min.	Max.	
1992	Cape Mendocino	7.01	10.36	53.34	151.11	1468.85	C(2) D(2)
1986	Chalfant Valley-01	5.77	10.54	10.54	202.59	279.58	D(2)
1986	Chalfant Valley-02	6.19	14.33	14.33	392.12	438.32	D(2)
2002	Denali, Alaska	7.9	290.70	293.06	10.02	22.39	E(6)
1999	Duzce, Turkey	7.14	24.26	206.09	24.73	144.64	C(2) E(2)
1976	Friuli, Italy-01	6.5	20.23	20.23	308.83	344.65	C(2)
1976	Gazli, USSR	6.8	12.82	12.82	596.70	703.92	C(2)
1999	Hector Mine	7.13	52.29	52.29	143.04	186.05	C(2)
1979	Imperial Valley-06	6.53	22.65	30.35	216.87	320.53	D(2) E(2)
1980	Irpinia, Italy-01	6.90	22.65	30.35	136.71	350.97	B(4)
1952	Kern County	7.36	43.39	43.39	153.00	174.41	C(2)
1995	Kobe, Japan	6.90	25.40	25.40	284.57	304.56	B(2)
1999	Kocaeli, Turkey	7.51	47.03	112.26	134.64	243.94	A(2) B(2) E(2)
1992	Landers	7.28	44.02	44.02	713.03	774.17	C(2)
1994	Little Skull Mtn,NV	5.65	14.12	30.17	116.71	208.81	B(3)
1989	Loma Prieta	6.93	16.51	114.87	94.64	388.07	B(3) D(2) E(8)
1990	Manjil, Iran	7.37	37.90	37.90	486.92	504.78	C(2)
1984	Morgan Hill	6.19	38.20	38.20	190.58	196.59	D(2)
1986	N. Palm Springs	6.06	6.28	6.28	201.08	214.06	D(2)
1985	Nahanni, Canada	6.76	6.80	6.80	959.25	1074.88	C(2)
2002	Nenana Mountain	6.70	275.28	277.70	7.08	10.73	E(8)
1994	Northridge-01	6.69	18.99	45.77	110.19	1554.79	A(4) B(8)
1971	San Fernando	6.61	31.55	31.55	145.57	148.98	C(2)
1986	San Salvador	5.80	9.54	9.54	398.66	600.51	D(2)
1987	Superstition Hills-02	6.54	29.91	29.91	113.76	152.94	D(2)
1978	Tabas, Iran	7.35	55.24	55.24	819.93	835.58	B(2)
1981	Westmorland	5.90	20.47	20.47	152.18	237.25	D(2)

2.3. Fatigue and Park and Ang Damage Indices

The global Fatigue Damage Index is defined here as:

$$F = \sum_{i=1}^{i=Nc} \frac{1}{a} S^b \tag{2.3}$$

where a and b are the fatigue constant and exponent, respectively; S is the amplitude of the ith cycle and Nc is the total number of displacement cycles. The fatigue damage is calculated for b = 3 which is in accordance with experimental results (Málaga-Chuquitaype C., 2011; Málaga-Chuquitaype C. and Elghazouli, A.Y., 2010). Besides a value of a = 1 was used here as the focus of the calculation is on generalized trends and relative differences between pinching and bi-linear models rather than on a precise quantification of the Fatigue Index which is case specific.

$$D = \frac{\delta_m}{\delta_u} + \frac{c}{F_y \delta_y} \int de \tag{2.4}$$

Where δ_m is the maximum displacement, δ_u is the ultimate monotonic displacement capacity of the structure, c is an empirical factor that defines the balance between the extreme displacement and the hysteretic energy $\int de$ terms, and F_y and δ_y are the yield force and yield displacement, respectively. In this study, c is set to 0.15 in accordance with conventional practice.

2.4. Scope of the Parametric Analysis

A statistical study was performed on the peak inelastic displacement demands for several SDOF systems with 5% viscous damping under constant strength ratio scenarios. The inelastic displacement ratio (C_R) is defined as the ratio between the peak lateral inelastic displacement ($\delta_{inelastic}$) and the peak lateral elastic displacement demand ($\delta_{elastic}$) on a SDOF with the same mass and initial stiffness:

$$C_R = \frac{\delta_{inelastic}}{\delta_{elstic}} \tag{2.5}$$

$\delta_{inelastic}$ is calculated from response history analyses on structures with constant relative strength in proportion to the strength required to keep the system elastic (F_y). The constant relative strength scenarios are characterized by the strength ratio R defined as:

$$R = \frac{mS_a}{F_y} \tag{2.6}$$

where m is the mass of the system and S_a is the acceleration spectral ordinate. Five values of lateral strength ratios R were considered (i.e. R = 1.5, 2, 3, 4 and 5). Parametric analyses were performed for 3 levels of pinching (i.e. P = 0.15, 0.3 and 0.6) representative of the range of pinching levels usually observed in partially-restrained connections.

