

# An Energy Spectrum Method for Collapse Evaluation of RC Moment Frame Structures

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## SUMMARY:

This paper presents a simple collapse evaluation procedure based on the energy concept that has been recently developed and successfully used by Goel et al., for design purposes, called the Performance-Based Plastic Design (PBSD) method.

Collapse evaluation of a structure generally requires intensive time-history analysis under a set of ground motions, each scaled to multiple levels of intensity, to obtain the collapse margin ratio (CMR). A number of methods have been used in current practice, such as incremental dynamic analysis (IDA) and collapse evaluation by adjusted collapse margin ratios (ACMR) as proposed in FEMA P695. In the PBSD method, the design base shear for selected hazard level is determined by equating the work needed to push the structure monotonically up to a target drift to the corresponding energy demand of an equivalent SDOF oscillator. It turns out that the same work-energy equation can also be used to estimate the collapse capacity of a given structure without the need for carrying out cumbersome and time-consuming incremental dynamic analysis.

In this approach the skeleton force-displacement (capacity) plot of the structure, such as that obtained from an inelastic pushover analysis, is converted into an energy-displacement plot ( $E_c$ ) which is superimposed over the energy demand plot ( $E_d$ ) for the specified ground motion. For collapse evaluation purpose, i.e., determination of CMR, the energy demand plot is scaled up to the point defining collapse limit on the  $E_c$  plot. The  $S_a$  value corresponding to collapse and thus CMR are then determined by equating the limiting values of  $E_d$  and  $E_c$ .

This method is applied to 4, 8, 12 and 20-story reinforced concrete moment frame structures which are adopted from FEMA P695 and redesigned by the PBSD method. The results are compared with those obtained by using IDA and ACMR. The results show that the collapse margin estimates obtained by the energy spectrum method are in good agreement with those by using IDA and ACMR. It is also noted that the CMR of PBSD frames calculated by both methods are much higher than those of the code compliant (baseline) frames.

*Keywords: Reinforced Concrete Moment Frames, Collapse Evaluation, Performance-Based Plastic Design.*

## 1. INTRODUCTION

Collapse evaluation of a structure generally requires intensive time-history analysis under a set of ground motions, each scaled to multiple levels of intensity, to obtain the collapse margin ratio (CMR). A number of methods have been used in current practice, such as incremental dynamic analysis (IDA) (Vamvatsikos and Cornell, 2002) and collapse evaluation by adjusted collapse margin ratios (ACMR) as proposed in FEMA P695 (2009). However, IDA requires a large number of inelastic analyses with representative ground motion time history data, and thus is time and resource-consuming.

This paper presents a simple collapse evaluation procedure based on the energy concept that has been recently developed by Goel et al. (2008), for design purposes, called the Performance-Based Plastic Design (PBSD) method. In the PBSD method, the design base shear for selected hazard level is determined by equating the work needed to push the structure monotonically up to a target drift to the corresponding energy demand of an equivalent ED-SDOF oscillator. The same work-energy equation can also be used to estimate the collapse capacity of a given structure without the need for carrying out

cumbersome and time-consuming incremental dynamic analysis.

