Performance based seismic design method considering incremental collapse prevention

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SUMMARY
This paper presents an alternative multilevel performance based seismic design method that explicitly includes the design for incremental collapse prevention. The objective of this method is to truly assure the accomplishment of a performance objective through effective control of damage occurring in the structure. The simplified approach of design for collapse consists of preventing a condition of dynamic instability under extreme demands. To show the validity of the method proposed, the design of a 10 - storey reinforced concrete frame and its validation through nonlinear step by step dynamic analyses is carried out.

Keywords: performance based seismic design, degrading behaviour, incremental collapse

1. INTRODUCTION

The goal of earthquake engineering is to assure adequate performance of a building throughout the seismic events that may occur during its entire lifespan through the accomplishment of prescribed performance levels, also known as limit states, for probabilistic seismic ground motion levels. The set of limit states to be satisfied for a given level of demand is known as performance objective, which is defined in accordance with the importance and function of the construction.

In recent years, several simplified Performance Based Seismic Design, PBSD, methods have been developed, whose objective is to truly assure the accomplishment of limit states through a better control of the factors that intervene in structural response. This effort is motivated by the fact that conventional seismic design methods, such as those contained in building codes, are unable to guarantee the fulfilment of a performance objective. However, existing simplified PBSD methods may be cumbersome to use in practical design and/or lack conceptual transparency, which is the reason why they have not yet been implemented in professional earthquake engineering practice.

Furthermore, although the performance based seismic design approach defines a collapse prevention limit state (FEMA 273, 1997) which accepts that a structure may undergo severe stiffness and strength degradation, none of the simplified PBSD procedures consider explicitly this instance of behaviour in their formulation. Due to the importance of collapse prevention of building structures, its implementation in simplified PBSD procedures is essential.

This research group has proposed an alternative performance based seismic design method that considers, in a more explicit manner, most factors that influence structural response, thus being able to assure the fulfilment of a performance objective involving two limit states (Barradas *et al.* 2010). This method is based on the definition of a bilinear design behaviour curve, *i.e.*, graphic representation of spectral displacement *vs.* spectral pseudoacceleration of the fundamental mode, from which design demands explicitly defined from the target displacements considered are obtained. Due to the transparency of its formulation, this design procedure is not only efficient, but also straightforward.

A modification of the aforementioned multilevel seismic design procedure, that includes the design for collapse prevention, is currently being investigated. At present, the proposed method considers the incremental collapse type. This new proposal consists of the definition of a design trilinear behaviour curve in which the last branch describes expected degrading behaviour under extreme demands, *e.g.*, collapse limit state. The present paper illustrates the general framework of the proposed method, focusing on the simplified collapse design approach considered, and a design application of a 10-storey frame with its correspondent validation through nonlinear dynamic analysis of the designed structure.

2. COLLAPSE PREVENTION IN PBSD

Structural collapse is defined as the local or global failure of the system due to the loss of its vertical load carrying capacity. Two primary types of global collapse can be identified: progressive and incremental. The first is defined as the total or disproportionate failure of a structural system due to the spread of an initial local failure, i.e., to other parts of the structure. Incremental collapse is caused by the severe deterioration of storey shear strength due to large displacements, i.e., dynamic instability (Ibarra, 2005).

The design for collapse prevention, as defined by FEMA 273 (1997), consists of assuring global stability of the building, considering severe degradation of its strength and stiffness, for the demand associated to the performance objective considered. Since the amount of damage expected would be such that the structure would be deemed non repairable, *i.e*., total economic loss is accepted, the goal of design for collapse prevention is to avoid gross human loss during low probability seismic events.

