Influence of modal regularity in the evaluation of reinforced concrete structures using simplified nonlinear methods

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SUMMARY
This paper presents the results of an ongoing investigation on a structural/demand condition, measured by a modal regularity index, which may lead to wrong results when simplified nonlinear analysis procedures are used for the seismic performance evaluation of structures responding in the nonlinear range of behaviour. It is shown that undesirable modal irregularity effects may be present in viaduct type bridges when the composition of the vibration modes of the bridge changes due to inelastic effects. Finally a modification to the application of a seismic evaluation procedure that eliminates the undesirable effects of modal regularity is proposed. To show the validity of the procedure proposed, the capacity curve of a bridge with ill regularity conditions is calculated and compared with the capacity curve calculated using nonlinear incremental dynamic analyses.

Keywords: modal regularity, modal spectral analysis and nonlinear statics methods

1. INTRODUCTION

Bridges are apparently simple structures whose performance under designed seismic demands should be satisfactory when correctly analyzed and designed. However, this expectation is not always fulfilled, partly because most of the methods and recommendations for the seismic analysis and design of structures have been developed considering building-like structures, resulting in many of the assumptions involved being indiscriminately accepted for other types of structures such as bridges. Bridges are structures that behave very differently to buildings, particularly when subjected to seismic demands consistent with limit states accepting structural damage, something that should be explicitly considered when the structure is analyzed. This drawback, however, is seldom recognized when using simplified procedures to evaluate the seismic performance of structures which accept the validity of the equal displacement rule, Veletsos and Newmark (1960), regardless of its limitations when applied, even with modification factors, to short period structures and/or structures built on soft soil conditions, and to limit states involving changes in the modal characteristics of the elastic structure.

Regarding this last limitation, recent literature on the use of Simplified Nonlinear Analysis Procedures, SNAPs, for the assessment and seismic design of bridges, often questions the validity of the results obtained with these procedures, in general attributed to a regularity condition characterized by changes on the modal shapes for different damage levels. Even though these definitions help to classify the degree of regularity in a structure, there is still no simplified nonlinear analysis procedure that eliminates or even diminishes its effects in the quality and even correctness of results.

This paper proposes a new definition of regularity of a structure responding in the inelastic range of behaviour characterized by an index, named modal regularity index, whose value defines the validity of results of existing SNAPs. It is demonstrated that this index becomes particularly important when validating the results of the seismic evaluation of bridges. Finally, to eliminate this limitation on the SNAPs previously developed by the senior author, a modification to the original procedure is proposed. To validate this modification an illustrative example is presented.

2. STRUCTURAL REGULARITY

The concept of regularity in structures has not been uniquely defined in existent codes for seismic design. This structural condition, particularly relevant in some types of bridges, has been marginally dealt with in codes such as AASTHO, (2006) where most viaduct type bridges are classified as irregular due to their mechanical and geometric characteristics, and the EC8-2 (CEN, 1994), where, the degree of regularity depends on the ductility factor, q, *i.e.,* the more ductile a bridge may be, the more irregular. Motivated by this limitation, some research efforts have recently been devoted to investigate the concept of structural regularity in bridges and its influence on approximate evaluation of seismic performance under earthquake action. their seismic behaviour and have proposed indices to characterize the regularity structures and help the analyst to decide which type of methods may be used for the evaluation or seismic design of bridges. The first research effort that deals with the limitations of the seismic analysis methods used for the evaluation and seismic design of bridges due to a structural regularity condition was published by Calvi *et al*. (1994), who proposed an elastic regularity index, I_R, to characterize this condition in bridges responding in the linear range of behaviour. This regularity index, I_R, combines the modal shapes of the deck alone with that of the whole bridge.

Using as reference the work of Calvi *et al*. (1994) there have been several proposals to define regularity indices where, besides the consideration of the participation of higher modes to performance the nonlinear behaviour of the structure is considered using pushover analysis, Isakovic, T. and Fischinger M. (2000). These authors have carried out extensive research on the influence of their definitions of regularity on the seismic behaviour of continuous viaducts, finding that the degree of regularity of a bridge varies with parameters such as: the ratio of the stiffnesses of the deck to the stiffness of the piers, the torsional sensitivity, the type of supports and the location of the most rigid piers. In their work they have proposed 2 different regularity indices, both termed "index" by the authors, one that covers the elastic behaviour and the other, the inelastic behaviour, both aiming to validate the use of unimodal methods and the use of the nonlinear static method, N2. Both indices are based on the comparison of normalized lateral deformed configurations obtained from two different lateral load patterns. For the inelastic index, the first deformed configuration is obtained using the N2 method with the prescribed lateral load pattern and the second deformed configuration using the load pattern defined from the first deformed configuration. The values of the indices proposed by Calvi *et al*., (1994) Isakovic, T. and Fischinger M. (2000) are such that a bridge with a regular behaviour has a regularity index close to one, while a bridge with an "index" that tends to zero will have an irregular behaviour inelastic.

