High-Fidelity Model-Based Acceleration Control of Shake Tables

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SUMMARY:
This study presents a control method called acceleration trajectory tracking control (ATTC) that improves the acceleration control performance of shake tables. The ATTC method consists of an acceleration feed-forward controller, a system dynamics command shaking, and intentional time-delay, a Kalman filter for displacement measurement, and an actuator displacement feedback controller. The ATTC method provides acceleration tracking capability as well as ensures stability of the system. Following the theoretical description of the ATTC method, an experimental investigation is presented. The ATTC method is successfully implemented in the control system for a uniaxial shake table at the Johns Hopkins University, and the experimental results show the superior performance of the ATTC method over the conventional displacement feedback with command shaping. Furthermore, repeatability of the ATTC method is experimentally verified.

Keywords: Shake table tests, acceleration control, model-based control, feed-forward control

1. INTRODUCTION

Shake tables provide the most direct experimental means for the performance assessment of structures subject to ground motion. At present, a large number and a wide variety of shake tables are in active use around the world (for example, a 1,200-tons payload extremely large shake table (Ohtani et al. 2003), an outdoor shake table (Van Den Einde et al 2004), a 6-degrees-of-freedom shake tables (Bruneau et al 2002), etc). The purpose of the control systems in shake tables is to reproduce reference accelerations at the table. In general, the reference accelerations are either recorded accelerations during earthquake, synthetic accelerations from attenuation and seismological study, or some sort of waveforms such as sinusoidal and random waves. However, acceleration control of shake tables is extremely difficult because of inherent nonlinearities in servo hydraulic systems (i.e., valve dynamics, oil flow, etc.), control-structure interactions, dynamics of the base support, etc. Although measured accelerations at the table can be used as the input acceleration in the performance assessment, it is essential to have an acceptable acceleration control capability to assess the true impact of the reference accelerations on structures.

Most shake tables are driven by servo hydraulic actuators to meet the large force and high velocity requirements for shaking the table with payload. Servo hydraulic actuators are in nature unstable. A closed-loop feedback control is required to stabilize the motion of the actuator piston. Due to the existence of an unobservable and marginally unstable mode in the acceleration measurement (e.g., constant velocity motions cannot be detected from the acceleration measurement; see the theoretical proof in the next section), acceleration feedback control of the hydraulic actuators is unstable, and cannot be adopted for the shake tables. In practice, actuator displacement is used as the feedback, and thus the reference input to the hydraulic actuators is also the displacement. This inability to directly control the acceleration with feedback makes the control systems for shake tables insensitive to unpredictable disturbances in acceleration.

In shake tables, the reference displacement inputs to hydraulic actuators are calculated from a double integration of the reference accelerations (Spencer and Yang 1998; and Twitchell and Symans 2003). Due to the aforementioned inherent nonlinearities in servo hydraulic systems, displacement feedback control does not ensure acceptable displacement control performance even if boundary conditions are ideal. Several researchers proposed control methods that compensate the actuator/system dynamics to improve the control performance. Spencer and Yang (1998) presented the transfer function iteration method employed in many commercial shake tables. The method is based on the iterative command shaping using the inverse of the transfer function from the actuator reference displacement to the measured acceleration. Twitchell and Symans (2003) proposed a simplified approach without iterations using an inverse of the transfer function from the reference displacement to the measured displacements. Those methods are basically command shaping, and can improve displacement control performance in both the time and frequency domains. However, improvement in the acceleration control performance is rather limited; while the frequency domain distortion (particularly magnitude of transfer function) can be reduced, acceleration tracking in the time domain is either not achieved or limited within the low frequency range.

This paper presents a method to improve the acceleration control performance for shake tables. The proposed method, called the acceleration trajectory tracking control (ATTC) method, combines an acceleration feed-forward, a displacement feedback, a command shaping, an intentional time delay component, and Kalman filter for the displacement measurement. Analytical open-loop transfer functions are developed to discuss the stability, controllability, and observability of servo hydraulic actuators. Then, the ATTC method is developed based on the system dynamics of shake table. The ATTC method is implemented in the uniaxial shake table at the Johns Hopkins University, and an experimental investigation is conducted to verify the performance of the ATTC method in acceleration control. The experimental results and their implication are discussed in this paper.