The approach recommended for collapse assessment in performance based earthquake engineering is the estimation of the probability that an intensity measure of the system, IM, exceeds the intensity associated to collapse, IM_C, as shown in Eqn. 2.1 (Ibarra, 2005). Spectral pseudoacceleration of the fundamental mode of a system, S_a, is commonly used as IM. The majority of the existing collapse assessment methods are conceived for the incremental type of collapse, *i.e.*, IM_C is the spectral acceleration that causes dynamic instability.

$$P[C|IM = im_i] = P[IM_c < IM = im_i] \tag{2.1}$$

The proposed method is consistent with such an accepted collapse assessment approach. Its primary objective is to guarantee that structures subjected to demands exceeding those considered in the performance objective stipulated in the Mexican Building Code (GDF, 2004) are dynamically stable, since such objective does not explicitly address degrading behaviour. Nonetheless, this methodology can also be used to design for a performance objective that includes a collapse prevention limit state. In this case, a rate of exceedence for such a limit state, and thus an IM_C, is defined explicitly. The strategy of the collapse design approach employed in the proposed method is to prevent dynamic instability through effective damage control in structural components, considering that the systems will experience severe strength-stiffness degradation when subjected to extreme demands.

3. GENERAL BEHAVIOUR OF DEGRADING SYSTEMS

Structural systems that exhibit large displacements due to intense ground motions will likely develop severe reductions of structural strength and stiffness. This may lead to the appearance of a negative slope in the strength-displacement plot of structural behaviour. The causes of severe degradation are associated to the geometry of the structure and the properties of the structural components, *i.e.*, P-Δ effects, local strength degradation of structural components and loss of structural integrity due to total failure of some elements.

The main issue regarding degrading structures is that they are susceptible to present instability during seismic loading, i.e., to be dynamically unstable. Dynamic instability is defined as a disproportionate response of the system due to a small increase in the magnitude of dynamic loading during a segment of time. Unlike the static case in which the sole existence of a negative value of tangent stiffness determines that the structure is unstable, in the dynamic case the inertia and damping forces can provide a stabilizing effect during intervals of time where the structure is deemed statically unstable (Bernal, 1998). However, the response of the system could be disproportionate for certain load intensity. The incremental collapse strength of a structure is, by definition, the demand associated to such condition.

The behaviour of a degrading MDOF system is very complex, and is dependent on many structural characteristics, such as the distribution of gravity loads and structural element stiffnesses along the height of the structure, the damage distribution developed and the level of local deformation of structural components. However, different authors denote that the properties of degrading SDOF systems provide a good insight in the understanding of MDOF structures (Bernal, 1998; Vamvatsikos, 2002). For this reason, several simplified collapse assessments have been developed on the basis of SDOF system behaviour, e.g., Adam *et al*. (2012).

The displacement based design seismic method developed by the authors of this paper considers a simplified approach to account for the degrading behaviour of structural systems due to incremental collapse under extreme loading. At present, the method considers only the decrease of stiffness in structures due to P-Δ effects. A study concerning the effect of accumulated damage in structural components on global behaviour of frames subjected to characteristic ground motions of the valley of Mexico is accordingly underway.

3. STIFFNESS-STRENGTH DEGRADATION DUE TO P-Δ EFFECTS

It is a recognized fact that gravity loads acting on the structure reduce its global lateral stiffness due to second order effects. Such reduction is usually not significant during the elastic stage of behaviour of a structure subjected to earthquake ground motion. However, during inelastic response, the deterioration of strength and stiffness caused by vertical loads can be substantial, as the stiffness in this stage of behaviour may be severely reduced, and thus, dynamic stability may occur. This point can be better understood through examination of Eqn. 3.1, which represents the characteristic value problem of a structure considering second order effects.

$$Det[|K - K_G| - \lambda_i|M|] = 0 \tag{3.1}$$

where: K is the stiffness matrix associated to the structural properties, K_G is the geometric stiffness matrix which accounts for second order effects, λ_i is the characteristic value, M is the mass matrix of the system.

If the effective stiffness, K_E, of the system, *e.g.,* the reduced stiffness due to second order effects [K-K_G], is positive definite, *i.e.*, statically stable, the resulting characteristic values are positive, thus, second order effects do not cause degradation. On the other hand, if the effective stiffness has at least a negative characteristic value K_E is statically unstable. However, as mentioned in the previous section, static instability is a necessary but not sufficient condition for the occurrence of dynamic instability, Bernal (1998).