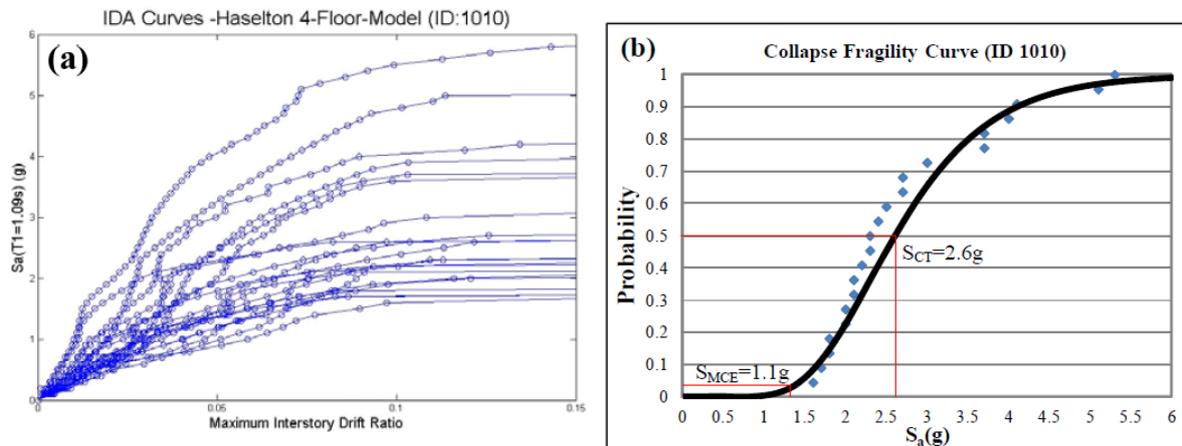
After a brief description of the proposed energy spectrum method, its application to 4, 8, 12 and 20-story RC special moment frame structures which are adopted from FEMA P695 and redesigned by the PBPD method is presented. The results are compared with those obtained from IDA. Evaluation of RC structures presents special challenge due to their complex and degrading (“pinched”) hysteretic behavior. This aspect is taken care of by making appropriate modification in constructing the energy demand curve,  $E_d$ . The results show that the collapse margin estimates obtained by the energy spectrum method are in good agreement with those by using IDA. This can be considered as a very good correlation between the results given by an approximate method with those from more intensive and precise time-history analysis.

## 2. COLLAPSE SAFETY ASSESSMENT IN FEMA P695

In FEMA P695, the collapse capacity of each RC special moment frame model is evaluated by nonlinear time-history analyses with 44 prescribed ground motions whose amplitudes are scaled to reflect specified intensities. In terms of the acceptance criterion of collapse safety assessment, it is expressed as a collapse margin ratio (CMR), which is the ratio of the median value of the collapse capacity to the intensity of the maximum considered earthquake (MCE). The CMR is calculated by:

$$CMR = \frac{S_{a-\text{median collapse capacity}}}{S_{a-MCE}} \quad (1)$$

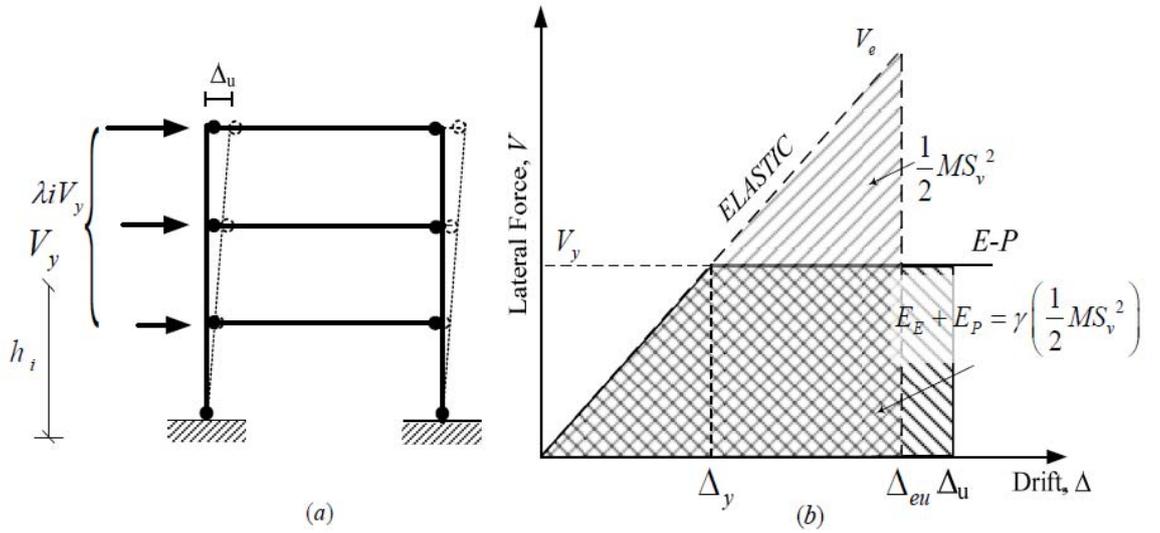
For determination of median value of the collapse capacity, the incremental dynamic analysis (IDA) is adopted in FEMA P695. IDA is a technique to systematically process the effects of increasing earthquake ground motion intensity on structural response up to collapse. As shown in Figure 1 (a) for a 4-story RC SMF, each curve presents the response of this structure to a ground motion whose intensity increased until collapse and each point on the curve presents the time-history analysis result recorded with the spectral intensity and peak interstory drift. Furthermore, the collapse fragility curve can be plotted by converting these IDA statistics with cumulative lognormal distribution function as shown in Figure 1 (b). Thus, the CMR can be obtained as the ratio of the median collapse intensity ( $S_{CT} = 2.6g$ ) to the MCE intensity of  $S_a = 1.1g$ , which is equal to 2.36. It is worth noting that modelling uncertainties and spectral shape effect are not considered in CMR. Therefore, adjusted collapse margin ratio (ACMR) is proposed in FEMA P695 to account for these effects.