Based on the same assumptions that consider existing regularity indices, Calvi *et al*. (1994) and Isakovic, T. and Fischinger M. (2000) state that a bridge is irregular if the higher modes significantly influence its calculated behaviour. Similarly, Maalek *et al.*, (2009) propose two inelastic regularity indices, FRI and SRI, both based on the comparison of transverse deformed patterns obtained with different lateral load distributions. To calculate these indices, analyses based on lateral pushover are used. The FRI index is based on the assumption that the degree of regularity of a bridge depends on the difference that may exist between the deformed bridge configuration and a completely regular deformed configuration, product of lateral pushover of the deck alone. The SRI index is defined in a similar way as the FRI, however for the calculation of the SRI the complete bridge is used to generate both deformed configurations. The first deformed configuration defined by a lateral pushover analysis carried out using a load pattern based only on the fundamental mode and the calculation of the second deformed configuration using a lateral load distribution obtained from a complete modal spectral analysis, using an accepted modal combination rule. Contrary to the interpretation of the indices previously described, the SRI and FRI indices for a bridge with irregular behaviour approach one, and the indices for a bridge with regular behaviour tends to zero.

3. MODAL REGULARITY

Considering that current seismic design regulations do not clearly define the concept of structural regularity for all kinds of structures and that existing definitions do not consider all factors that may cause the simplified methods of nonlinear analysis to give erroneous seismic performance results for an irregular structure, this paper proposes a new definition of structural regularity, named modal regularity, which considers the evolution of the modal shapes during the elastic and inelastic response of a structure produced by a particular seismic demand. The effects of modal regularity on the seismic performance of a structure appear in the results of SNAPs when there is a change in the modal shapes, from one damage state to another. Accordingly, these effects only occur in the calculation of the inelastic performance of the structure. The degree of modal regularity can vary from a slight change in the modal shapes to the inversion of participation factors and mode shapes. The effect of modal regularity can occur in the calculation of the seismic behaviour of any type of structure; however it is more evident in the case of viaduct type bridges where changes in modal composition may be manifested immediately after the yielding of the first pier. This condition does not generally occur in buildings where the effects of modal regularity are only evident when the structure presents considerable damage under earthquake action situation that rarely occurs in code designed buildings and this explains why analysts do not consider this effect in the application simplified methods of nonlinear analysis.

Although modal regularity is not the only factor that causes the performance calculated using SNAPs to differ from the performance calculated with a method considered as "exact", this factor may be the cause of erroneous results if not properly considered in the analysis procedure. Some of the issues that significantly influence the modal regularity of viaduct type bridges are:

1. The ratio of superstructure and substructure stiffnesses.
2. The ratio of the stiffness of adjacent piers.
3. Location of the stiffer pier.
4. The damage model.
5. The characteristics of the seismic demand.

This paper proposes two similar modal regularity indices, I_{MR}, both giving practically equal results aimed to provide the analyst with information to identify, quickly and easily, the degree of regularity of a structure and to decide which simplified procedure of nonlinear analysis is convenient to use. The magnitude of these indices varies in the range of 0 to 1; so that a bridge with a regular behaviour has a regularity index close to one and a bridge with irregular behaviour an index close to zero. The modal regularity indices proposed in this paper are based on basic concepts of structural dynamics considering inelastic behaviour by using modal analysis at different damaged states and may be calculated using Eqn. 1, 2 and 3.

$$I_{MR} = \sum_{j=1}^{n} \alpha \Phi_i^j \, \Phi_{i-1}^j \tag{1}$$

$$I_{MR} = \sum_{j=1}^{n} \alpha \left| \frac{\Phi_i^j}{\sum_{j=1}^{n} \left|\Phi_i^j\right|} - \frac{\Phi_{i-1}^j}{\sum_{j=1}^{n} \left|\Phi_{i-1}^j\right|} \right| \tag{2}$$

$$\alpha^j = \frac{\Gamma_i^j S_{ai}^j}{\sum_{j=1}^{n} \Gamma_i^j S_{ai}^j} \tag{3}$$

where:

Φ_i^j: Normalized modal shape of mode *j* and event *i*.