On the basis of the median acceleration response spectra for the different soil classes here considered, a clear distinction is made between moderately stiff to stiff soils sites (Classes A, B, C and D in Table 1) and soft soils sites (Class E in Table 1) for the purposes of presentation and discussion in this paper. The effects of structural model characteristics and levels of strength demands are discussed below with emphasis on peak inelastic displacements, Fatigue Damage and Park and Ang Damage indexes.

3. ASSESSMENT OF INELASTIC DISPLACEMENT DEMANDS

Mean inelastic displacement ratios were computed by averaging the results for each period, strength ratio and hysteretic model. Figs. 2 and 3 present mean inelastic displacement ratios for moderately stiff to stiff soils sites and for the different structural systems considered. The effects of structural model characteristics, strength demand and soil conditions on inelastic displacement ratios are discussed in detail elsewhere (Málaga-Chuquitaype and Elghazouli, 2012).

The curves for C_R presented in Fig. 2 follow the general trends observed by other researchers where inelastic displacement ratios increase as the structural period tends to zero. Based on the negligible variability in displacement ratios as a function of R observed for pinching systems, Fig. 3 presents their mean displacement ratios normalized over bi-linear models for different levels of pinching averaged over all R values (i.e. 2, 3, 4 and 5). It can be observed from Fig. 3 that there is some degree of dependence of C_{PR}/C_R ratios on the level of pinching, particularly for relatively stiff systems. As expected, the displacement amplification of short-period pinching models built on moderate to stiff soils with respect to bi-linear predictions tend to in-crease for lower values of *P* owing to the reduced energy dissipation in systems with higher pinching levels.

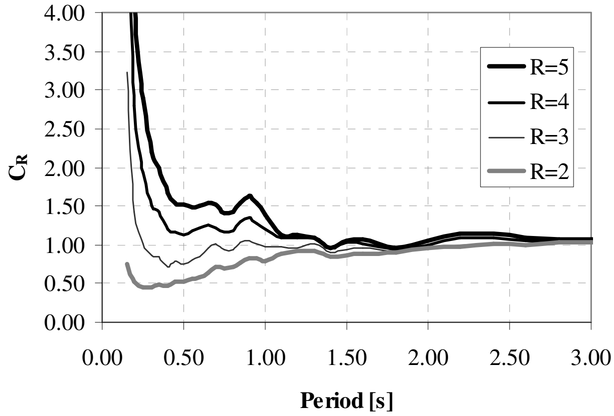


Figure 2. Mean inelastic displacement ratios for moderately stiff to stiff soil ground-motions on bi-linear systems

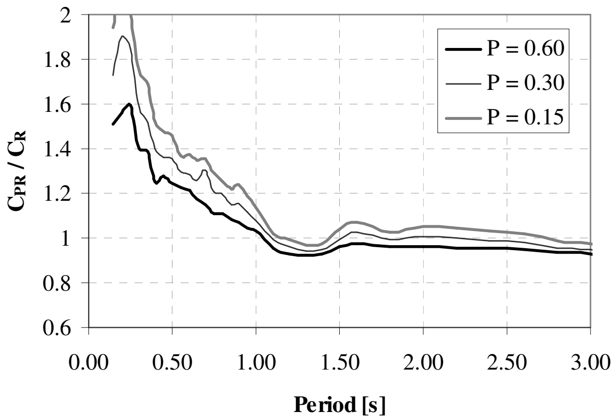
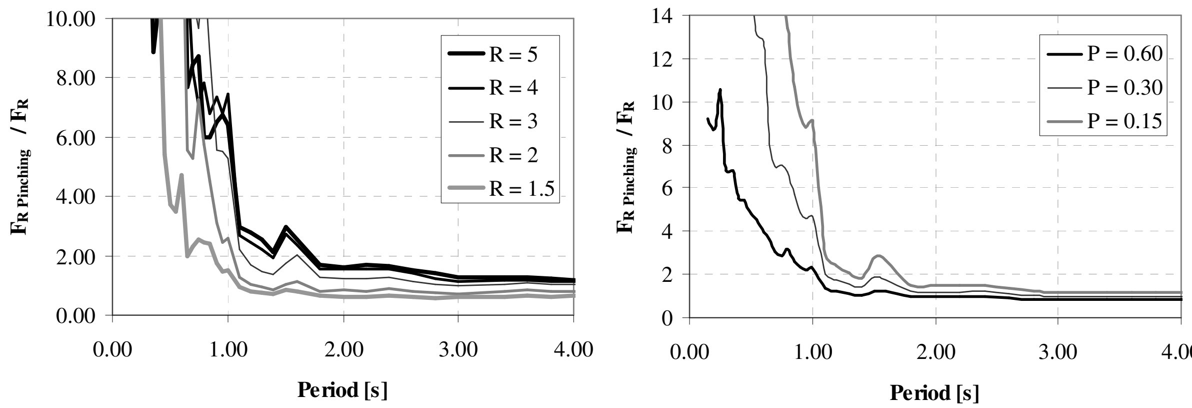


