Plunging breaker model of tsunami with Arbitrary Lagrangian Eulerian approach

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SUMMARY:
Plunging breaker on beaches constitute one of the most energetic events in the coastal environment. A better understanding and modeling of the kinematics of breaking waves is of prime importance for coastal engineering problems.
In this paper, A numerical scheme is developed to predict the wave of an unsteady, incompressible viscous flow with free surface. The method involves a two dimensional finite element (2D), in a vertical plan. The Naiver-Stokes equations for conservation of momentum and mass for Newtonian fluids, continuity equation, and full nonlinear kinematic free-surface equation, were used as the governing equations. The mapping developed to solve highly deformed free surface problems common in waves formed during wave propagation, transforms the run up model from the physical domain to a computational domain with Arbitrary Lagrangian Eulerian (ALE) finite element modeling technique.

Keywords: Plunging breaker, solitary wave, Naiver-Stokes equations, Arbitrary Lagrangian Eulerian.

1. INTRODUCTION

The base of knowledge on the dynamics of ocean surface waves has experienced a substantial growth during the recent decades. Two major restoring forces for ocean waves are wind and earthquake. Wind generates waves, by horizontal momentum transfered to surface currents. Large submarine earthquakes or landslides generate long "tsunami" waves with no specific wave periods. These waves occur quite frequently in nature. These long waves are often studied because of their disastrous consequences to coastal regions. There have been several disasters caused by long water waves in coastal regions in Japan, USA and elsewhere. One of the most recent was the Nicaraguan disaster of September 2, 1992. On this date a submarine earthquake generated such a large tsunami that it caused significant damages along the Nicaraguan coast. The observed height was as high as 10 meters. With its high destructive power, that tsunami invaded the area, severely damaged the coast and sand beach, brought down walls of houses and left at least 167 people dead. In 2004, the earthquake that generated the great Indian Ocean tsunami is estimated to have released the energy of 23,000 Hiroshima-type atomic bombs, according to the U.S. Geological Survey (USGS). More than 150,000 people were dead or missing and millions more were homeless in 11 countries. In the last disaster (2011), an 9.0 magnitude earthquake and a 10 meter high tsunami wave hit northeastern Japan and thousands of people have have died and hundreds of thousands are affected.
Breaking of a waves starts when the maximum wave steepness is reached. The breaking wave height and the breaking wave angle are fundamental parameters required in coastal. Also, breaking waves have a strong affect on the hydrodynamic behavior of ship wakes as well as on the structural behaviour of offshore structures. Depending on the way in which they break, breaking waves have been classified as spilling, plunging, surging or collapsing. Plunging breakers are major causes of the overturning of ships in rough seas. Before Longuet and Cokelet (1976), most of the numerical computations had succeeded only in integrating the equations of motion up to the instant when the

surface became vertical. Carmo, Santos and Barthelemy (1993) presented a numerical model for surface wave propagation based on the nonlinear dispersive wave approach described by Boussinesq equation. Chubarov and Shokin (1987) presented the numerical modeling of long wave propagation, in particular for tsunami waves, in the framework of non-linear dispersion models of the Boussinesq and Korteweg-de Vries type using finite difference method. Goring and Raichlen (1992) examined propagation of long waves past a step and onto a horizontal shelf analytically and experimentally using Boussinesq equation. Hibberd and Peregrine (1979) obtained a numerical solution based on shallow water wave equation to describe the behavior of a uniform bore over a sloping beach and the subsequent run-up. Zelt (1991) parameterized the wave breaking with an artificial viscosity term in the momentum equation.
The numerical solution for wave breaking and run-up has been developed using three defined theories: (1) Lagrangian; (2) Eulerian; (3) Arbitrary Lagrangian-Eulerian description. For breaking waves in particular, the Arbitrary Lagrangian-Eulerian is superior in terms of handling high distortion in the grids. The Lagrangian description of the fluid motion, in which the mesh of the domain moves with the fluid, is preferred when there is only small distortion in the grids. The Eulerian description of the fluid motion, in which the mesh of the domain is spatially fixed, is preferred for any flow where the mesh motion is highly controlled if required and where there is no multivalued point among the grids. To have a versatile description of the fluid domain, it is necessary to have a method with the benefits of both Lagrangian and Eulerian descriptions, without their deficiencies. Such a method, developed in the last two decades, is the "Arbitrary Lagrangian- Eulerian formulation" in which grid points may be moved with the fluid in normal Lagrangian description. This method allows the grids move independent of the fluid motion.