Γ_i^j: Modal participation factor of mode *j* and event *i*.

S_{ai}^j: Spectral pseudo acceleration corresponding to mode *j* and event *i*

4. PROPOSED SEISMIC EVALUATION PROCEDURE

The seismic evaluation procedure proposed in this paper preserves the essence of the original procedures developed by Requena and Ayala (2000) and Mendoza and Ayala (2011), however this modified procedure eliminates the pathogenic effects of modal regularity found in the seismic evaluation of reinforced concrete bridges using SNAPs. It consists of the construction of the capacity curve of the structure using a sequence of evolutionary modal spectral analyses when subjected to defined seismic actions of increasing intensity. For each modal spectral analysis associated to each increment of earthquake intensity the behaviour curves for each participating mode, also known as modal capacity curves, are defined including only events with the same modal configuration. The total response of the structure is obtained by adding the participation of all modes, using a conventional modal superposition rule.

4.1. Seismic assessment

The seismic performance of a bridge is defined using the independent modal behaviour curves constructed branch by branch, calculating as many performance points as damage states generated in the structure until the total intensity of the demand is reached. Each performance point is defined by a modal spectral analysis considering a structure with the damage corresponding to each branch. The details of procedure for calculating the seismic performance is described in the following steps:

1. Define the seismic demand. The method proposed is developed considering that the seismic demand applied to the structure is given by a design spectrum or even the response spectrum of a single record.
2. Define the capacity of the structural elements by calculating their moment- curvature diagrams.
3. Calculate the elastic branch of the capacity curve. This branch is defined by calculating the characteristic displacement and base shear in the structure corresponding to the intensity of seismic demand required to produce the first damage. This intensity is defined by calculating a scaling factor, S_f^1, for each of the bridge piers and by choosing the lowest of all define the yield point of the capacity curve. The scaling factor is calculated with Eqn. 4.

$$S_f^1 = \frac{M_y - M_G}{M_{DS}} \tag{4}$$

   where, M_y is the yield moment, M_G is the gravitational moment and M_{DS} is the internal moment, obtained with modal spectral analysis.
4. Extract from the elastic branch the capacity curve the elastic branches of the modal behaviour curves for all modes that contribute to the performance of the structure.
5. Define a new model of the structure considering the damage due to the increment of seismic demand used for this branch.
6. Once the performance of the new model of the structure is in this inelastic branch, the required increment of the demand intensity, sufficient to generate a subsequent damage state is calculated and the location of a fictitious[1] performance point along this branch of the capacity curve is defined. The scaling factor necessary for calculation of the increment of demand intensity, Eqn. 5:

$$S_f^2 = \frac{M_y - M_{ac}}{M_{DS}} \tag{5}$$

   where M_{ac} is the accumulated moment of the pier.
7. Calculate the first fictitious performance points on the independent modal behaviour curves. To define the location of these points on the modal behaviour curves modal regularity is considered by adding only events with similar modal configurations.
8. Modify the location of the otherwise fictitious performance point along the inelastic branch of the capacity curve. This correction is necessary to consider the dissipation of energy due to hysteresis, Mendoza and Ayala (2011).

[1] performance point calculated without considering energy dissipation due to hysteresis.

9. Calculate the performance points for subsequent stages of damage by repeating the calculation process from step No. 5.

The method ends when a predefined target displacement is reached or the structure presents a collapse mechanism. Fig. 1 shows schematically the steps involved in the application of this procedure.