**Figure 1.** Incremental dynamic analysis (a) IDA plot of  $S_a$  versus maximum interstory drift, (b) collapse fragility curve

### 3. ENERGY BALANCE CONCEPT IN PERFORMANCE-BASED PLASTIC DESIGN

Performance-Based Plastic Design (PBPD) method, which accounts for inelastic structural behavior directly, and practically requires no or little iteration after initial design, has been developed by Goel et al. By using the concept of energy balance applied to a pre-selected yield mechanism with proper strength and ductility, structures designed by the PBPD method can achieve more predictable performance under strong earthquake ground motions. It is important to select a desirable yield mechanism and target drift as key performance limit states for given hazard levels right from the beginning of the design process. The distribution and degree of structural damage are greatly dependent on these two limit states. Determination of the design base shear for given hazard level is a key element in the PBPD method (Goel and Chao, 2008). It is calculated by equating the work needed to push the structure monotonically up to the target drift to that required by an equivalent elastic-plastic single degree of freedom (EP-SDOF) system to achieve the same state, Figure 2.



**Figure 2.** Energy balance concept in PBPD (Lee and Goel, 2001).

Assuming an idealized E-P force-deformation behavior of the system, the work-energy equation can be written as:

$$(E_e + E_p) = \gamma \cdot \left( \frac{1}{2} M \cdot S_v^2 \right) = \frac{1}{2} \gamma \cdot M \cdot \left( \frac{T}{2\pi} S_a \cdot g \right)^2 \quad (2)$$

where  $E_e$  and  $E_p$  are, respectively, the elastic and plastic components of the energy needed to push the structure up to the target drift,  $S_v$  is the design pseudo-spectral velocity,  $M$  is the total seismic mass of the system,  $\gamma$  is the energy factor (Lee and Goel, 2001), and  $T$  is the fundamental period. The energy factor is defined as the ratio of the energy required by the inelastic system to that of the equivalent elastic system and is given by:

$$\gamma = \frac{2\mu - 1}{R_y^2} \quad (3)$$

where  $\mu$  is the ductility ratio and  $R_y$  is the yield force reduction factor. The energy factor can be computed for a given ductility level using a suitable  $R_y$ - $\mu$ - $T$  relationship such as the one developed by

Newmark and Hall (1982). For seismic design purposes, a target ductility level can be selected and the energy factor can be computed.