2. FREE SURFACE HYDRODYNAMIC FLOW

An accurate description of the equations, governing the medium, is a vital part of any problem solving process. In this chapter, the basic equations used to solve the incompressible free surface viscous fluid are introduced, and then the derivation process of discretized equations, in the generalized coordinate system, is presented.

2.1. Problem Formulation

The physical domain V surrounded by a piecewise smooth boundary S is shown in Figure 2.1. This domain is occupied by a viscous incompressible fluid with the coefficient of constant kinematic viscosity of v and the specific mass of p. The problem under consideration is the unsteady motion of a surface wave under gravity. Two-dimensional unsteady incompressible viscous flow is considered. The governing equations are expressed by the unsteady Navier-Stokes equation and the equation of continuity. Let the rectangular coordinates be denoted by x, y and the corresponding velocity components be denoted by u,v. As a result, the equations of conservation of momentum and mass, for incompressible Newtonian fluids, in the arbitrary Lagrangian-Eulerian form are given as follows:

$$\begin{aligned}
&\frac{\partial \bar{u}}{\partial \bar{t}}\Big|_{\xi,\eta} + (\bar{u}-\bar{w}_u)\frac{\partial \bar{u}}{\partial \bar{x}} + (\bar{v}-\bar{w}_v)\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{x}} + \bar{\nu}\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) \\
&\frac{\partial \bar{v}}{\partial \bar{t}}\Big|_{\xi,\eta} + (\bar{u}-\bar{w}_u)\frac{\partial \bar{v}}{\partial \bar{x}} + (\bar{v}-\bar{w}_v)\frac{\partial \bar{v}}{\partial \bar{y}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{y}} + \bar{\nu}\left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right) - \bar{g} \qquad (2.1) \\
&\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0
\end{aligned}$$

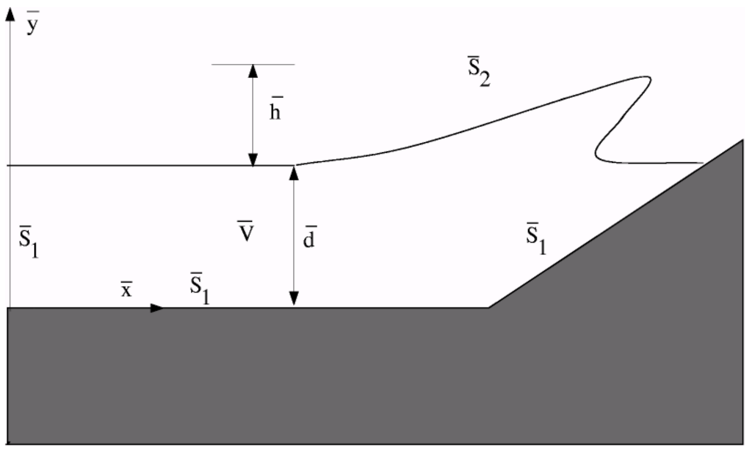

Figure 1.1. Mathematical models for non-linear response history analysis

where w_u and w_v are the mesh velocities in x and y directions. The boundary $\bar{S}$ consists of two types of boundaries: one is the $\bar{S}_1$ on which velocity is given; the other is the free surface boundary $\bar{S}_2$ on which the surface force is specified. The boundary conditions can be expressed as the followings,

$$\begin{aligned} \bar{u} &= \hat{\bar{u}} \quad on \quad \bar{S}_1 \quad (-\frac{1}{\bar{\rho}}\bar{p} + 2\bar{v}\frac{\partial \bar{u}}{\partial \bar{x}}).n_{\bar{x}} + \bar{v}(\frac{\partial \bar{u}}{\partial \bar{y}} + \frac{\partial \bar{v}}{\partial \bar{x}}).n_{\bar{y}} = \hat{\bar{c}}_x \quad on \quad \bar{S}_2 \\ \bar{v} &= \hat{\bar{v}} \quad on \quad \bar{S}_1 \quad \bar{v}(\frac{\partial \bar{u}}{\partial \bar{y}} + \frac{\partial \bar{v}}{\partial \bar{x}}).n_{\bar{x}} + (-\frac{1}{\bar{\rho}}\bar{p} + 2\bar{v}\frac{\partial \bar{v}}{\partial \bar{y}}).n_{\bar{y}} = \hat{\bar{c}}_y \quad on \quad \bar{S}_2 \end{aligned} \tag{2.2}$$