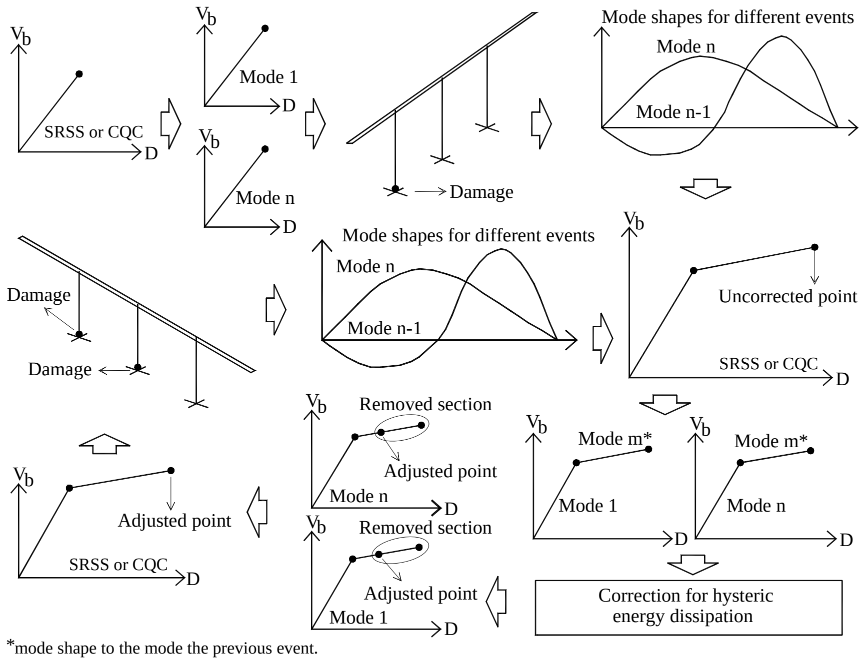

Figure 1. Steps involved in the procedure to evaluate the performance of an irregular bridge.

5. MODAL REGULARITY IN THE SEISMIC PERFORMANCE OF VIADUCTS

Viaducts are geometrically simple structures whose nonlinear seismic performance may be complex and unpredictable, particularly when using SNAPs, even for structures considered regular in geometry and governed by only one mode. To illustrate the participation of higher modes in the response of viaducts with and without incursions into the nonlinear range of behaviour and the influence of changes in the modal compositions between the undamaged and damaged states on the calculation of the modal regularity index 20 bridge models were investigated. Accordingly, the modal shapes and modal participation factors of each model, 10 considered short with a total length of 200m and divided into 4 equal spans, and 10 considered long with a total length of 600 m and divided into 7 equal spans, were calculated. The seismic demand used corresponded to the response spectrum of the E-W acceleration record at the Takatori Station during the January 17, 1995 earthquake in Kobe, Japan.

To demonstrate the influence of modal regularity on the results of SNAPs and validate the proposed procedure, 4 bridges were analyzed, 2 regular in geometry (V232P and V313P) and 2 with irregular geometry (V213P and V133P). In the nomenclature used to define the characteristics of the bridges, the letter "V" indicates the type of bridge (V: Viaduct), the letter "P" the type of support at bridge abutments, (P: hinge), and the intermediate numbers, VxxxP the ratios of pier heights to a basic height of 7m. (see Fig. 2). For all modal analyses total bridge mass was considered concentrated at nodes located along the deck assuming that the piers weighted 10.1 t/m and the deck 20.2 t/m. Table 1 shows the mechanical and geometrical properties of the bridges studied.

Table 1. Mechanical properties of the materials

	f'_C (Mpa)	E_C (Mpa)	γ_C (kN/m^3)	A (m^2)	I_X (m^4)
Deck	27	25000	25	6.97	88.46
Pier	27	20000	25	2.2	7.8

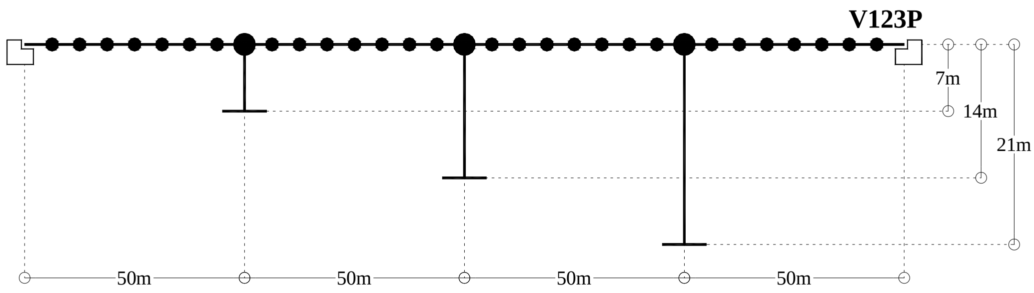

Figure 2. Bridge with irregular geometry

5.1. Modal analysis results

Figs. 3a and 3b show the modal participation factors of the modal analysis carried out to the 20 reinforced concrete viaduct type bridges considering undamaged and damaged states. These figures show the significant influence that higher modes may have on the seismic performance of continuous viaducts and that there is not a dominant fundamental mode which governs the performance of such structures, even in the elastic range of behaviour. Furthermore, these figures also show that the participation factors defining the influence of different modes can vary significantly from one viaduct to another, characteristic shown for both short and long viaducts.