2. SYSTEM DYNAMICS OF SHAKE TABLES

Shake tables are comprised of electrical and mechanical components that have unique dynamic characteristics. System dynamics of the shake tables are subject to the serial, parallel and feedback connections of those components, and are complex and nonlinear. In addition, due to the control-structural interactions (Dyke et al. 1995), the system dynamics are influenced by the dynamics of test specimens that are placed on the shake table. While the displacement feedback with the conventional proportional-integral-differential (PID) controller provides a reasonable displacement tracking, acceleration tracking is still poor due to the slowness and time lag associated with the displacement feedback (Twitchell and Symans 2003). To facilitate the acceleration control design and stability assessment, this section briefly derives the dynamics and interactions of the governing components in shake tables.

2.1. Servo valve and oil flow in hydraulic actuators

We consider the relationship between an electrical command to a servo valve and oil flow in the actuator chambers. The electrical command changes the position of the valve spool that regulates the oil flow into the actuator chambers. The oil flow is often modeled proportional to the valve command with a constant time-delay (Dyke et al. 1995; and Conte and Trombetti 2000). The first-order approximation model of the transfer function from the valve command u to the oil flow q can be expressed as:

$$H_{qu}(s) = \frac{q(s)}{u(s)} = \frac{k_v}{1+\tau s} \tag{2.1}$$

where k_v is the valve gain; τ is the time delay of the servo valve; and s is the Laplace variable.

Oil flow in the actuator chambers is the driving source of hydraulic actuators. However, flow rate is also affected by the dynamics of the actuator piston. From the equilibrium in flow rate q, the first-order governing oil flow equation can be obtained as

$$q(t) = A\dot{x}_a(t) + k_e F_a(t) + \frac{V}{4\beta A}\dot{F}_a(t) \tag{2.2}$$

where A is the piston area; x_a is the actuator displacement; k_e is the flow-force coefficient; F_a is the actuator force; V is the volume of the chamber; and β is the bulk modulus of the fluid. For more details, see Conte and Trombetti (2000). The first, second, and third terms are those from piston movement, oil leakage, and change in chamber volume, respectively. Taking the Laplace transform of Eq (2), transfer function from the oil flow q to the actuator displacement x_a is obtained as follows:

$$H_{x_a q}(s) = \frac{x_a(s)}{q(s)} = \frac{H_{x_a F_a}}{AsH_{x_a F_a} + k_e + k_1 s} \tag{2.3}$$

where $k_1 = V/4\beta A$, and $H_{x_a F_a}$ is the transfer function from the actuator force F_a to the actuator displacement x_a.

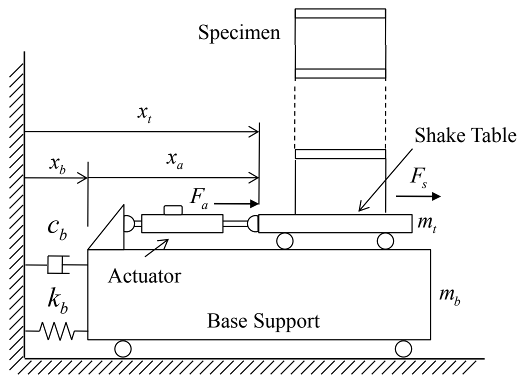

Figure 1. A schematic of a uniaxial shake table.

2.2. Equation of motions for shake tables

To establish the actuator transfer function, we consider the equations of motion for shake tables. Figure 1 shows a schematic of a uniaxial shake table including a shake table, a hydraulic actuator, a base support, and a test specimen. Force resistance of the base support is modeled as a combination of a linear spring k_b and a dashpot c_b for the sake of simplicity. The equations of motion for the shake table and the base support can be expressed as:

Shake Table: $m_t\ddot{x}_t(t) - F_a(t) - F_s(t) = 0$ (2.4)

Base Support: $m_b\ddot{x}_b(t) + c_b\dot{x}_b(t) + k_b x_b(t) + F_a(t) = 0$ (2.5)

where m_t and m_b are the mass of the table and the base support, respectively; x_t, x_a and x_b are the displacement of the shake table, the actuator, and the base support, respectively; and F_s is the base shear of the specimen. The actuator, table and base support displacements hold the following relationship.