A negative characteristic value λ_i of the ith mode is the slope of a degrading stage of behaviour of the corresponding curve of spectral pseudoacceleration *vs.* spectral displacement. The associated characteristic vector, Φ_{in}, is the modal shape considering the effects of diminution of strength and stiffness due to gravity loads. This shape depends on the distribution of storey stiffnesses throughout the height of the system, thus, it is evident that the distribution of inelasticity is directly related to the displacement configuration in a degrading response phase. Furthermore, due to the characteristics of its deformed shape, the first mode is considered to be the critical case for second order effects; hence, the degrading behaviour in a structure caused by P-Δ can be characterized by the properties of the first mode only.

In accordance with these concepts, the simplified collapse approach employed in the proposed design method consists of controlling structural damage to prevent dynamic instability, accepting the possibility that degrading behaviour may occur under extreme demands due to the effect of gravity loads during inelastic response. The level of degradation is measured by the characteristic value of the first mode associated to a damaged state, which can be obtained from modal analysis of a simplified structural model that represents such state. The proposed method describes the behaviour of a structural system through a plot of spectral pseudoacceleration *vs.* spectral displacement; hence, the level of degradation, *i.e.*, negative stiffness, of the postcapping stage is represented graphically in such curve.

4. PROPOSED MULTILEVEL PERFORMANCE BASED SEISMIC DESIGN METHOD

4.1. Reference system

The fundamental basis of the proposed PBSD method is the concept of reference system, defined as a SDOF oscillator with trilinear behaviour associated to the properties of the fundamental mode of a MDOF structure, from which its elastic and inelastic response may be approximated. It is evident that this approach is applicable for structural systems whose response is primarily defined by one mode throughout the entire seismic event, i.e., a structure that satisfies modal regularity (Ayala *et al.*, 2012).

Under the acceptance of this assumption, structural behaviour can be characterized through a trilinear plot of spectral pseudoacceleration, S_a, *vs.* spectral displacement, S_d, of the fundamental mode of a structure (see Fig. 1). In the framework of the proposed method, such plot is referred to as a behaviour curve (Ayala, 2001). This curve can be transformed to and from the capacity curve of the structural system by the means of basic structural dynamics concepts (Freeman, 1984), or directly from modal analysis of simplified linear models that represent each stage of behaviour.

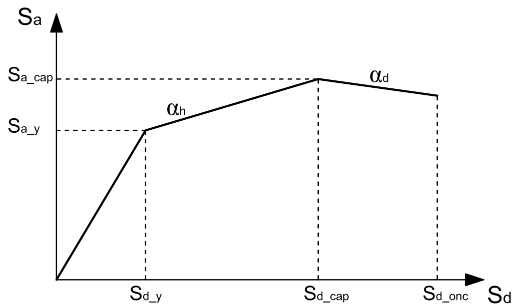


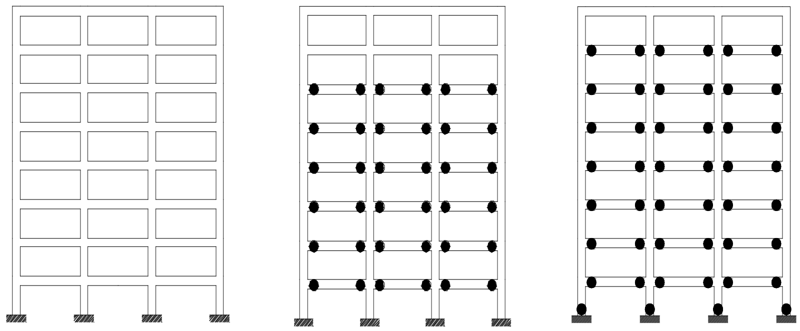
Figure 1. Trilinear behaviour curve

The points that define the behaviour curve are called characteristic points: yield (S_{d_y}, S_{a_y}), capping (S_{d_cap}, S_{a_cap}) and onset of collapse (S_{d_onc}, S_{a_onc}). The first branch of the curve represents the properties of the elastic stage of behaviour. The second branch represents the properties of the hardening stage associated to the damage distribution developed up to the capping point. The third branch describes the degrading stage of behaviour, *i.e.*, negative slope, corresponding to a more severe damage distribution that is associated to a higher level of demand. The slopes of the branches are the squared circular frequencies, *i.e.*, stiffnesses of the reference system, corresponding to each stage of behaviour, ω_E^2, $\alpha_h\omega_E^2$, $\alpha_d\omega_E^2$. The ratios α_h and α_d are the postyielding and postcapping stiffnesses normalized to the elastic stiffness. The strength level of structural members that are expected to develop inelastic behaviour is associated to the yield strength per unit mass, R_y, *i.e,* S_{a_y}. The cap strength per unit mass, R_{cap}, i.e., S_{a_cap}, is directly related to the demand of the elements that behave elastically until the capping displacement is reached.