Solution of the work-energy equation gives the required design base shear,  $V_y$ , as:

$$\frac{V_y}{W} = \frac{-\alpha + \sqrt{\alpha^2 + 4\gamma C_e^2}}{2} \quad (4)$$

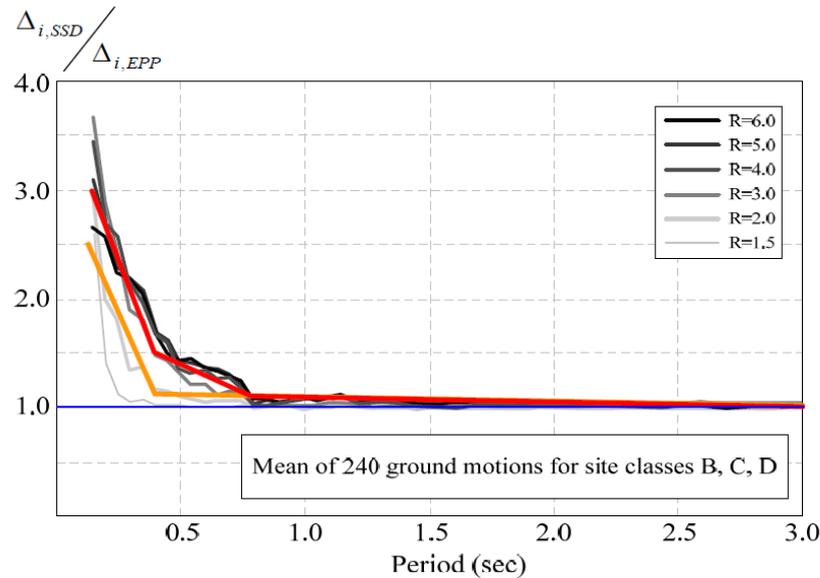
where  $W$  is the seismic weight of the structure,  $C_e$  is normalized design pseudo acceleration ( $S_a/g$ ) and  $\alpha$  is a parameter given by:

$$\alpha = \left( \sum_{j=1}^n \lambda_j h_j \right) \frac{\theta_p 8\pi^2}{T^2 g} \quad (5)$$

$\theta_p$  is the target plastic story drift, and  $h_i$  is the height from the ground to floor level  $i$ , and  $\lambda_i$  is the lateral force distribution factor.

#### 4. MODIFICATION FOR RC STRUCTURE IN PBDP METHOD

RC structures do not possess full EP hysteretic property but degrading strength and stiffness (pinched) behavior instead. Thus, the following modification has been applied which shows good promise. This modification is based on consideration of the effect of degrading hysteretic behavior on peak displacement. Investigators have studied the effect of degrading hysteretic behavior of SDOF systems on resulting peak displacements. Approximate expressions have been proposed for modification factors to account for this effect, e.g., factor  $C_2$  in FEMA 440 (2008), Figure 3.



**Figure 3.** Mean displacement ratio of SSD to EPP models ( $C_2$ ) computed with ground motions recorded on site classes B, C, and D. (FEMA 440, 2006)

## 5. VERIFICATION OF ENERGY BALANCE CONCEPT IN TERMS OF DESIGN BASE SHEAR

The validity of estimating design base shear based on PBPD energy balance concept can be further verified by the static pushover analysis. Liao et al. (2010) applied the PBPD approach to redesign four baseline RC SMF (4, 8, 12 and 20-story) as used in the FEMA P695. The pushover curves for the 20-story baseline and PBPD frames in Figure 4 show that, even though the design base shear for the baseline code compliant frame is smaller than that of the PBPD frame by 41%, the ultimate strength of these two are almost equal. That is mainly due to the fact that the design of the baseline frame was governed by drift which required major revision of the member sizes after having been designed for required strength. That iteration step is not needed in the PBPD method because it accounts for inelastic structural behavior and target drift directly, and practically eliminates the need for assessment or iteration after initial design. It also shows the accuracy of estimation of design base shear by using PBPD energy balance concept.