$$x_t(t) = x_a(t) + x_b(t) \tag{2.6}$$

From Eqs (4)-(6) and further derivation, the transfer function from the actuator force to the displacement can be obtained as:

$$H_{x_aF_a}(s) = \frac{x_a(s)}{F_a(s)} = \frac{1}{\left(m_t + H_{F_sa_t}\right)s^2} + \frac{1}{m_bs^2 + c_bs + k_b} \tag{2.7}$$

where $H_{F_sa_t}$ is the transfer function from the table acceleration to the base shear of the payload.

2.3. Open loop transfer functions

An open-loop transfer function from the electrical valve command to the actuator displacement can be obtained by substituting Eq (2.7) into Eq (2.3) and multiplying Eqs (2.1) and (2.3):

$$H_{x_au}(s) = \frac{x_a(s)}{u(s)} = H_{x_aq}(s)H_{qu}(s) = \frac{k_v}{s(1+\tau s)} \cdot \frac{\left(m_b + m_t + H_{F_sa_t}\right)s^2 + c_bs + k_b}{D} \tag{2.8}$$

$$D = s\left(k_e + k_1s\right)\left(m_t + H_{F_sa_t}\right)\left(m_bs^2 + c_bs + k_b\right) + A\left\{\left(m_t + m_b + H_{F_sa_t}\right)s^2 + c_bs + k_b\right\} \tag{2.9}$$

In the same manner, the open-loop transfer function from the electrical valve command to the shake table acceleration can be given by.

$$H_{a_tu}(s) = \frac{a_t(s)}{u(s)} = s^2\frac{x_t(s)}{u(s)} = \frac{k_vs}{1+\tau s} \cdot \frac{\left(m_b + H_{F_sa_t}\right)s^2 + c_bs + k_b}{D} \tag{2.10}$$

It should be noted from Eqs (2.8) and (2.10) that because of the dynamics of the base support, the double differentiation of the actuator displacement does not yield to the table acceleration, that is,

$$H_{a_tu}(s) \neq s^2H_{x_au}(s) \tag{2.11}$$

The above relationship implies that the actuator displacement tracking does not necessarily ensure the reference acceleration tracking at the shake table.

3. ACCELERATION TRAJECTORY TRACKING CONTROL

As mentioned earlier, displacement feedback controls are limited in term of acceleration tracking in shake tables. A discrepancy between the reference and the measured acceleration is particularly prominent in a high frequency because of the poor phase characteristics in the displacement control. This section investigates the open- and closed-loop stabilities of shake tables using the transfer functions obtained in the previous section, and then introduces a possible control method for acceleration tracking control for shake tables.

3.1. Stability, controllability, and observability of shake tables

Prior to the development of control methods, we consider the stability of the open-loop transfer functions. For the sake of simplicity, we consider an shake table without test specimens (i.e., $H_{F_sx_t} = 0$). Without losing generality, the open-loop transfer functions from the electrical command to the actuator displacement and to the table acceleration are simplified as:

$$H_{x_a u}(s) = \frac{k_v}{s(1+\tau s)} \cdot \frac{(m_t + m_b)s^2 + c_b s + k_b}{\bar{D}(s)} \tag{3.1}$$

$$H_{a_t u}(s) = \frac{k_v s}{(1+\tau s)} \cdot \frac{m_b s^2 + c_b s + k_b}{\bar{D}(s)} \tag{3.2}$$

$$\bar{D}(s) = m_t s(k_e + k_1 s)(m_b s^2 + c_b s + k_b) + A\{(m_t + m_b)s^2 + c_b s + k_b\} \tag{3.3}$$

Based on the linear systems theory, the dynamics of the hydraulic actuator in shake tables described in Eq (12) can be expressed using six state variables; the order of the denominator in the transfer function is six. If all of the six poles (roots of the denominator) lie in the left side of the complex plane, systems are guaranteed to be stable. As seen in Eq (3.1), the actuator displacement open-loop transfer function has a marginally stable pole at the origin (s=0). Note that the rest of the poles ($-\tau$ and roots of $\bar{D}(s)$) are stable. Practically, the pole at the origin is unstable; as s gets close to zero, the output goes to infinity (unbounded instability). However, the pole at the origin is observable in the displacement measurement and more importantly controllable from the valve command. Therefore, this practically unstable pole can be stabilized by introducing a closed-loop feedback with the displacement measurement.