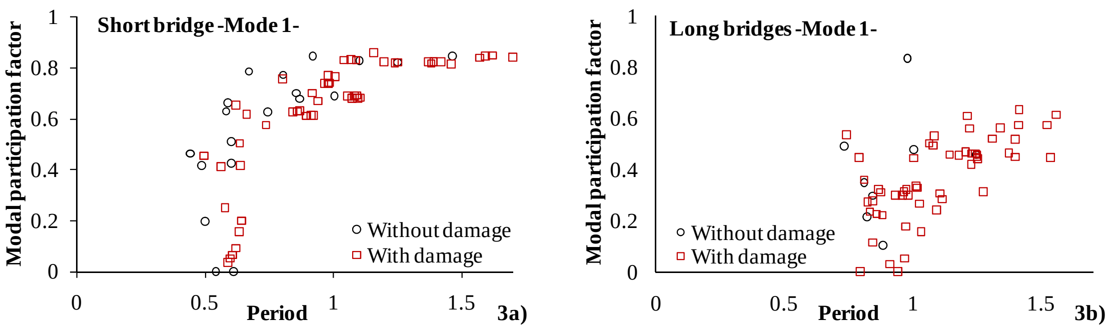

Figure 3. Influence of the higher modes in the response of the structure.

Fig. 4 shows the modal participation factors from the undamaged to the damaged state together with their corresponding modal regularity indices for 4 reinforced concrete bridges, 2 with irregular geometry (V213P and V113P) and 2 with regular geometry (V232P and V313P). It may be observed that geometric regularity is not a determining factor in predicting modal regularity of the viaducts. Figs. 4a to 4c show that the modal participation factors for the three bridges change considerably from the undamaged to the damaged state, and that their modal regularity indices tend to be very low, even for the viaduct with the regular geometry. Fig 4d shows that the modal participation factors have a stable behaviour from the undamaged to the damaged state and that their modal regularity indices are close to one.

Figs. 5a to 5d show the modal shapes of the bridges V232P, V213P, V313P and V113P for two different damage states. Figs. 5a and 5b show that the behaviour of the modal shapes is not constant in the course of the response of the bridge. The observed changes from the undamaged to the damaged state involve not only changes in the shape but also in the order of the modes, producing differences in the corresponding participation factors particularly for the modes that most significantly contribute to seismic performance. Fig. 5c shows that in spite of the changes in modal shapes from the undamaged to the damaged state, not sufficient to be considered a change in the order of the modal shapes, the value of the modal regularity index is low enough for the structure to be considered as irregular. Finally, Fig. 5d shows the case of a bridge with regular geometry where the changes in modal shapes and the corresponding participation factors for the undamaged and damages states are minimal and therefore the modal regularity index is close to unity.