where the superscript caret denotes a function which is given on the boundary and n_x and n_y symbolize the direction cosines of the outward normal to the boundary with respect to co-ordinate x and y. Top Equations can be rendered dimensionless by introducing the following variables:

$$\bar{x} = x\bar{d}, \bar{y} = y\bar{d}, \bar{p} = p\bar{\rho}\bar{g}\bar{d}, \bar{u} = u(\bar{g}\bar{d})^{1/2}, \bar{v} = v(\bar{g}\bar{d})^{1/2}, \bar{t} = t(\frac{\bar{d}}{\bar{g}})^{1/2} \tag{2.3}$$

Using these transformations, the Equations 2.1 and 2.2 are modified as follows:

$$\begin{aligned} &\frac{\partial u}{\partial t}\Big|_{\xi,\eta} + (u - w_u)\frac{\partial u}{\partial x} + (v - w_v)\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{\mathrm{Re}}(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) \\ &\frac{\partial v}{\partial t}\Big|_{\xi,\eta} + (u - w_u)\frac{\partial v}{\partial x} + (v - w_v)\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{\mathrm{Re}}(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}) - 1 \\ &\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \end{aligned} \tag{2.4}$$

$$\begin{aligned} u &= \hat{u} \quad on \quad \bar{S}_1 \quad (-p + \frac{2}{\mathrm{Re}}\frac{\partial u}{\partial x}).n_x + \frac{1}{\mathrm{Re}}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}).n_y = \hat{c}_x \quad on \quad \bar{S}_2 \\ v &= \hat{v} \quad on \quad \bar{S}_1 \quad \frac{1}{\mathrm{Re}}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}).n_x + (p + \frac{2}{\mathrm{Re}}\frac{\partial v}{\partial y}).n_y = \hat{c}_y \quad on \quad \bar{S}_2 \end{aligned} \tag{2.5}$$

2.2. Free Surface Formulation

On a fluid surface we have

$$\overline{F} = \overline{h}(x,t) + \overline{d} - \overline{y} = 0 \quad on \quad \overline{S}_2 \tag{2.6}$$

where h is the position of the free surface. The kinematic condition associated with the fluid free surface can be defined as

$$\frac{D\overline{F}}{D\overline{t}} = 0 \tag{2.7}$$

Using arbitrary Lagrangian-Eulerian definition , Equation 2.7 can be modified as,

$$\frac{\partial \overline{F}}{\partial \overline{t}}\Big|_{\xi,\eta} + (\overline{u} - \overline{w}_u)\frac{\partial \overline{F}}{\partial \overline{x}} + (\overline{v} - \overline{w}_v)\frac{\partial \overline{F}}{\partial \overline{y}} = 0 \tag{2.8}$$

By plugging into Equation 2.7, we obtain,

$$(\frac{\partial \overline{h}}{\partial \overline{t}} - \frac{\partial \overline{y}}{\partial \overline{t}})\Big|_{\xi,\eta} + (\overline{u} - \overline{w}_u)\frac{\partial \overline{h}}{\partial \overline{x}} + (\overline{v} - \overline{w}_v)(-1) = 0 \tag{2.9}$$

Utilizing the dimensionless form of the free surface kinematic equation can be written as,

$$(\frac{\partial h}{\partial t} - \frac{\partial y}{\partial t})\Big|_{\xi,\eta} + (u - w_u)\frac{\partial h}{\partial x} + (v - w_v)(-1) = 0 \tag{2.10}$$

Equation 2.10 can be more simplified as,

$$\frac{\partial h}{\partial t}\Big|_{\xi,\eta} + (u - w_u)\frac{\partial h}{\partial x} - v = 0 \tag{2.11}$$

3. NUMERICAL ANALYSIS

The numerical model is based on a finite element method for the spatial discretization of partial differential equations. This method is implemented using weighted residual variational technique for the solution approach within each element.