On the other hand, the pole at the origin is cancelled out in the table acceleration open-loop transfer function, Eq (3.2), due to the zeros from the double integration of s. This pole-zero cancellation makes the pole at the origin unobservable in the acceleration measurement. In other words, the pole at the origin cannot be controlled and stabilized based on the acceleration measurement. Thus, acceleration feedback control for shake tables is unstable and practically not feasible. Therefore, this study explores acceleration control methods without an acceleration feedback control.

3.2. Acceleration trajectory tracking control

The goal of control design is to obtain the configuration, specification, and identification of key parameters of a proposed system to produce the desired output. With focus placed on the control performance and the stability, this paper proposes a control method called acceleration trajectory tracking control (ATTC) that consists of an acceleration feed-forward and a displacement feedback control loops. The displacement feedback loop incorporates a system dynamics command shaping, an intentional systems time delay, and a Kalman filter. Figure 2 shows a schematic of all of the components in the proposed control method. Details and roles of each component are given below.

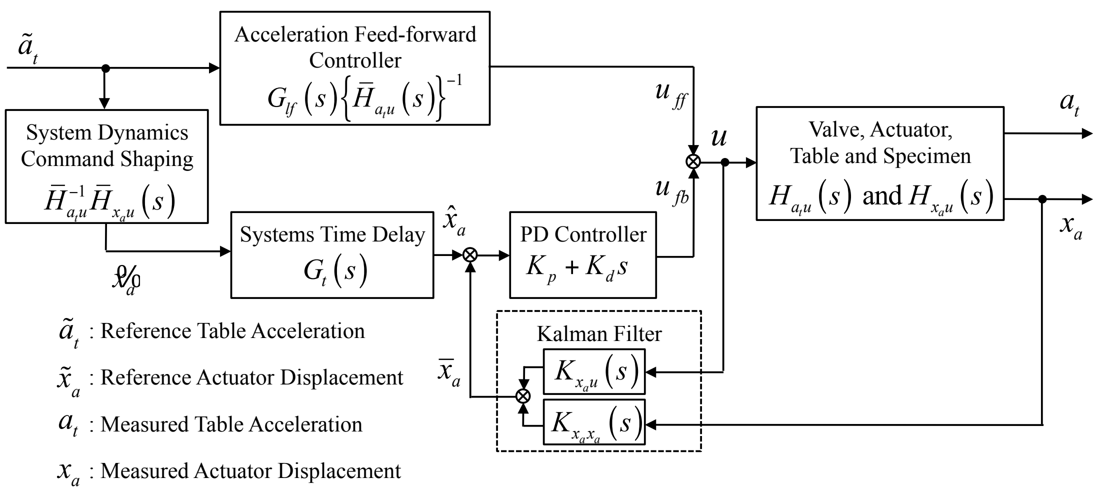

Figure 2. A block diagram of the acceleration trajectory tracking controller.

3.2.1. Acceleration feed-forward using the pseudo inverse of the table acceleration transfer function
Acceleration feed-forward controller is adopted to generate the primary driving command for the reference acceleration, compensating the dynamics from the valve to the table acceleration. The feed-forward controller consists of a lowpass filter G_{lf} and the inverse of the approximated table acceleration transfer function $\bar{H}_{a_t u}$. Roles of the lowpass filter are (i) to make $G_{lf}\bar{H}_{a_t u}^{-1}$ strictly proper so that it can be realized in a state space model; and (ii) to reduce the frequency contents that are higher than the frequency range of interest. The relationship between the reference acceleration and the valve command from the feed-forward controller is given as:

$$u_{ff} = G_{lf}\bar{H}_{a_t u}^{-1}\tilde{a}_t \tag{3.4}$$

Note that the computation of the feed-forward command can be performed off-line.