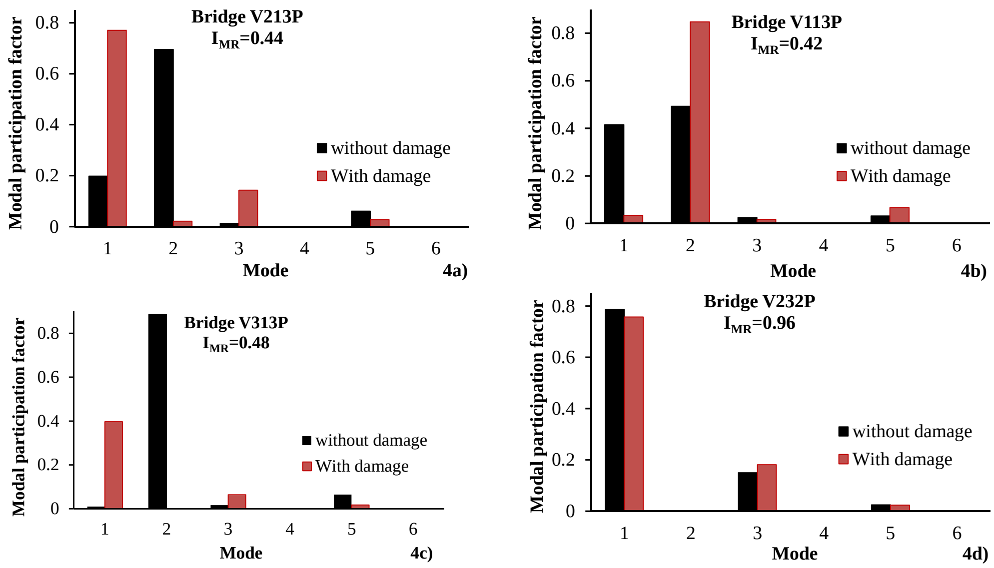

Figure 4. Modal participation factors of a state of damage to another

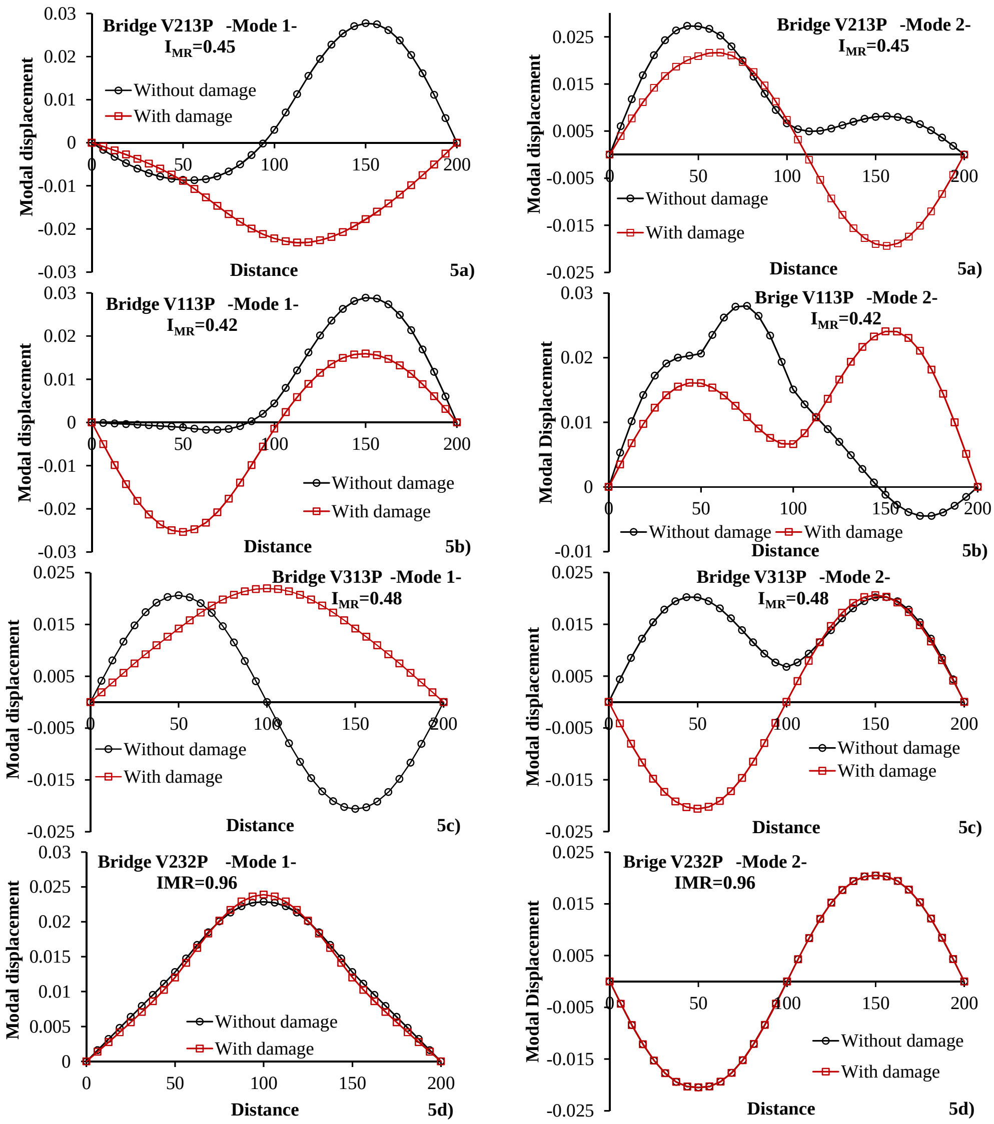

Figure 5. Modal shapes of bridges V213P, V313P,V133P and V232P.

