Dam-Reservoir interaction analysis based on Scaled Boundary Finite Method and Finite Element Method

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SUMMARY:
This paper proposes and validates a new formulation on the dynamic response of dam-reservoir system, including water compressibility, absorption of the boundary of the reservoir, the earthquake excitation from different directions and the radiation condition boundary at infinity. The impounded water is modeled by Scaled Boundary Finite Element Method, and discretization only at the boundary coincide with the dam face is needed, which is less than the Boundary Element Method, in addition no fundamental solution required. For the dam body, the tradition approach called Finite Element Method is employed. Coupled with the dam and the impounded water by the same displacements at the dam face, dynamic response of the dam-reservoir system including dam flexibility can be evaluated. To verify the newly developed method, the numerical examples including gravity dams are employed. In particular, the characteristics of the dam flexibility on the dynamic response of the dam-reservoir system are investigated.

Keywords: dynamic response; water compressibility; dam flexibility; boundary absorption; gravity dam

1. INTRODUCTION

Dam reservoir interaction has been receiving significant attention all the time because the safety of dams during earthquakes is extremely important for life and property at the downstream. To analysis the dam-reservoir systems, the problem of hydrodynamic pressures generated on the upstream face of the dam must be determined. It is known that the excellent piece of Westergaard's work [1] in 1933 on analysis of hydrodynamic pressures was the earliest study. In Westergaard's study, the structure was assumed to be a straight rigid dam with a vertical upstream face, and the vibrations in the earthquake were assumed horizontal in a direction perpendicular to the dam face, besides, the compressibility of the water was also neglected. In fact, even today, dam design in many countries utilizes these or similar results. Continuing with the assumption that the gravity dam is rigid, Chopra [2] has been given the more complete and comprehensive analyses of hydrodynamic pressure on the dam face due to horizontal as well as vertical components of ground motion, and the water compressibility was also considered. With Finite Element Method (FEM) developing, numerical method is to simulate the dam-reservoir systems. In the framework of the FEM [3-7], the hydrodynamic pressures are computed by different kinds of numerical methods, and it also can be considered the water compressibility and the absorption of the reservoir bottom. The numerical methods are effective not only for the gravity dams, but also for the arch dams.

For all the numerical methods of the FEM, they are necessary to discretize the whole or the part of the dam-reservoir systems. As the infinity of the reservoir, it is necessary to discretize the reservoir as long as possible for the finite element method analysis. As a result, the computational efforts and the memory of the computer are required much. To overcome this defaults, the truncation boundary of the reservoir is assumed, and it must be satisfied the radiation condition at the boundary to keep the energy dissipation. The pioneer study is by Sommerfeld in 1949, and then Sharan[8] developed the Sommerfeld boundary condition, he proposed a damper technique to model the effects of radiation damping in the finite element analysis of hydrodynamic pressures, and Sharan's boundary condition boundary is found to be very effective and efficient. It also can be selected the truncation boundary at a relatively very short distance even half of the dam height away from the structure, which can greatly reduce the cost of computation. In spite of the proposed boundary conditions seem efficient, the fluid domain is to be discretized, the computational effort is also very large for the 300m graduate concrete dams. As a result, the boundary element method is applied in the dam-reservoir systems to calculate the hydrodynamic pressures, it discretizes the boundary of the domain only, and this can greatly reduce the computation. Human et al [9] applied the boundary element to compute the hydrodynamic pressures subjected to a harmonic ground motion, and it was also successful in dealing with cases of the shape of infinite reservoirs and modelling the energy loss in the waves moving to the infinity in the truncation boundary. Unfortunately, the boundary element method is only effective for simple shapes of the reservoir because the complex fundamental solutions are difficult to get.

In recent studies, Bounanani et al [10] derived hydrodynamic pressures generated on the rigid gravity dam face in the frequency domain, and it was considered the flexibility of the reservoir bottom. After that, Bounanani [11] gave the hydrodynamic pressure considered the flexibility of the dam by applying model analysis method, which was in the finite element method framework, the results were also included the effects of water compressibility and the flexibility of the reservoir bottom. To conclude, they were obtained the relative complete results in the frequency domain.