Figure 3. Mean inelastic displacement ratios of pinching systems normalized by mean displacement ratios of bi-linear systems. Average for all strength values on moderately stiff to stiff soils as a function of pinching factor (*P*)

4. ASSESSMENT OF FATIGUE DAMAGE

Figs. 4 and 5 present the results of mean Fatigue Damage of pinching models normalized by the corresponding Fatigue Damage indices in bi-linear systems for stiff and soft soils, respectively. It can be observed from Fig. 4(a) that for stiffer structures, with periods equal or shorter than 1 second, Fatigue Damage in pinching systems can be significantly higher than the corresponding damage levels expected in bi-linear structures. Also, the relative fatigue damage (between pinching and bi-linear systems) is observed to increase with increasing levels of strength demand (higher values of R). The dependence of fatigue damage on the level of pinching can be studied with reference to Fig. 4(b) which presents relative Fatigue Indexes for structures with three pinching factors for a constant strength demand of R = 3. It can be observed from Fig. 4(b) that there is significant dependence of fatigue damage ratios on the level of pinching, especially for short period structures.

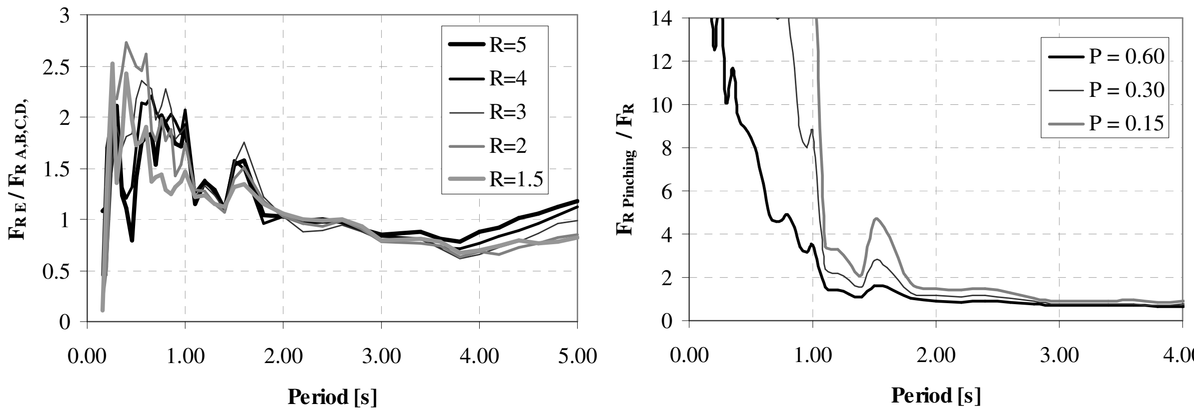
Similar trends as those identified for moderate to stiff soils (Fig. 4) are observed for structures on soft soils (Fig. 5), although increased variability with respect to R is evident in the case of soft soils. In addition, for structures with initial periods longer than 2 seconds the Fatigue Damage indexes of pinching systems in soft soils are lower than those observed in bi-linear models. Similarly, the levels of fatigue in pinching short period structures on soft soils are not as severe as those on stiffer soil conditions.