Comparing the modal regularity index (I_{MR}) proposed in this paper with the classification proposed by AAHSTO, (2006), it was observed that there is no agreement between the two proposals, even for bridges with geometric regularity and this is because the AASHTO code only considers information on the differences of geometric and mechanical properties between adjacent spans, not directly related to the modal regularity of a structure which depends on several other factors. The classification proposed by AASHTO only gives consistent results for bridges, regular in geometry, that approximately have constant modal participation factors for the undamaged and damaged states, and a modal regularity index not smaller than 0.85.

5.2. Seismic assessment results

For the validation of the proposed seismic evaluation procedure it was necessary to choose a seismic demand that produced a significant influence of the fundamental mode of the considered examples. The seismic demand used corresponded to the same EW component of the Takatori Station acceleration record. To carry out the evolutionary modal spectral analyses, the commercial structural analysis program SAP2000, (CSI, 2000), was used, and for the incremental dynamic analyses, the program DRAIN 2DX, Prakash *et al*. (1992).

Fig. 6 shows the capacity curves for a viaduct of regular geometry (V232P) obtained using the simplified nonlinear analysis procedure considering the hysteretic energy dissipation as proposed by Mendoza and Ayala (2011) and the nonlinear incremental dynamic analyses. This figure shows that for demands of equal intensity there is a similar correspondence between base shears and displacements for both methods, *i.e.,* the performances obtained with the simplified procedure are approximately the same as the "exact" performances. With this example it is shown that the simplified procedure without modification provides satisfactory approximations to the performances obtained using reference nonlinear incremental dynamic analyses, for viaducts with regular geometry, with constant modal participation factors throughout their response and a modal regularity index close to 1.

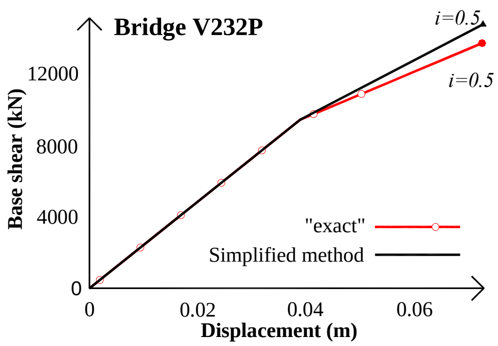

Figure 6. Correction for dissipation of hysteretic energy of the capacity curve of bridge V232P.

Fig. 7 shows a comparison between the capacity curves for a viaduct of regular geometry, V313P, and another irregular, V213P, both obtained with the simplified procedure proposed by Mendoza and Ayala (2011) without modification, and with reference nonlinear incremental dynamic analyses. During the construction of the capacity curves with both procedures it is observed that the displacements and base shears produced by seismic demands of equal intensity do not correspond, i.e., the shapes of the capacity curves approximately coincide, however the performances associated to both curves are different. This example demonstrates that geometric regularity is not a sufficient characteristic to guarantee a correct approximation to performance as the simplified procedure without modification, *i.e.,* without explicit consideration of modal regularity, is in general, not able to provide approximate performances close to those obtained with a method considered as "exact".

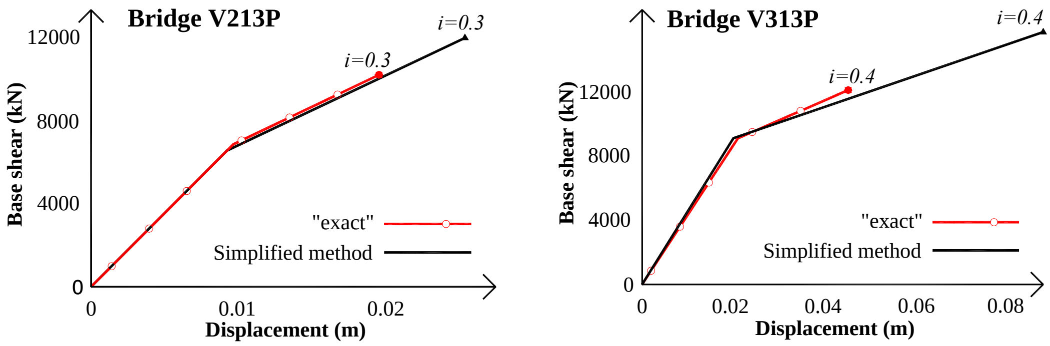

Figure 7. Capacity curves of the bridges V213P V313-corrected and hysteretic energy dissipation.