As numerical methods developing, a semi-analytical method called Scaled Boundary Finite Element Method (SBFEM) comes about. SBFEM was originally brought forward by Wolf and Song [12] in 2000 and had a lot of successful applications in numerical simulation in several fields, such as structural dynamics, soil-structure interaction [13] [14] and fracture modelling in concrete structures.

This paper proposes a novel approach based on SBFEM to calculate the hydrodynamic pressures which can conveniently simulate water compressibility, absorption of reservoir sediments and the flexibility of the dam. In addition, it's worthy mentioning that the discretization is required merely at the boundary of the fluid domain, and the radiation condition is satisfied automatically at infinity, both contribute to great reduction in computational efforts. Besides, it is also no foundation solution required and can be effective for the all kinds of the reservoir shape.

2. DAM-RESERVOIR SYSTEMS

2.1. Basic formulas of the dam body

For the dam-reservoir systems subjected to the ground motion and considered the hydrodynamic force on the upstream, the displacement is governed by the motion equation based on finite element method in the time domain

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=-[M]\{\ddot{u}_g\}+\{F_p(u,u_g)\} \tag{2.1}$$

where $[M]$, $[C]$ and $[K]$ are the mass matrix, damping matrix and stiffness matrix, respectively. $\{u\}$, $\{\dot{u}\}$ and $\{\ddot{u}\}$ denote the displacement, velocity and acceleration of the nodes on the dam, respectively. It should be noted that, in the right hand of the equation, $\{\ddot{u}_g\}$ is the acceleration of the ground motion, $\{F_p(u,u_g)\}$ is the hydrodynamic force generated on the upstream of the dam.

In order to solve Equation(2.1), the interaction between dam body and the impounded water in the reservoir called the hydrodynamic force must be obtained. For the rigid dam, there is no relative displacement in the dam body, so the hydrodynamic pressures are only depended on the frequency of the ground motion; but for the flexibility dam, hydrodynamic pressures are related to the motion of the dam body, therefore, hydrodynamic pressures are coupled with the response of the structure. How to calculate the coupled hydrodynamic pressures is the key to solve the response of the system, and the detail is in the following sections.

2.2. Dam-water interaction

2.2.1. Assumptions

To calculate the hydrodynamic pressures, it is necessary to have the following assumptions, many of which have been discussed in the references.

1. The reservoir bottom is horizontal, and the depth of the reservoir is keeping a given value, the water extends to infinity in the upstream direction, i.e. the bottom and the surface is parallel.
2. The gravity dam is flexibility and the upstream face of the dam is vertical (see **Figure 1**).
3. The effects of waves at the free surface of the reservoir are ignored.
4. The excited ground motion is assumed to reach all nodes of the base at the same time.
5. The motion of the system is considered as two dimensional.
6. Water is assumed to an inviscid and linear fluid domain.

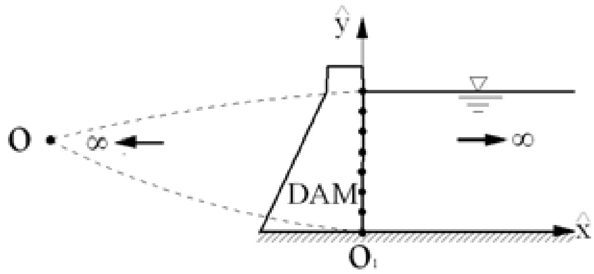


Figure 1. The sketch map of the gravity dam-reservoir systems

2.2.2. Governing equation and the boundary condition equations of the water

With the assumptions list above, the hydrodynamic pressures are governed by the wave equation called Helmeholze equation.

$$\nabla^2 p - \frac{1}{c^2}\ddot{p} = 0 \tag{2.1}$$

where ∇^2 is the Laplacian operator, p is the hydrodynamic pressure, dots denote derivatives with respect to time, c is the velocity of the compression waves. It is mentioned that c will to be infinity when the compressibility of the water is ignored, and Equation (2.1) is to be Laplace equation, so the hydrodynamic pressures are in depended of the frequency.