3.1. Basic concept

In the temporal discretization, the total time t is divided into a number of short time increments Δt. Each time point is denoted by n. Velocity and pressure at the nth time point can be defined as:

$$\begin{aligned} u^n &= u(x,y,t^n) = u(\xi,\eta,t^n) \quad & p^n &= p(x,y,t^n) = p(\xi,\eta,t^n) \\ v^n &= v(x,y,t^n) = v(\xi,\eta,t^n) \quad & h^n &= h(x,t^n) = h(\xi,t^n) \end{aligned} \tag{3.1}$$

where x and y denote the coordinate at the nth time point in the physical domain. The parameters ξ and η are the fixed coordinate at the nth time point in the reference domain. Velocity and pressure at time point $n+1$ can be defined subsequently as:

$$
\begin{aligned}
&u^{n+1}=u(x,y,t^{n+1})=u(\xi,\eta,t^{n+1}) \quad p^{n+1}=p(x,y,t^{n+1})=p(\xi,\eta,t^{n+1}) \\
&v^{n+1}=v(x,y,t^{n+1})=v(\xi,\eta,t^{n+1}) \qquad h^{n+1}=h(x,t^{n+1})=h(\xi,t^{n+1})
\end{aligned}
\tag{3.2}
$$

In the Eulerian treatment, the spatial differentiation can be approximated in the form:

$$
\frac{\partial u}{\partial t}\Big|_{\xi,\eta}=\frac{u^{n+1}-u^{n}}{\Delta t} \quad \frac{\partial v}{\partial t}\Big|_{\xi,\eta}=\frac{v^{n+1}-v^{n}}{\Delta t} \quad \frac{\partial h}{\partial t}\Big|_{\xi,\eta}=\frac{h^{n+1}-h^{n}}{\Delta t}
\tag{3.3}
$$

With substituting Equations, the equations of motion, continuity and kinematic boundary condition can be discredited into,

$$
\begin{aligned}
&\frac{u^{n+1}-u^{n}}{\Delta t}=-(u^{n}-w_{u}{}^{n})\frac{\partial u^{n}}{\partial x}-(v^{n}-w_{v}{}^{n})\frac{\partial u^{n}}{\partial y}-\frac{\partial p^{n+1}}{\partial x}+\frac{1}{\mathrm{Re}}(\frac{\partial^2 u^{n}}{\partial x^2}+\frac{\partial^2 u^{n}}{\partial y^2}) \\
&\frac{v^{n+1}-v^{n}}{\Delta t}=-(u^{n}-w_{u}{}^{n})\frac{\partial v^{n}}{\partial x}-(v^{n}-w_{v}{}^{n})\frac{\partial v^{n}}{\partial y}-\frac{\partial p^{n+1}}{\partial x}+\frac{1}{\mathrm{Re}}(\frac{\partial^2 v^{n}}{\partial x^2}+\frac{\partial^2 v^{n}}{\partial y^2})-1 \\
&\frac{\partial u^{n+1}}{\partial x}+\frac{\partial v^{n+1}}{\partial y}=0 \\
&\frac{h^{n+1}-h^{n}}{\Delta t}=-(u^{n+1}-w_{u}{}^{n+1})\frac{\partial h^{n}}{\partial x}+v^{n+1}
\end{aligned}
\tag{3.4}
$$

The boundary conditions corresponding are described by:

$$
\begin{aligned}
&u^{n+1}=\hat{u} \quad (-p^{n+1}+\frac{2}{\mathrm{Re}}\frac{\partial u^{n+1}}{\partial x}).n_{x}+\frac{1}{\mathrm{Re}}(\frac{\partial u^{n+1}}{\partial y}+\frac{\partial v^{n+1}}{\partial x}).n_{y}=\hat{c}_{x} \\
&v^{n+1}=\hat{v} \quad \frac{1}{\mathrm{Re}}(\frac{\partial u^{n+1}}{\partial y}+\frac{\partial v^{n+1}}{\partial x}).n_{x}+(p^{n+1}+\frac{2}{\mathrm{Re}}\frac{\partial v^{n+1}}{\partial y}).n_{y}=\hat{c}_{y}
\end{aligned}
\tag{3.5}
$$

The analysis procedure presented here involves computing the unknown variable u^{n+1} , v^{n+1} , p^{n+1} , h^{n+1} which satisfy Equations 3.3 and 3.4 and the boundary conditions defined in Equations 2.4, starting from the known variable u^{n} , v^{n} , p^{n} , h^{n}.