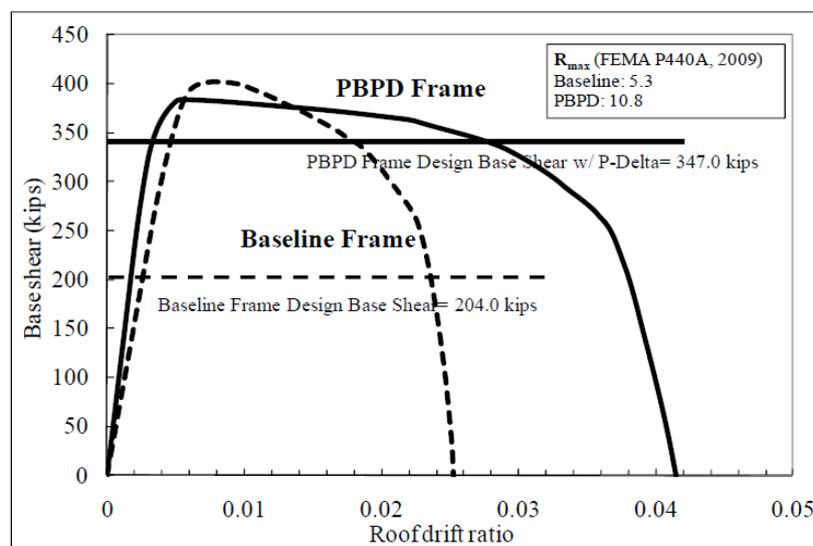


Figure 4. Pushover curves for 20-story baseline and PBPD frames

## 6. COLLAPSE EVALUATION BASED ON ENERGY BALANCE CONCEPT

In the previous section the energy-based PBPD method was presented and discussed in the context of design of new structures for a target maximum drift. Therefore, with other terms being known, the design base shear is determined by solving the work-energy Equation (2). It turns out that the same energy equation can also be used for evaluation purposes, where the structure is defined, including its force-displacement characteristics, and the goal is to estimate the collapse margin ratio for a given structure. Similar concept has been successfully applied to seismic performance evaluation to predict the expected maximum displacements for a given seismic hazard (Leelataviwat et al., 2007).

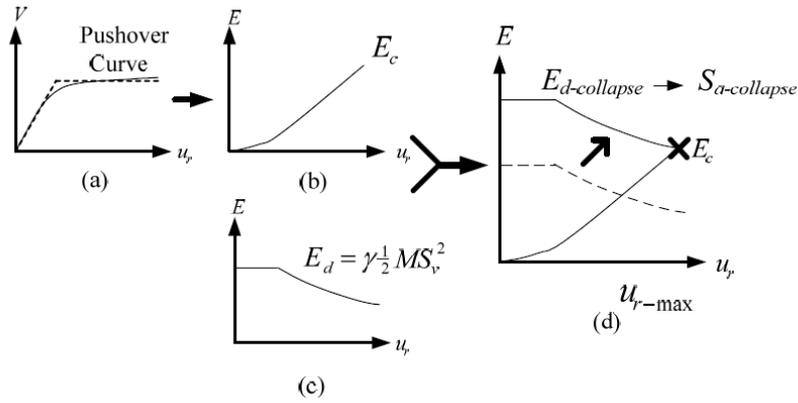
Static pushover method has been widely accepted as a useful tool for performance-based seismic design and evaluation of structures (FEMA 440, 2008). Since its introduction to the engineering community, the pushover analysis method has been a subject of extensive research and several new approaches have been proposed. Recent notable modifications include adaptive load patterns and multiple modal analysis procedures. In most cases, the behavior of the structure is characterized by the capacity curve which is represented by a plot of the base shear versus the roof displacement. The capacity curve is used to establish an equivalent SDOF system.

In order to use the energy concept for evaluation purposes, the right hand side of Equation (2) can be

viewed as energy demand for the given seismic hazard,  $E_d$ , and the left hand side as energy capacity of the given structure,  $E_c$ . Both these quantities vary with displacement.

The value of  $S_a$  corresponding to collapse limit can be obtained by either solving the work-energy equation analytically, or graphically by constructing the two energy curves as a function of the reference displacement and determining their intersection at the end point of  $E_c$ .