To solve the Equation (2.1) with numerical method, it is necessary to discretize the domain of the dam-reservoir system. For the Scaled Boundary Finite Element Method, the scaling centre is selected at the infinity of the downstream direction, because the reservoir bottom is parallel to the surface, the interface is only to be discretized and no foundation solution required, which can be greatly decreased the computational efforts.

Besides, the solution of the Equation (2.1) should be satisfied the boundary conditions as following, at the dam surface

$$p_{,n} = -\rho \ddot{u}_n \tag{2.2}$$

where ρ denotes the density of the water, and $\ddot{u}_n$ is the normal acceleration of the dam surface. The subscribe n of p is the outward normal derivative.

At the bottom, considering the absorption of the bottom by Fenves and Chopra[5],

$$p_{,n} = -\rho \ddot{v}_n - q\dot{p} \tag{2.3}$$

in which $\ddot{v}_n$ is the normal acceleration of the bottom, and q is the coefficient of the absorption, it can be determined by the following equation

$$q = \frac{1}{c}\frac{1-\alpha}{1+\alpha} \tag{2.4}$$

where α is the reflecting coefficient of the reservoir sediment, and its range is from 0 to 1. When α is 0, it denotes the perfect reflecting bottom condition while it is the perfect absorption bottom condition when α is 1.

At the infinity of the upstream, as mentioned in above, it is can be automatically satisfied the radiation condition by SBFEM.

By employing the weighted residual method on the governing equation and the boundary condition equations above, the hydrodynamic pressures in the frequency domain based on the SBFEM can be obtained, the solution can be expressed as Equation(2.5), and the detail is in the reference [15].

$$\{\overline{F}_p(\omega)\} = -[A][M^a(\omega)](\{\overline{\ddot{u}}_g(\omega)\} + \{\overline{\ddot{u}}(\omega)\}) \tag{2.5}$$

where $[A]$ is the conversion matrix of area, $\{\overline{\ddot{u}}_g(\omega)\}$ is the acceleration of the ground in the frequency, $\{\overline{\ddot{u}}(\omega)\}$ is the relative acceleration in the dam body subjected the ground motion, and $[M^a(\omega)]$ is the characteristic matrix based on SBFEM, $\{\overline{\ddot{u}}_g(\omega)\}$, $\{\overline{\ddot{u}}(\omega)\}$, and $[M^a(\omega)]$ are all depend on the frequencies in the frequency domain.

2.3. The couple of the dam-reservoir system

Substituting Equation (2.5) into Equation(2.1), the motion equation of the dam-reservoir system is obtained in the frequency domain, and it can be expressed in Equation(2.6).

$$(-\omega^2([M]+[M^b(\omega)]) + (1+i\eta)[K])\{\overline{u}(\omega)\} = -([M]+[M^b(\omega)])\{\overline{\ddot{u}}_g(\omega)\} \tag{2.6}$$

$$[M^b(\omega)] = [A][M^a(\omega)] \tag{2.7}$$

where η is the damping ration of the complex stiffness, and i is the unit imaginary number, $[M^b(\omega)]$ is the matrix of the added mass. Equation (2.6) clearly expresses the uncoupled motion governing equation of the dam-reservoir interaction in the frequency domain. Obviously, For Equation(2.6), only the matrix of $[M^b(\omega)]$ is unknown. But it can be obtained from the hydrodynamic pressures gained by the SBFEM approach and expressed as Equation(2.7). From Equation(2.6), the dynamic response of the dam including the hydrodynamic effects can be obtained. Substituting Equation (2.7) into Equation(2.6), the hydrodynamic pressures considering the flexibility of the dam are solved.