(a) Mean Fatigue Damage Index ratio between pinching systems and bi-linear systems for P=0.30

(b) Mean Fatigue Damage Index ratio between pinching systems and bi-linear systems for R=3

Figure 4. Fatigue Damage Index ratios for moderately stiff to stiff soils



(a) Mean Fatigue Damage Index ratio between pinching systems and bi-linear systems for P=0.30

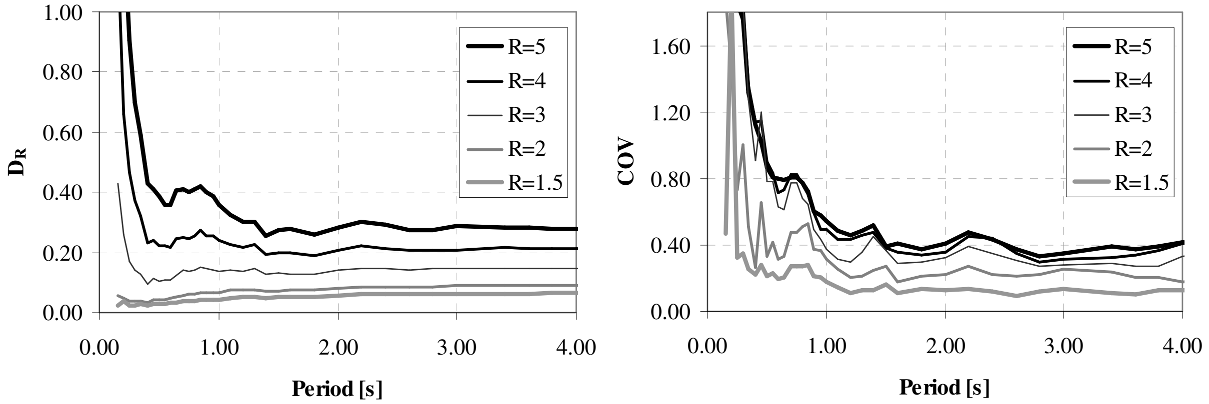
(b) Mean Fatigue Damage Index ratio between pinching systems and bi-linear systems for R=3

Figure 5. Fatigue Damage Index ratios for soft soils

5. ASSESSMENT OF PARK AND ANG DAMAGE INDEX

Fig. 6 presents the estimations of Park and Ang damage index for bi-linear SDOF models under different strength demands. A clear dependence of the Park and Ang Index on the lateral strength ratio R is evident in Fig. 6(a) with higher strength demands causing more pronounced damage over the full range of periods here studied. Likewise, the dispersion, quantified here by means of the coefficient of variation (COV), is observed to increase with the strength ratio in Fig. 6(b). A decrement in COV values with increasing period is also evident from Fig. 6(b) up to approximately 1 second while the levels of dispersion remain approximately constant for structures of longer fundamental periods.

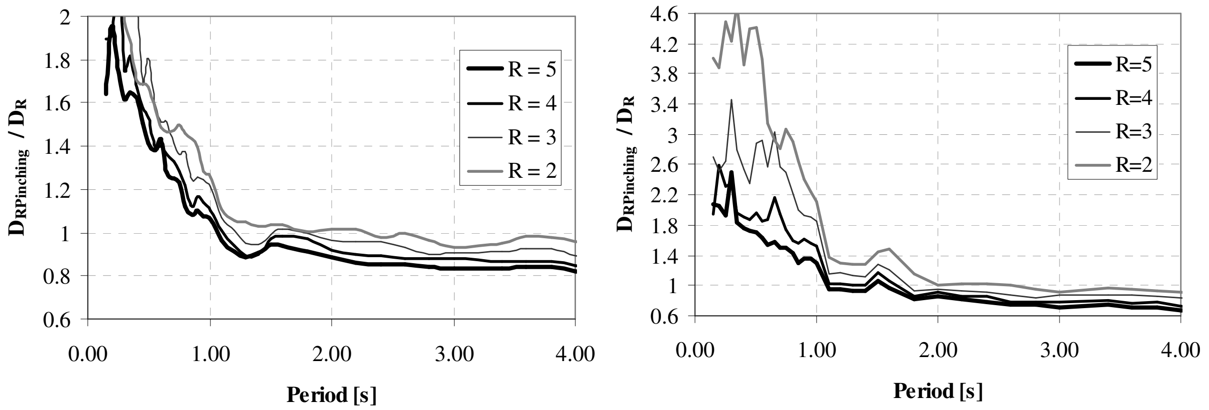
Fig. 7 depicts the results of mean Park and Ang Index in pinching systems with P = 0.3 normalized by mean Park and Ang Indexes of the corresponding bi-linear systems for stiff and soft soils sites. It can be observed from Fig. 7 that for periods longer than 1 second the results of bi-linear and pinching systems with R = 2 are broadly similar whereas lower levels of damage would be expected in pinching structures for higher R values in both stiff and soft soils. This inverse relationship between mean Park and Ang damage ratios and strength demand levels is maintained for shorter period structures albeit the higher damage exhibited by pinching models. With respect to bi-linear systems, short period pinching structures can reach Park and Ang damage index values of 2 or 4.6 times the expected values on bi-linear systems for stiff and soft soils sites, respectively. Additionally, the period range at which structural failure (e.g. D = 1) is attained is increased from T < 0.25 seconds in bi-linear systems to around T < 0.8 seconds in pinching systems built on soft soils while remaining around T < 0.3 seconds for stiffer soil conditions. Similar results were obtained for other levels of pinching.