4.2. Simplified linear models

It is assumed that the structural properties in each stage, *i.e.*, stiffness and deformed shapes, can be approximated using simplified linear models associated to the corresponding state of damage (see Fig. 2). The elastic stage model of the structure is defined considering the initial stiffness of structural elements; *e.g.*, cracked inertias for concrete structures and nominal inertias for steel ones. The hardening stage model is a replica of the elastic model where inelastic action is represented as simple hinges, in accordance with the damage distribution present in such stage of behaviour. The degrading stage model is similar to the inelastic one, but is defined for the corresponding damage distribution, which is, evidently, more severe than that of the previous stage, since it is associated to a higher level of demand. Under this assumption, the corresponding branches of the behaviour curve can be obtained from modal analysis of such models.

In the hardening and degrading models, inelastic action is represented by assigning simple hinges at the points where damage occurs, *i.e.*, moment releases in the corresponding bending axis. This simplification implies that the hardening stiffness of all plastic hinges is null and that every point of inelastic response is at a loading phase of the local hysteretic behaviour. Such conditions seldom occur in actual structural behaviour. However, the results of the studies performed by this research group, and the investigation carried out by other authors such as Sullivan (2010), show that assuming a value of zero for local strain hardening provides a good approximation of the global response, at least in first mode dominant structures. For the case of column base yielding, which is expected to occur under high seismic demands, a simplified damage model is yet to be defined, since this condition will influence the structural response in a more significant manner, *e.g.*, the structural period of a stage of behaviour that involves structural damage may be overestimated.



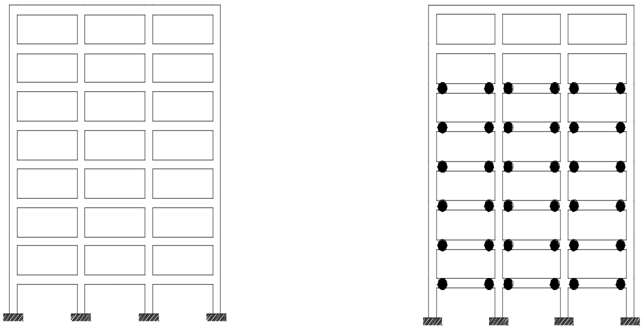
a) b) c)

Figure 2. Damage distributions associated to the three stages of behaviour: a) elastic; b) hardening; c) degrading

4.3. Contribution of higher modes

The behaviour curve represents solely the properties of the fundamental mode of a MDOF system. The contribution of higher modes in the elastic and hardening stage of the structure is obtained through the superposition of two modal spectral analyses of the corresponding simplified linear models. This procedure is based on the assumption that the elastic and hardening properties of all modes are directly related to the corresponding stiffness of each stage of behaviour (see Fig. 3).

In accordance with such assumption, the capacity curve of the system can be defined from the results of modal spectral analyses of both simplified models. Hence, the yield displacements and yield base shear is that obtained from the modal spectral analysis of the elastic simplified linear model; the displacement and base shear associated to the capping point is the sum of the results of modal spectral analyses of the elastic and hardening simplified linear models.



$[K_E] \rightarrow |\omega_E^2| \rightarrow [\Phi_E]$ $\qquad$ $[K_H] \rightarrow |\omega_H^2| \rightarrow [\Phi_H]$

Figure 3. Dynamic properties of the elastic and hardening stages of behaviour

It should be noted that modal combination rules used in modal spectral analysis, *e.g.*, CQC, SRSS, are conceived for linear MDOF systems, thus, its application on structures with nonlinear behaviour is theoretically inconsistent. However, such rules provide a good approximation of the maximum response in first mode dominant nonlinear structures. Furthermore, this approach allows for a straightforward and simple design procedure.