3. NUMERICAL EXAMPLES

In order to exam the effective of the proposed method, the Pine Flat gravity dam is selected, the dam-reservoir system is assumed to be a two-dimensional system. The dam is of 121.92m height, the shape of the cross-section is a triangle with the slope of 1:0.8 of the downstream, and the upstream of the dam is vertical. **Figure 2** gives the FEM model of the Pine Flat dam, it is meshed by 192 4-node plane strain elements. A dam Poisson’s ratio 0.2 and mass density 2400 kg/m^3 are adopted. To assess the influence of dam stiffness, the modulus of elasticity 35GPa is considered. A constant structural hysteretic damping ration 0.1 is selected. The full reservoir is assumed, and the impounded water is assumed compressible, with a velocity of pressure waves 1440m/s and a mass density 1000kg/m^3. The earthquake exciting is in the direction of the stream. To assess the flexibility of the reservoir bottom, the reflecting coefficient is selected as 1.0, 0.6, 0.2. For the sake of contrast, **Figure 3** and **Figure 5** give the results of the rigid dam. In **Figure 3** and **Figure 4**, the X-coordinate is the dimensionless frequency ($\Omega = \omega / (\pi c / (2H))$), H is the depth of the reservoir; the Y-coordinate is the hydrodynamic pressure at the dam heel in the frequency, respectively.

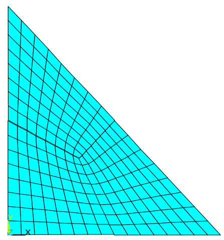
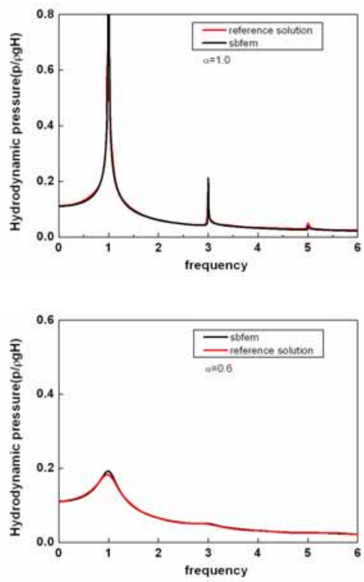
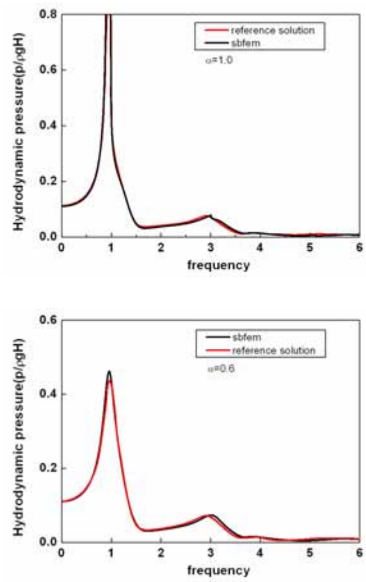
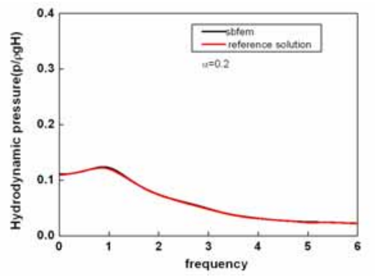


Figure 2. Pine Flat gravity dam[15]

From the results of the flexible dam (**Figure 4** and **Figure 6**), the solution by the present approach is good agreement with the reference [11] solution, it is clearly demonstrate the proposed approach is effective to solve the dam-reservoir interaction in the frequency domain. For the rigid dam (**Figure 3** and **Figure 5**), not only the frequency response functions but also the heigtwise distribution of the normalized hydrodynamic pressures are excellent agree with the closed form solution in reference [10]. In this example, the semi-unbounded reservoir is only discretized as 17 nodes on the interface by SBFEM approach compared with the FEM discretized the whole of the fluid domain and the BEM discretized the whole boundary of the fluid domain, it greatly decreases the computational efforts and the memory of the computer and it also keep the high computational accuracy.







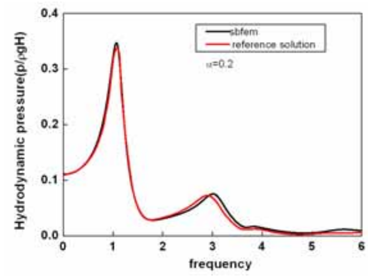
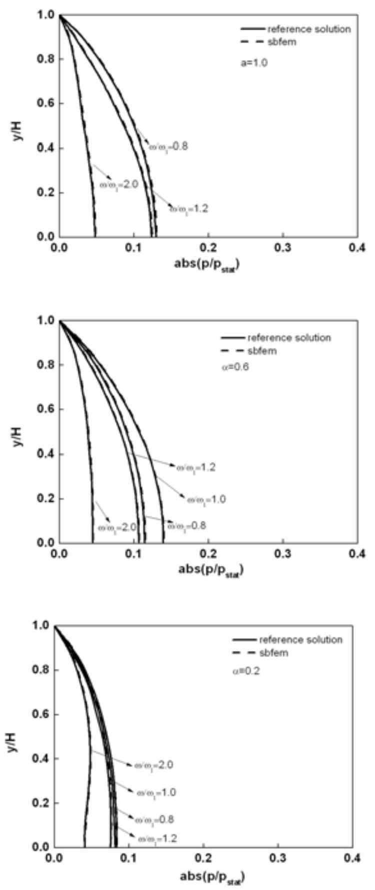