3.2.2. System dynamics command compensator
A displacement feedback is used to stabilize the shake table. The reference actuator displacement is computed from the reference table acceleration based on the system dynamics compensator instead of the double integration. The computed, reference actuator displacement should provide better prediction of the actuator displacement for the reference acceleration, and the relationship is given as:

$$\tilde{x}_a = \bar{H}_{x_a u}\bar{H}_{a_t u}^{-1}\tilde{a}_t \tag{3.5}$$

where $\tilde{x}_a$ is the reference actuator displacement; and $\bar{H}_{x_a u}$ is the approximated open-loop transfer function from the valve command to the actuator displacement.

3.2.3. Intentional system delay
A time delay is intentionally introduced between the reference table acceleration and the reference actuator displacement. The purpose of the intentional delay is to ensure the acceleration feed-forward loop is the driving source for the reference acceleration and that the displacement feedback loop serves only to provide the stability (prevent drift) of the table. The intentional system time delay is given as:

$$\hat{x}_a = G_t(s)\tilde{x}_a = \frac{1-\tau s}{1+\tau s}\tilde{x}_a \tag{3.6}$$

where $\hat{x}_a$ is the delayed, reference actuator displacement.

3.2.4. Kalman filter
A Kalman filter is employed to reduce noises in the actuator displacement without time delay. The model-based filtering effectively reduces noise in the displacement feedback loop so the displacement loop does not introduce high frequency disturbance in the valve command. Note that the valve command for high frequency acceleration is generated from the acceleration feed-forward controller.

The relationship between the valve command u, the measured actuator displacement x_a, and the estimated actuator displacement $\bar{x}_a$ are given by:

$$\bar{x}_a = K_{x_a u}(s)u + K_{x_a x_a}(s)x_a \tag{3.7}$$

where $K_{x_a u}$ and $K_{x_a x_a}$ are the Kalman gains from the valve command and the measured actuator displacement to the estimated actuator displacement, respectively. Those Kalman filter model and gains are determined from the open-loop transfer function from the valve command to the actuator displacement with measured displacement noise.

3.2.5. Displacement feedback with PD controller
The displacement feedback loop herein is included to stabilize the shake table. More specifically, it is intended to stabilize the pole at the origin, but not to affect the tracking of displacement and acceleration. In other words, the displacement feedback is just to prevent the drift of the shake table. The displacement feedback loop with a proportional and a derivative controller can be expressed as:

$$u_{fb} = \left(K_p + K_d s\right)\left(\hat{x}_a - \overline{x}_a\right) \tag{3.8}$$

where K_p and K_d are the proportional and the derivative gains, respectively; and u_{fd} is the valve command from the displacement feedback. In the implementation, the proportional gain needs to be tuned relatively low to reduce the influence on the system dynamics at the high frequency.

3.3. Overall transfer functions

The command to the servo valve is a sum of the feed-forward and feedback terms as:

$$u = u_{ff} + u_{fb} \tag{3.9}$$

The overall system transfer functions from the reference table acceleration $\tilde{a}_t$ to the measured table acceleration a_t and from the reference actuator displacement $\tilde{x}_a$ to the measured actuator displacement x_a can be obtained using Eqs (3.4-9) and further derivation.

$$H_{a_t\tilde{a}_t}\left(s\right) = \frac{a_t}{\tilde{a}_t} = \frac{G_{lf} H_{a_t u} \overline{H}_{a_t u}^{-1}\left(1 + G_t\left(K_p + K_d s\right)\overline{H}_{x_a u}\right)}{1 + \left(K_p + K_d s\right)\left(K_{x_a u} + K_{x_a x_a} H_{x_a u}\right)} \tag{3.10}$$

$$H_{x_a\tilde{x}_a}\left(s\right) = \frac{x_a}{\tilde{x}_a} = \frac{H_{x_a u} \overline{H}_{x_a u}^{-1}\left(G_t^{-1} + \left(K_p + K_d s\right)\overline{H}_{x_a u}\right)}{1 + \left(K_p + K_d s\right)\left(K_{x_a u} + K_{x_a x_a} H_{x_a u}\right)} \tag{3.11}$$

4. EXPERIMENTAL SETUP

The proposed acceleration trajectory tracking control method is implemented in the control system for the recently constructed uniaxial shake table at the Johns Hopkins University.