4.4. Inclusion of P-Δ effects

Second order effects are directly obtained from the solution of the characteristic value and vector equation including the geometric matrix, i.e., Eqn. 3.1. Thus, the analysis of the simplified models, at least those corresponding to hardening and degrading, must be performed using a program that includes the geometric matrix formulation. In addition, it is preferable that it calculates negative characteristic values to define the stiffness of the degrading branch.

4.5. Design procedure

The proposed design method consists of the definition of a trilinear design behaviour curve (R *vs* d, i.e., S_a vs S_d), that satisfies the considered performance objective. The elastic branch is defined in accordance to the initial stiffness required to accomplish the displacement associated to the service performance level, s. The properties of the hardening branch are determined in such a way that the displacement for the ultimate performance level, u, is satisfied, considering an accepted damage distribution for such level of demand. The degrading branch is defined according to the magnitude of gravity loads acting on the structure and a damage distribution that is more severe than that of the hardening branch, since it is associated to a higher level of demand.

The application procedure for the design of a frame structure is summarized in the following steps:

1-Predimensioning of structural elements based on the judgement and practical experience of the structural engineer or a rough design of the structure using a force based method, from which a realistic stiffness distribution throughout the height of the structure is defined.

2-Modal spectral analysis of the structural model using the design spectrum for service from which the maximum interstorey drift may be defined, adjusting, if necessary, the preliminary design in such a way that the resulting drift is less or equal than that prescribed for this limit state. This analysis provides the fundamental period of the structure, T_e, and the corresponding displacement, d_s, of the simplified reference system.

3- Definition of an acceptable damage distribution for the ultimate performance level, e.g., strong column-weak beam considering no yielding at column bases.

4- Modal analysis of the structural model corresponding to the ultimate performance level, *i.e.*, hardening model, representing the structural damage as hinges at the ends of the structural elements. In a similar way to step 2, the hardening period, T_h, is obtained, and, consequently, the postyielding to elastic stiffness ratio, α_h.

5- In a similar manner to step 2, definition of the allowable displacement of the reference system for the ultimate performance level, d_{u_all}, from the deformed shape of the hardening model, in such a way that the maximum storey drift of the structure does not exceed the maximum prescribed for this performance level.

6- Once the target modal displacements for the service and ultimate performance levels are defined, definition of the yield characteristic point associated, d_y, according to the desired design ductility for the ultimate performance ratio, μ_u.

7- Definition of the inelastic displacement, d_i, of the structure with period T_e from the displacement spectrum for the ultimate limit state, corresponding to μ_u and α_h. This displacement is compared with the maximum allowed, d_{u_all}. If the displacement is less or equal to that allowed, the properties of the structure are maintained, hence, $d_i = d_u$. Otherwise, the distribution of damage and/or the ductility are modified until this condition is satisfied. If the choice of the designer is to match the target displacements of both the ultimate and service limit states to the corresponding maximum allowable values, *i.e.* $d_{u\_all}=d_u$, the initial stiffness should also be modified to achieve such condition. In this case, iteration will almost certainly be required.

8- Definition of the yield strength, R_y, of the structure with period T_u, from the ultimate strength per unit mass spectrum for the ultimate limit state, corresponding to μ_u and α_h.

9- Definition of the cap strength per unit mass, R_{cap}, according to the intensity associated to initiation of degrading behaviour, *e.g.*, the demand correspondent to a given exceedence rate for collapse design, considering a design capping ductility, μ_{cap}.

10- Definition of a damage distribution for the degrading stage in a similar way to what is described in step 3. This distribution must contain that associated to the ultimate limit state, e.g., strong column-weak beam considering yielding at column bases.

11- Execution of the modal analysis of the structural model corresponding to the degrading stage, representing the structural damage as hinges at the ends of the structural elements, from which the degrading characteristic value, λ_d, and, hence, the degradation stiffness ratio, α_d, are obtained.