To solve Equations, the fractional method is employed. This method is one of the earliest and the most widely used method for solving fluid dynamic problems. In this method, by discretizing the equations of motion, the intermediate velocity can be obtained. However, this velocity may not satisfy the equation of continuity. To correct the obtained intermediate velocity, a correction potential should be introduced. The Poisson equation for the correction potential can be derived by trying to satisfy the equation of continuity. By solving the resultant Poisson equation, the correction velocity vector can be obtained.

$$
\begin{aligned}
&\frac{\tilde{u}^{n+1}-u^{n}}{\Delta t}=-(u^{n}-w_{u}{}^{n})\frac{\partial u^{n}}{\partial x}-(v^{n}-w_{v}{}^{n})\frac{\partial u^{n}}{\partial y}-\frac{\partial p^{n+1}}{\partial x}+\frac{1}{\mathrm{Re}}(\frac{\partial^2 u^{n}}{\partial x^2}+\frac{\partial^2 u^{n}}{\partial y^2}) \\
&\frac{\tilde{v}^{n+1}-v^{n}}{\Delta t}=-(u^{n}-w_{u}{}^{n})\frac{\partial v^{n}}{\partial x}-(v^{n}-w_{v}{}^{n})\frac{\partial v^{n}}{\partial y}-\frac{\partial p^{n+1}}{\partial x}+\frac{1}{\mathrm{Re}}(\frac{\partial^2 v^{n}}{\partial x^2}+\frac{\partial^2 v^{n}}{\partial y^2})-1 \\
&\frac{\partial u^{n+1}}{\partial y}-\frac{\partial v^{n+1}}{\partial x}=\frac{\partial \tilde{u}^{n+1}}{\partial y}-\frac{\partial \tilde{v}^{n+1}}{\partial x}
\end{aligned}
\tag{3.6}
$$

This equation implies that

$$u^{n+1} = \tilde{u}^{n+1} + \frac{\partial \phi}{\partial x} \quad v^{n+1} = \tilde{v}^{n+1} + \frac{\partial \phi}{\partial x} \tag{3.7}$$

where ϕ is a scalar which is referred to as the correction potential. By taking the partial derivation on both sides with regard to x and y respectively and adding them together, we have

$$\frac{\partial u^{n+1}}{\partial x} + \frac{\partial v^{n+1}}{\partial y} = \frac{\partial \tilde{u}^{n+1}}{\partial x} + \frac{\partial \tilde{v}^{n+1}}{\partial y} + \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \tag{3.8}$$

With substituting the equation of continuity, the equation for ϕ can be derived as

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = -\frac{\partial \tilde{u}^{n+1}}{\partial x} - \frac{\partial \tilde{v}^{n+1}}{\partial y} \tag{3.9}$$

With substituting, the equation of controlling the evolution of pressure can be expressed

$$\frac{\partial p^{n+1}}{\partial x} = \frac{\partial p^{n}}{\partial x} - \frac{1}{\Delta t}\frac{\partial \phi}{\partial x} \quad \frac{\partial p^{n+1}}{\partial y} = \frac{\partial p^{n}}{\partial y} - \frac{1}{\Delta t}\frac{\partial \phi}{\partial y} \quad p^{n+1} = p^{n} - \frac{1}{\Delta t}\phi \tag{3.10}$$

In order to implement a numerical solution procedure for the arbitrary Lagrangian Eulerian formulation, the momentum equation and the incompressibility constraint of the Navier-Stokes problem are analyzed using a procedure which consists of six separate phases. Let u^n , v^n , p^n , h^n be the velocities, pressure and wave height fields at time tn, where $t^{n+1} = t^n + \Delta t$. From u^n , v^n , p^n , h^n and the boundary specifications, the fields u^{n+1} , v^{n+1} , p^{n+1} , h^{n+1} are calculated.

4. TRANSFORMATION OF THE BASIC EQUATIONS INTO THE MAPPED COORDINATE SYSTEM

The computation of the propagation of free surface waves involves computational boundaries that do not coincide with coordinate lines in physical space. For the finite element method, such problem requires a complicated interpolation function on the local grid lines which results in the local loss of accuracy in the computational solution. Such difficulties require a mapping or transformation from physical space to a generalized space. This transformation simplifies the problem of highly deformed air-fluid interface that arises in the analysis of wave breaking. This mapping transforms the wave propagation model from the physical domain, (x, y) to a computational domain, (ξ, η). The use of generalized coordinates implies that a distorted region in physical space, such as breaking wave, is mapped into a rectangular region in the generalized coordinate space, where the unknown interface coincides with a coordinate line as in Figure 4.1.