The uniaxial shake table consists of a 1.2 m x 1.2 m aluminum sliding table mounted on two linear guides with high-precision, low-friction, linear ball-bearings. The shake table is driven by a 27 kN hydraulic actuator manufactured by Shore Western, Inc. The specifications of the shake table are: maximum displacement of +-7.6 cm, maximum velocity of +-5.1 cm/s, maximum acceleration of 3.8 g; and maximum payload of 0.5 ton. The operational frequency range of the simulator is 0.1-150 Hz.

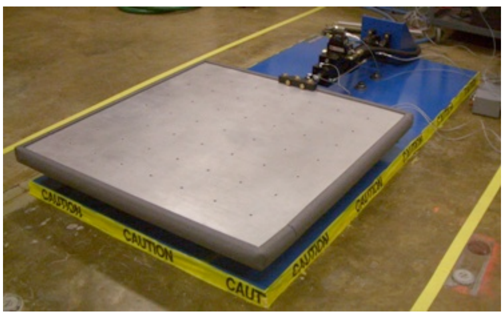

Figure 3. Uniaxial shake table at the Johns Hopkins University.

5. EXPERIMENTAL VERIFICATION

To verify the performance of the ATTC method in acceleration control, an experimental investigation was conducted. This paper presents the initial experimental work that did not include payloads. Using a band-limited random signal of which frequency ranges between 0.1 Hz and 30 Hz, system dynamics of the uniaxial shake table are experimentally obtained. Figure 4 shows the open-loop transfer functions from the valve command to the actuator displacement and the table acceleration.

The experimentally obtained open-loop transfer functions were approximated in a form of rational polynomial functions, employing the least square curve fitting method. The polynomial orders used in the approximation functions are the same as the ones in the analytical models: that is, orders for the numerator and denominator for the transfer function to the table acceleration are 3 and 5, respectively. And the orders for the numerator and denominator for the transfer function to the actuator displacement are 2 and 6, respectively. The approximated analytical models for the table acceleration and the actuator displacement exhibit excellent agreement with the experimental results in both the amplitude and the phase characteristics.

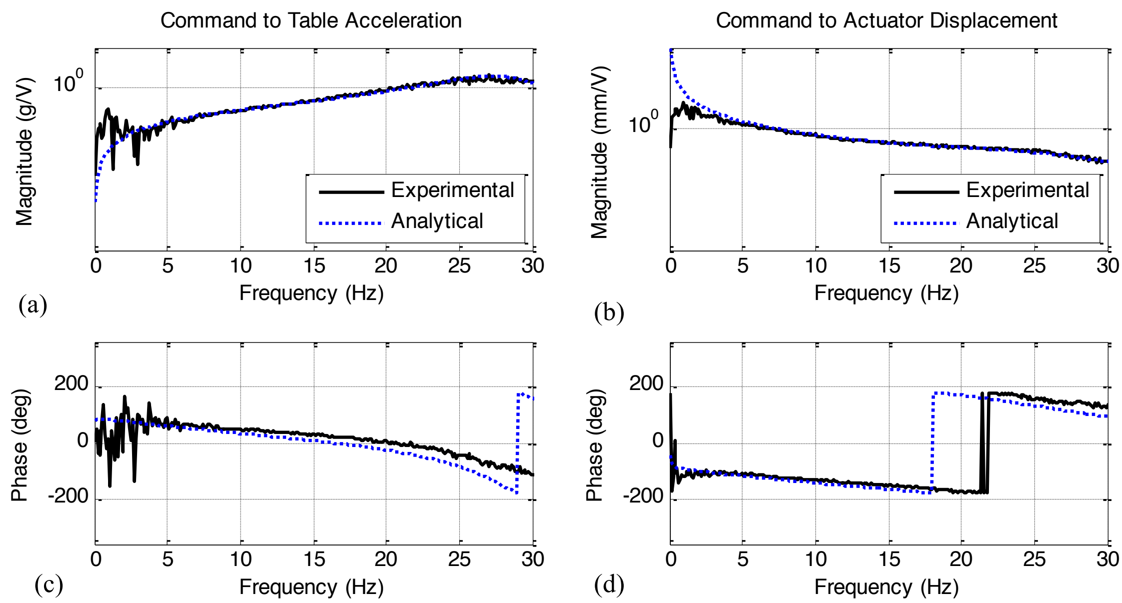

Figure 4. System dynamics: (a) and (c) are the magnitude and phase of the open-loop transfer function from the valve command to the actuator displacement, respectively; and (b) and (d) are the magnitude and phase of the open-loop transfer function from the valve command to the table acceleration, respectively.