12- Definition of the required cap strength, R_{cap_req} from the strength per unit mass spectrum for collapse design, correspondent to μ_{cap} and α_d. This value is compared with the capping strength of the behaviour curve, R_{cap}. If $R_{cap} \leq R_{cap\_req}$, the properties of the structure are maintained. Otherwise, the damage distribution corresponding to the degrading stage and/or the design capping ductility are modified to achieve such condition.

13- Definition of a plot of the behaviour curve once the characteristic points are defined (see Fig. 4).

14- Modal spectral analysis of the elastic and hardening models considering the demands obtained from the behaviour curve. The design forces of the structural elements are obtained from the sum of the results of such analyses.

15- Design of structural elements in accordance with the applicable building code.

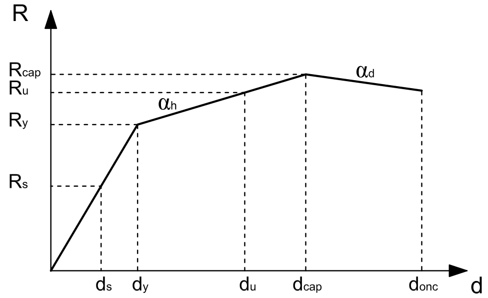


Figure 4. Design behaviour curve

5. DESIGN EXAMPLE

The design of a 10-storey reinforced concrete frame and its validation are shown. To demonstrate that the proposed method provides a good approximation of the results obtained via nonlinear step by step dynamic analysis of the designed structure, the design was performed using response spectrum obtained from the SCT-EW record of the 1985 Michoacán Earthquake. The demand considered for service is the elastic spectrum of this record divided by seven as stipulated in the Appendix A of the Mexico City code (GDF, 2004), and for the ultimate limit state the spectrum of the same record. Since there is no stipulation of a seismic demand corresponding to a true collapse limit state, in this paper, the demand used for collapse design was the response spectrum associated to the same record scaled by 1.5.

5.1. Description of the structure and allowable displacements

The nominal properties of the materials used in the design are: compressive strength of concrete, $f_c'= 2.5\times10^6$ kg/m^2; modulus of elasticity of concrete, $E_c= 2.2\times10^9$ kg/m^2; yield strength of steel, $f_y= 4.2\times10^7$ kg/m^2 and modulus of elasticity of steel, $E_s= 2.1\times10^{10}$ kg/m^2. The masses of floors one to nine are 7×10^3 kg·s^2/m and of floor ten (roof), 5.5×10^3 kg·s^2/m. A uniform gravitational load on beams of 1×10^3 kg/m for all floors and vertical nodal loads of 1.2×10^4 kg on every joint are considered.

In accordance with the Mexico City code, the allowable drifts for service and ultimate limit states considered in this example are 0.002 and 0.02, respectively. No limitation on storey drifts or roof displacements was considered for collapse design since the objective of this approach is the prevention of dynamic instability.

5.2. Analyses performed

The application of the method was carried out using the program SAP 2000 V.14 (CSI, 2006) which is able to calculate and provide negative characteristic values of a structure. The nonlinear dynamic analyses were performed with IDARC2D V.7.0, Reinhorn *et al.* (2009) using the following considerations: a) bilinear non-degrading hysteretic model considering a hardening stiffness of 5%, since the effect of local degradation is not a part of this study; b) proportional damping matrix; c) axial load-moment interaction considered; d) calculation of second order effects; e) the strength of elements considered was that obtained directly from the design procedure without any standardization, in order to determine if the demands obtained from the method are able to fully assure the assumed damage distributions.

5.2. Results and evaluation

The designed structure, which satisfies the performance objective and collapse prevention for the intensity considered, has the following properties: $T_e= 1.60$ s; $\alpha_h= 0.25$; $\alpha_c= -0.025$; $d_s= 0.04$ m; $d_u= 0.36$ m; $\mu_u= 3.00$; $\mu_{cap}= 3.50$. The comparison between the results obtained from the proposed method and nonlinear dynamic analysis for the performance objective considered are shown in Figs. 5 and 6. As can be observed, the maximum displacements and storey drifts obtained from nonlinear analysis correspond well with those expected. Therefore, the performance objective considered is fully satisfied, corroborating the good prediction of the desired damage distribution of the ultimate limit state, i.e., strong column-weak beam with no inelastic behaviour at column bases.