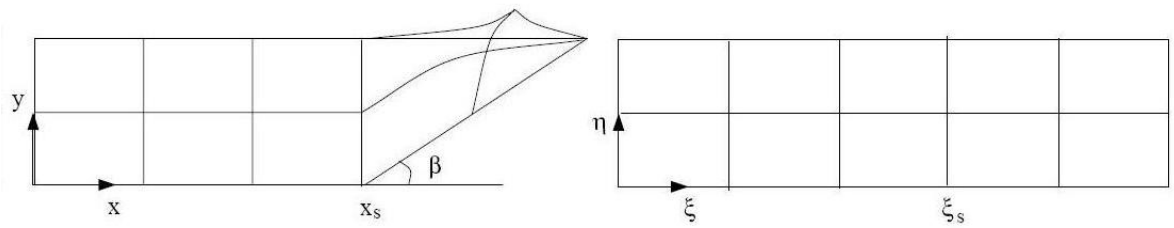

Figure 4.1: The computational grid is shown mapped back to the physical space.

Since the interior points in the computational domain form a regular grid and the boundaries coincide with coordinate lines, the determination of $x(\xi,\eta)$, $y(\xi,\eta)$ is easier than working in the irregular physical domain. Referring to the physical and computational meshes pictured in Figure 4.1, the following mapping, can be established.

$$x=\sum_{i=1}^{3}(\xi+h\alpha_i)F_i(\eta)$$
$$y=\eta(1+h)+(1-\eta)(\xi-\xi_s)\tan\beta \tag{4.1}$$

where ξ_s is the starting point of the slope. The function $F_i(\eta)$ are interpolation function. Employing three point interpolation, we have:

$$F_1(\eta)=1-3\eta+2\eta^2$$
$$F_2(\eta)=4\eta-4\eta^2 \tag{4.2}$$
$$F_3(\eta)=-\eta+2\eta^2$$

5. SOLITARY WAVES PROPAGATION

Solitary waves are believed to represent a good model for both tsunamis and extreme design waves. A solitary wave is essentially a wave that has infinite length lying entirely above the still water level and propagates at constant velocity without a change in form if the stillwater depth is uniform. According to this characteristic that the wave keeps its initial form without deformation, the Eulerian Lagrangian description of fluid motion is employed here to solve the problem. In this description, the particles are followed in y direction in Lagrangian manner and the coordinate is fixed in the x direction. Although, the solitary waved can be readily produced in the laboratory, in what it appears to be pure form, many numerical methods have failed to establish a wave of permanent shape. A definition sketch for the solitary wave is shown in Figure 4.2.

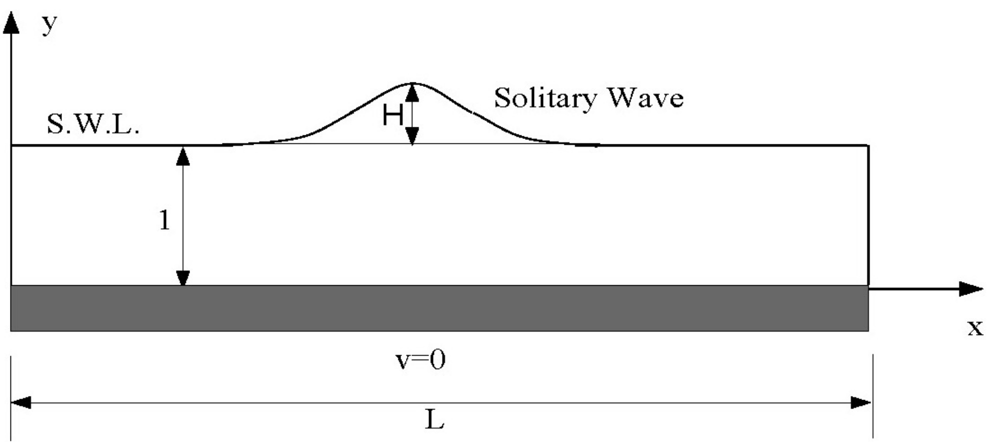

Figure 5.1: Definition sketch for propagation of a solitary wave in constant depth