(a) Mean Park and Ang Damage Index

(b) Coefficient of Variation of Park and Ang Damage Index

Figure 6. Park and Ang Damage Index of bi-linear systems on stiff to moderately stiff soils

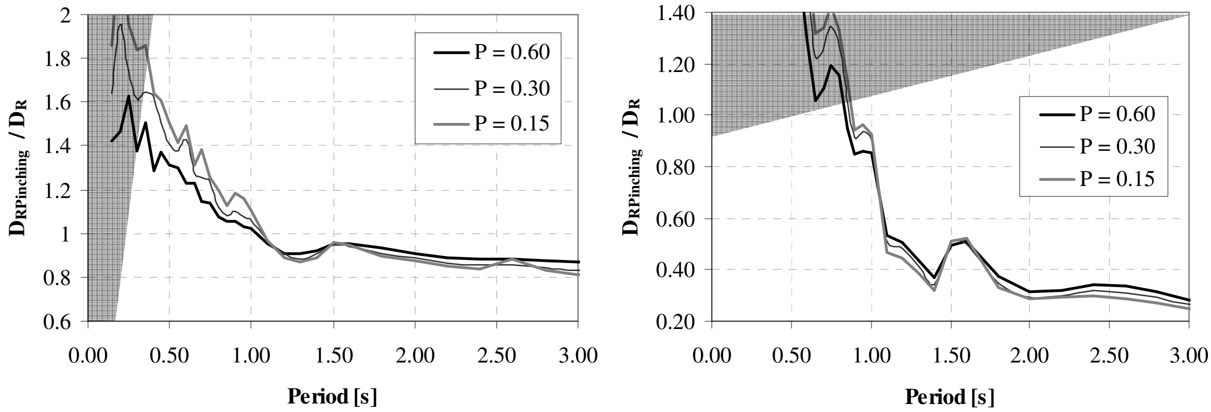


(a) Moderately stiff to stiff soils

(b) Soft soils

Figure 7. Mean Park and Ang Damage Index of pinching systems with P = 0.3 normalized by mean Park and Ang Damage index of bi-linear models

The influence of the level of pinching on the expected Park and Ang damage indexes can be evaluated with reference to Fig. 8 that presents the results of normalized index values for R = 5 and stiff and soft soils sites. The shaded areas in Fig. 8 represent areas of structural failure (e.g. D = 1). In the case of moderately stiff to stiff soils sites, the level of pinching has some degree of influence on the expected relative Park and Ang damage indexes for shorter period structures (T < 1 second) with higher damage expected on structures with higher levels of pinching owing to their reduced energy dissipation capabilities. Similar trends are observed for softer soils although the differences between structures with varying pinching factors are notably smaller. Importantly, the values of Park and Ang damage indexes attained in pinching structures of longer initial period (T > 1second) are significantly smaller than those observed in their bi-linear counterparts with values as low as 30% of those expected in bi-linear systems for soft soil conditions.



(a) Stiff to moderately stiff soils (b) Soft soils

Figure 8. Mean Park and Ang Damage Index of pinching systems normalized by mean Park and Ang Damage index of bi-linear models for R = 5

6. INFLUENCE OF POST-ELASTIC HARDENING

Fig. 9 presents the mean Fatigue Damage Index values of pinching systems normalized by the corresponding values of bi-linear models for structures with different strain hardening coefficients (Eq. 2.2). The increment in average Fatigue Damage Index with decreasing period previously identified for bi-linear and pinching SDOF models is also evident for all levels of strain hardening studied in Fig. 9. Although the increments associated with the highest strain hardening value here studied (*Sh* = 15%) lead to smaller Fatigue Damage indexes in the short period range, no direct relationship between post-elastic stiffness and reduction of Fatigue Damage Index can be identified.