The response of the structure under the extreme demand considered was evaluated via incremental dynamic analysis, IDA, (Vamvatsikos, 2002). The results obtained indicate that the structure developed the desired collapse mechanism, strong column-weak beam with yielding at column bases, and was able to remain dynamically stable. Fig. 7 depicts the displacement *vs* base shear plot up to the corresponding response of the structure for the collapse design intensity.

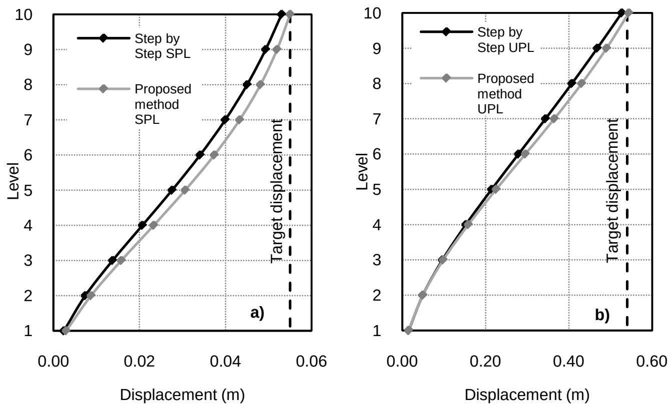


Figure 5. Comparison of displacements: a) service; b) ultimate

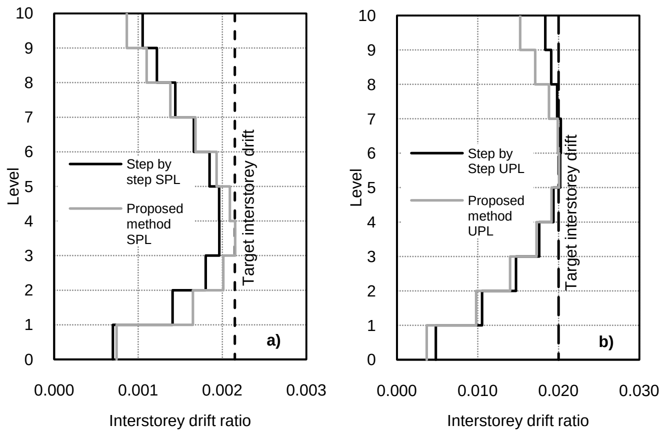


Figure 6. Comparison of interstorey drifts: a) service; b) ultimate

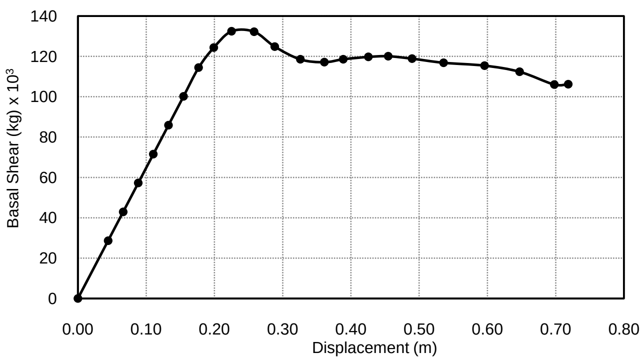


Figure 7. Capacity curve obtained from Incremental Dynamic Analysis

7. CONCLUSIONS

A proposed multilevel performance based seismic design method that includes design for prevention of incremental collapse has been presented. As can be observed from the results obtained from the design example, it provides excellent control of damage distributions and, hence, of storey drifts and displacements. For these reasons, the method guarantees the fulfilment of a performance objective and the assurance of dynamic stability for a considered intensity exceedence.

ACKNOWLEDGEMENTS
The sponsorship of the National Council of Science and Technology of the project No. 82839, "Development of conceptual, theoretical models and simplified methods for the seismic evaluation and design of structures based on performance" and the scholarship of the first author during his graduate studies are acknowledged.

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