There are three theoretical solutions of the solitary wave equations. Boussinesq obtained an analytical solution for the wave profile, wave propagation speed, and the water particle velocities. McCowan carried out the solution to the first order approximations to determine the wave characteristics. Laitone's solution is similar to Boussinesq but with higher order terms. The initial condition is assumed as Laitone's first approximation that are in the following dimensionless form:

$$
\begin{aligned}
&h = H\,\mathrm{sec}h[x\sqrt{.75h_0}\,] \quad && v = yh\sqrt{3H}\,\tanh[x\sqrt{.75h_0}\,] \\
&c = \sqrt{1+h} && p = 1+h-y \qquad (5.1) \\
&u = h &&
\end{aligned}
$$

where h, y, p, v, u, c denote normalized wave celerity, velocities in x and y directions, pressure, water depth and wave height from the stillwater surface respectively. H stands for maximum initial wave height of the incident solitary wave.

6. RESULTS

The propagation and deformation of a solitary wave over bathymetry topography is investigated under the more sophisticated conditions than before. The arbitrary Lagrangian-Eulerian description is examined where the spatial coordinates are moving with the velocity and the computation is done in the reference coordinate system ξ and η. The reason for the selecting of arbitrary Lagrangian-Eulerian description for modeling of solitary breaking wave is force the model to cope with a various of wave profiles, specially with a multi valued surface elevation. As it was mentioned, breaking happen in different forms and the most common ones are plunging breakers, where a jet is thrown forward from the crest region, and bore breaker, where a region near the shoreline has short steep turbulent front. Most of the solitary waves with finite amplitude are subject to break as they climb up a mild sloping beach. Researchers have expended considerable effort to obtain the conditions where the solitary waves do not break as they propagate over uneven bottom. According to their findings, the waves propagating under the following conditions do not break as they approach the shoreline.
- The small amplitude waves; In this case the wave has a very small amplitude or in other word the wave profile has a very gentle slope. Shallow water waves, long waves, with some wave length much higher than the constant water depth are grouped in this category.
- Steep Beach; When the slope of a beach is steep and not mild.
- Shallow water region; The waves moving in shallow water region dissipate most of their energy because of the bottom friction. Except the aforementioned conditions, in most cases the waves propagating towards bathymetry topography will break. To trace free surface waves in breaking zone, researchers have adopted a variety of techniques including considering artificial viscosity in the kinematic, or dynamic free surface equations, putting artificial viscosity in the momentum equations or smoothing free surface. In the present algorithm, none of the above assumptions has been used and comparisons with the numerical results calculated by Heitner (1969) and Zelt (1991) and the experimental results obtained by Camfield and Street (1967) have been carried out.

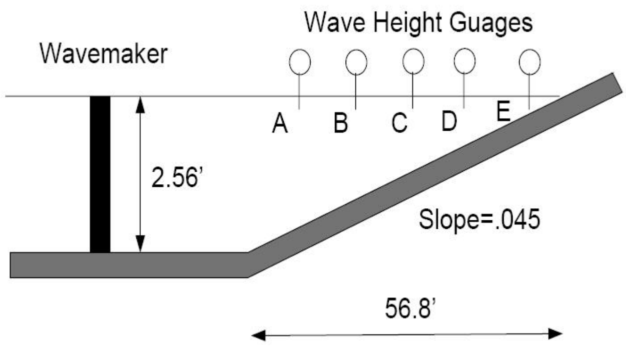

Figure 6.1. Definition sketch showing the schematic of slope and the location of gauges.

In the experimental frame of Camfield and Street (1967), the solitary wave is generated in a rectangular tank with constant depth by a vertical piston-type wave plate whose motion is controlled through a hydraulic electronic system at one end of the tank. On the other side of tank the test area is located that consists of a linear sloping beach for letting the incident wave to deform and if needed, to break. The water surface height is measured continuously by several electrical gauges that are located along the test area. Figures 6.1 shows the layout for this test. The first comparison is done with the experimental data of Camfield and Street (1967) and the numerical results for the propagation and breaking of a solitary wave with H/d =0.078 in constant depth d = 2.5 and slope is equal to .045. The results for this comparison have been shown in Figure 6.2. The solid line presents the current solution, the dash line is the experimental results obtained by Camfield and Street (1967).