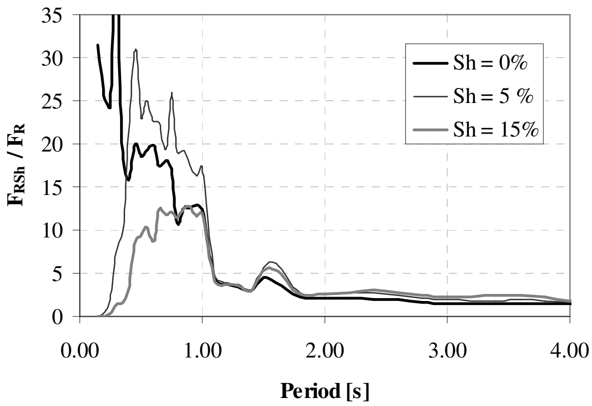


Figure 9. Mean Fatigue Damage Indexes for pinching structures normalized by mean Fatigue Damage Indexes in by-linear systems for different strain hardening coefficients R = 5.

The mean Park and Ang Damage index values for pinching systems normalized by the corresponding values on bi-linear models are presented in Fig. 10 for different strain hardening coefficients. Contrary to Fatigue Damage, a clear influence of the level of strain hardening is evident for Park and Ang Damage Index values in the short period range. The inclusion of moderate levels of stain hardening substantially reduce the values of Park and Ang Damage in pinching systems as compared with bi-linear models with higher strain hardening leading to lower estimations of damage. On the other hand, the response in the long period range remains largely insensitive to the inclusion of varying levels of hardening stiffness.

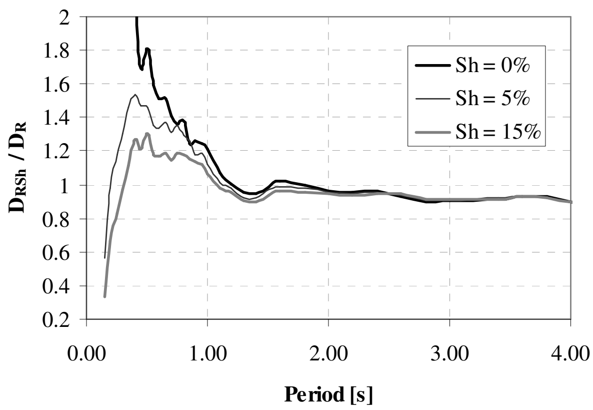


Figure 9. Mean Park and Ang Damage Indexes for pinching structures normalized by mean Park and Ang Damage Indexes in by-linear systems for different strain hardening coefficients R = 5.

7. CONCLUSIONS

This paper has examined the inelastic displacement response of steel structures of known levels of strength when subjected to a relatively large number of ground-motions. Bi-linear SDOF systems representative of moment resisting structures were analyzed as well as Modified Richard-Abbott models typical of partially restrained (PR) frames. The influence of model characteristics and level of inelastic behaviour has been discussed. The study revealed key differences in the inelastic deformation demands between bi-linear and pinching models, particularly in the relatively short period spectral region. The ratio between the overall yield strength and the strength during pinching intervals was found to be the main factor governing the inelastic deformations of pinching models when compared with bi-linear model predictions. Pinching models can exhibit higher displacement demands that may reach more than double the peak displacements estimated through bi-linear models for relatively stiff structures.

With regards to Fatigue Damage Indexes, it was shown that fatigue damage in stiff pinching systems can be significantly higher than the corresponding damage levels expected in bi-linear structures and that this increases with increasing levels of strength demand (higher values of R). Additionally, significant dependence of fatigue damage ratios on the level of pinching was observed, especially for short period structures. On the other hand, an inverse relationship between mean Park and Ang damage ratios and strength demand levels was identified for short period structures. With respect to bi-linear systems, short period pinching structures can reach Park and Ang damage index values of 2 or 4.6 times the expected values in the short period range. Importantly, the values of Park and Ang damage indexes attained in pinching structures of longer initial period (T > 1second) are significantly smaller than those observed in their bi-linear counterparts with values as low as 30% of those expected in bi-linear systems for soft soil conditions. Finally, although no clear relationship between different levels of post-elastic stiffness and Fatigue Index was identified, the mean Park and Ang Damage index values for pinching systems normalized by the corresponding values on bi-linear models show a clear influence of the level of strain hardening, especially for structures of fundamental periods shorter than 1 second.

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