With the experimental and approximated analytical transfer functions and experimentally identified system delay of 0.01 sec in valve dynamics, the shake table acceleration transfer function was established. The Kalman filter gains are determined based on the high frequency noise reduction and the time delay, accounting for the experimental transfer function of the shake table and the noise level in the displacement measurement. Then, the PD gains in the displacement feedback loop are selected based on the transfer function and a numerical time-history analysis such that the displacement loop provides stability, but does not influence high frequency response of the shake table. The shake table acceleration transfer function in the ATTC method was shown in Figure 5. To evaluate the performance of the ATTC method in comparison with the conventional displacement feedback with command shaping (DFCS), a shake table acceleration transfer function of the DFCS was also plotted in Figure 5. The ATTC method shows better performance than the DCFS in the transfer function over a wide range of frequency. In particular, the ATTC method is superior to the DFCS method in phase characteristic.

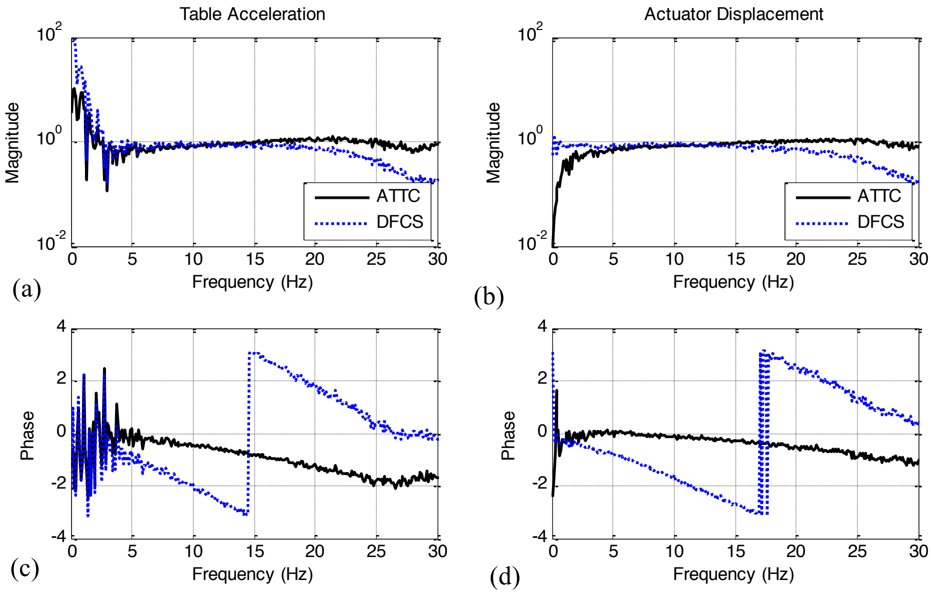

Figure 5. Closed-loop transfer functions: (a) and (c) are the magnitude and phase of the transfer function from the reference acceleration to the measured acceleration, respectively; and (b) and (d) are the magnitude and phase of the transfer function from the reference displacement to the measured displacement, respectively.