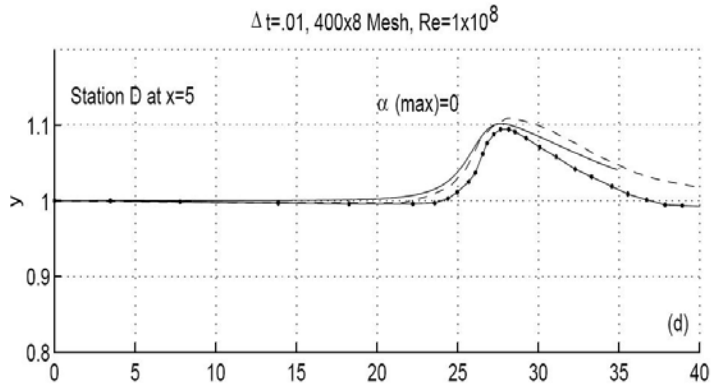

Figure 6.2. Comparison the free surface profiles of the present numerical model with experimental data obtained by Camfield and Street.

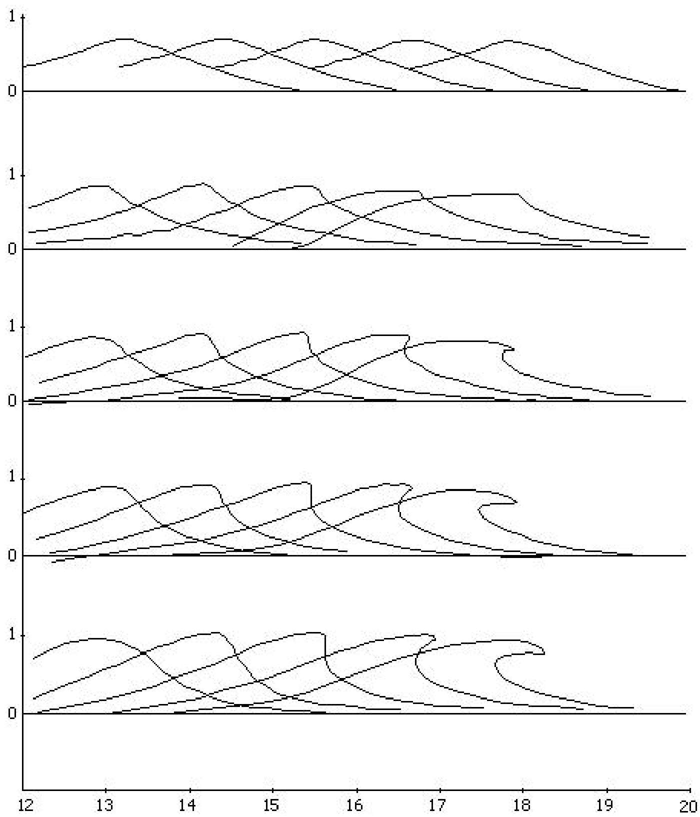

Figure 6.3.: Solitary wave modelling with present numerical model with amplitude a=0.6; 0.7; 0.8; 0.9; 1.0 from top to bottom.

7. CONCLUSION

The model is validated by comparing numerical results with theoretical solutions and with results obtained numerically or experimentally. When compared with known results for wave speed, results from the method agree excellent, when results for fluid velocities are compared with the analytical solution the agreement is found to be good. Overall, the conformity between the available data and the computations is well and in most cases the numerical model gives excellent results. The model can be employed in any geometry, under complicated boundary conditions, and with arbitrary bathymetry, without any additional computational effort. The method is tested on a free, steady wave of finite amplitude, and then applied to unsteady waves and is found to give excellent agreement with independent calculations based on the other existing theories. The steepening and overturning of freely running waves is presented and it is demonstrated that the surface remains rounded till well after the overturning takes place for periodic waves. The steepening of the forward face of a solitary wave is almost up to the instant when the free surface becomes vertical. In case of the periodic wave, the model was able to demonstrate the multivalued surface elevation when the steepening of the forward face of the wave almost pass the vertical position and plunging breaker is occured. Although more overturning is not achieved, the computational procedure can be modified so as to cope with the complete plunging breaker. Therefore, the problem of breaking waves can conveniently be studied using the present method for periodic and solitary waves because of its validity for all types of waves with different wavelengths.

One of the most advantages of the present study is in its capability to cope with a wide range of problems, with free surface or without it, long waves or short waves, small amplitude theory or finite amplitude theory. It is almost a general method to handle different aspects of fluid mechanics problems. Another advantage of the present study is that no smoothing or artificial viscosity is applied. The model's convergence is satisfactory and in contrast to most of the other methods, there was no need to damp the hump artificially.

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