Figure 5 presents a graphical illustration of the evaluation process. Lateral force-displacement plot for the given structure is shown in Figure 5(a), where  $V$  represents the total force (base shear), and  $u_r$  the roof displacement, used as reference displacement. This plot can be obtained by a static pushover analysis by applying either an appropriately selected force or displacement pattern. It is common to plot total force versus roof displacement, but it can be done for any other floor or story level from which the force or displacement at other levels can be determined. The energy capacity curve,  $E_c - u_r$ , can be generated as a function of  $u_r$ , by calculating the work done by lateral forces up to the displacement at each level corresponding to  $u_r$ , Figure 6(b). Next, the energy demand,  $E_d$ , can be calculated for varying values of  $u_r$  for different hazard levels and plotted as shown in Figure 5(c). By scaling up the  $E_d$  to the intersection with the limit of  $E_c$ , where the energy demand and capacity become equal at collapse limit, gives the corresponding  $S_{a-collapse}$ , as shown in Figure 5(d). Therefore, CMR can be easily obtained by dividing  $S_{a-collapse}$  by  $S_{a-MCE}$ .



**Figure 5.** Proposed energy-based evaluation method for MDOF systems: (a) Push-over curve, (b) Energy-displacement capacity diagram, (c) Energy demand diagram, and (d) Determination of  $S_{a-collapse}$

As mentioned earlier, the modification in PBPB for RC structures is necessary due to stiffness and strength degrading hysteretic behavior. This aspect is taken care of by making appropriate modification in constructing the energy demand curve,  $E_d$ . As described in Section 3,  $C_2$  factor method was implemented in Equation (2) for modification of design target drift for an equivalent non-degrading system. Thus, the energy demand for the given hazard for a RC structure,  $E_d$ , can be expressed as shown in Equation (6).

$$E_d = \frac{1}{2} \gamma^* \cdot M \cdot \left( \frac{T}{2\pi} S_a \cdot g \right)^2 = \frac{1}{2} \cdot \frac{2 \frac{u_r}{u_y} - 1}{(R_\mu^*)^2} \cdot M \cdot \left( \frac{T}{2\pi} S_a \cdot g \right)^2 \quad (6)$$

## 7. EXAMPLES

This energy spectrum method is applied to 4, 8, 12 and 20-story RC SMF structures which are adopted from FEMA P695 and redesigned by the PBPB method. Calculated values of CMR by using energy

spectrum method for the baseline and PBPD frames are listed in Table 1, which reflect much enhanced margin against dynamic instability (collapse) of PBPD frames over the baseline frames. CMR obtained by IDA and ACMR of the baseline frames are also summarized in Table 1. The results show that the collapse margin estimates obtained by the energy spectrum method are in good agreement with those by using IDA and ACMR. It is also noted that the CMR of PBPD frames are much higher than those of the baseline frames in both methods.

**Table 1.** CMR and ACMR of PBPD and baseline frames with energy spectrum method and IDA

	<b>PBPD frame</b>	<b>Baseline frame</b>		
	CMR by <b>Energy spectrum method</b>	CMR by <b>Energy spectrum method</b>	CMR by <b>IDA (FEMA P695)</b>	ACMR (FEMA P695)
4-story	3.82	3.33	2.36	3.53
8-story	4.49	2.11	1.63	2.58
12-story	3.21	1.03	1.59	2.54
20-story	4.06	1.86	1.98	2.96

## 8. SUMMARY AND CONCLUSIONS

The PBPD method is a direct design method which uses pre-selected target drift and yield mechanism as key performance objectives, which determine the degree and distribution of expected structural damage. The design base shear for a specified hazard level is calculated by equating the work needed to push the structure monotonically up to the target drift to the energy required by an equivalent EP-SDOF to achieve the same state.

It was shown in this brief paper that the basic work-energy equation in the PBPD method can also be easily used for collapse evaluation purposes where the goal is to determine the spectral intensity  $S_a$  at collapse for a given structure. Collapse margin ratio can be quickly obtained by dividing  $S_{a-collapse}$  by  $S_{a-MCE}$  without the need for carrying out cumbersome and time-consuming incremental dynamic analysis. The results as presented in this paper showed excellent agreement with those obtained from more elaborate inelastic time-history analyses, such as IDA and ACMR. The results also show much enhanced margin against dynamic instability (collapse) of PBPD frames over the baseline frames.

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