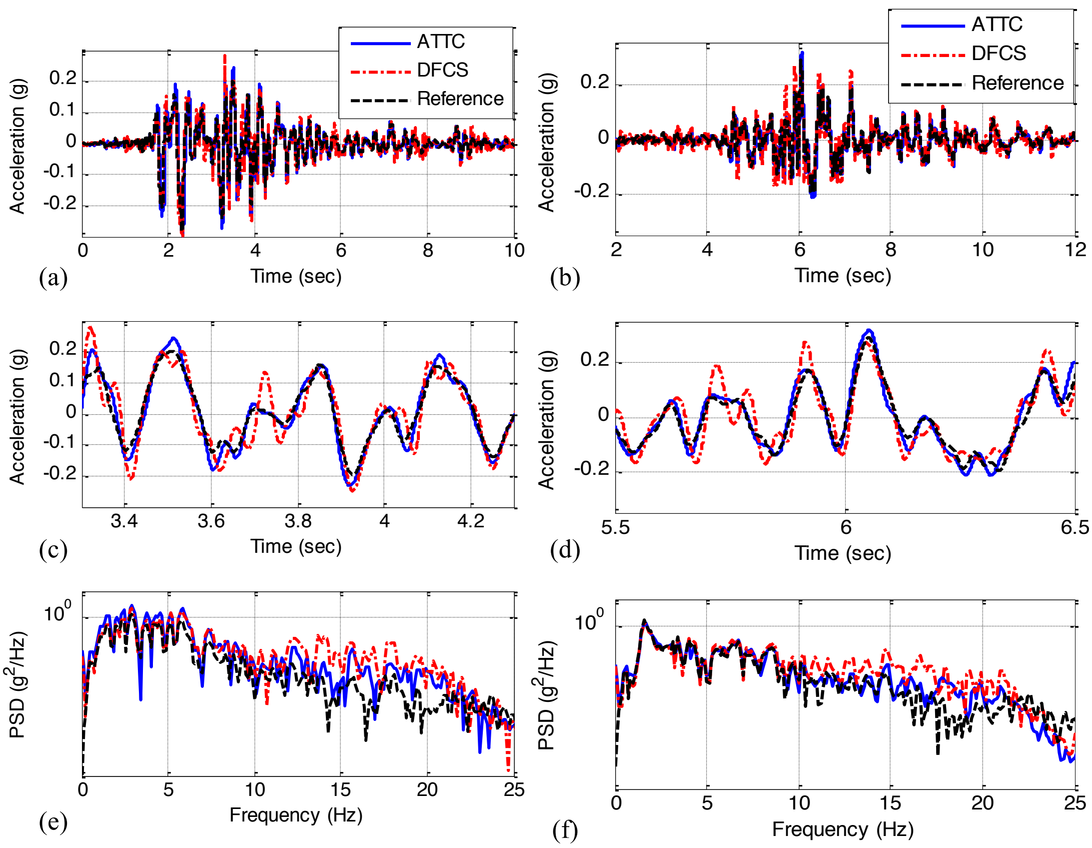

Figure 6. Comparison of the acceleration among the ATTC, DFCS, and reference: (a), (c), and (e) are a wide and a narrow views of the acceleration tracking, and the acceleration power spectral density for Kobe earthquake, respectively; (b), (d), and (f) are a wide and a narrow views of the acceleration tracking, and the acceleration power spectral density for Loma Prieta earthquake, respectively.

Acceleration tracking performance of the ATTC method was experimentally investigated for earthquake records. Figures 6 shows the comparison between the ATTC method and DFCS method in the acceleration time histories and the acceleration power spectral density for scaled Loma Prieta and Kobe earthquake records. The ATTC method exhibit smooth trajectory tracking to the reference earthquake not only in a long time scale (see Figures 6 (a) and (b)) but also in a short time scale (see Figures 6 (c) and (d)). On the other hand, while the DFCS method shows reasonable trajectory tracking in a long time scale, it shows poor performance in a short time scale. This high-frequency pitching is associated with the displacement feedback, and the results agree with studies in literature (e.g., Twitchell and Symans 2003). The ATTC method also shows better performance than the DFCS method in the frequency domain, particularly in high frequency range (see Figures 6 (e) and (f)).

6. CONCLUSIONS

This paper introduced an acceleration trajectory tracking control (ATTC) method for shake tables. The ATTC method incorporates an acceleration feed-forward, a system dynamic command shaping, an intentional time-delay, a Kalman filter for the displacement measurement, and a displacement feedback loop. Theories behind the ATTC method including the stability were discussed, and then the table acceleration transfer function was analytically derived.

The ATTC method was successfully implemented in the control system for the uniaxial shake table at the Johns Hopkins University. Experimental investigation demonstrated the superior performance of the ATTC method over the conventional displacement feedback with command shaping (DFCS) method that is used in commercial shake table control. The ATTC method shows excellent performance in acceleration trajectory tracking in a wide range of frequency without pitching that can be found in the DFCS method.

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