Fig. 8 shows a comparison between the capacity curves for the viaduct regular in geometry but with modal irregularity, V313P, obtained with the simplified procedure with modifications proposed in this paper and the corresponding one obtained using nonlinear incremental dynamic analyses. With this example it is shown that the performances obtained with the simplified method with modifications tends to approach the performances obtained with the "exact" method., *i. e*., the proposed procedure leads to performances close to those considered as "exact".

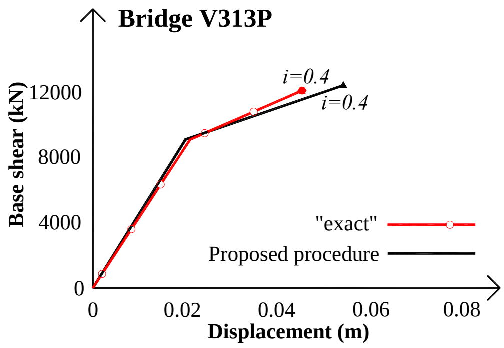

Figure 8. Capacity curves for bridge V313P

7. CONCLUSIONS

This paper presented the results of an ongoing research on the evaluation of the influence of changes in the composition and characteristics of the modal shapes of viaduct type reinforced concrete bridges responding in the inelastic range of behaviour and on the definition of a modal regularity index needed to validate the performance results obtained using nonlinear simplified methods of seismic analysis. For the cases where the SNAPs lead to wrong results due to the regularity of the structure, this paper suggests a modification to the simplified nonlinear analysis procedure that gives correct approximate seismic performance. From the results of the modal analyses performed on 20 models of viaducts considering their undamaged and damaged states and the seismic evaluation of the bridges, V213P, V232P and V313P, studied in this investigation the following conclusions may be extracted:

1. Modal regularity is an important characteristic needed when simplified methods of evaluation of nonlinear seismic performance of structures, such as reinforced concrete viaducts, are used.
2. Changes in the modal composition for structures for undamaged and damaged states may significantly affect the correctness of the performance results of simplified methods of nonlinear analysis. The seismic performance of viaduct type bridges generally involves the contribution of higher modes, contrary to what generally happens in building structures where the fundamental mode dominates the seismic response of the structure, even in the nonlinear range of behaviour. It was shown that the number of modes that significantly participate to the

seismic response of a continuous viaduct is proportional to the number of piers, i.e., the larger number of spans, the larger the number of participating modes.

3. The parameters given by the code AASHTO (2006) to define the regularity of bridges can provide satisfactory results as long as the structures do not present changes in modal composition at any stage of their inelastic response. It is shown that the regularity of viaduct type bridges does not only depend on the geometric and mechanical properties of adjacent piers, but most importantly on the changes of modal shapes and participation factors associated to their undamaged and damaged states.
4. The modal regularity index, I_{RM}, proposed in this paper can be used as an aid for the analyst to decide whether the simplified methods of nonlinear seismic analysis are suitable for the seismic evaluation of structures such as viaduct type bridges.
5. The deformed lateral configuration calculated by linear or nonlinear static analyses is an unreliable parameter to define the regularity of viaduct-type bridges. In both cases it is important to validate the use of a particular modal combination rule as these rules are normally tested on the seismic evaluation of buildings and not of bridges.
6. The values of the modal regularity indices calculated for the examples used in this paper to characterize regularity are congruent with the performance results obtained with the SNAPs. Based on these results, a structure can be considered regular when its modal regularity index is larger or equal than 0.85; if this index is smaller, the structure is classified as irregular requiring the use of the SNAP proposed in this paper for the calculation of "accurate" performances or alternatively a procedure using nonlinear incremental dynamic analyses.
7. The evaluation procedure proposed in this paper produces approximations of seismic performances of viaducts close to those obtained with a method considered as "exact".

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