Figure 3. Frequency response functions of rigid dam for normalized hydrodynamic pressures

Figure 4. Frequency response functions of flexible dam for normalized hydrodynamic pressures



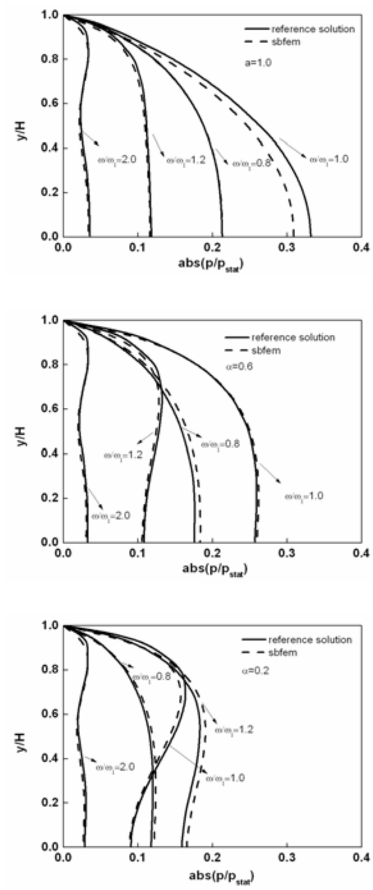


Figure 5. Heightwise distribution of normalized hydrodynamic pressure on rigid dam upstream

Figure 6. Heightwise distribution of normalized hydrodynamic pressure on flexible dam upstream

The results show the compressibility of the water is very important for the hydrodynamic pressures. Ignoring the compressibility, hydrodynamic pressures are independent of the frequency of the excitation and the response curve is a straight line. Obviously, this can be decreased the hydrodynamic pressures during the low frequencies range, but in the high frequencies range, it is opposite. The previous investigations have not concluded the compressibility of the water is benefit or not of the response of the dam, but it is very important to consider its effects from the all the results.

From the contrast of the frequency response curves in **Figure 3** and **Figure 4**, the flexibility of the dam affects dramatically the hydrodynamic pressures. Because of the flexibility of the dam, the hydrodynamic pressures have had the remarkable increase during the first two resonant frequencies range (the resonant frequency of the reservoir is determined by $\omega_n = (2n-1)\pi c/(2H)$). Very clearly, the dam body has resonance nearby the low resonant frequency, but during the high frequencies range, there is acceleration in the dam body which is in the opposite direction of the exciting, and this can decrease the effects of the dam flexibility.

The frequency response curves (**Figure 3** and **Figure 4**) and the results of the heightwise distribution (**Figure 5** and **Figure 6**) can be concluded that the absorption of the reservoir bottom affects the hydrodynamic pressures generated on rigid dam surface or on flexible dam surface. With the reflecting coefficient decreasing, the hydrodynamic pressures are also decreasing. As a result, the absorption of the reservoir bottom is important of the dynamic interaction of the dam-reservoir system, and how to reasonably determine the reflecting coefficient can give better results of the dam-reservoir system subjected to the earthquakes.

Figure 5 and **Figure 6** shows that the flexibility of the dam affects the distribution of the hydrodynamic pressures on the dam face. Its effects are increasing with the reflecting coefficient of the reservoir decreasing, particularly during the high frequencies range. The flexibility of the dam changes the distribution shape of the hydrodynamic pressure on the rigid dam face. For the rigid dam, the shape is a parabola; but for flexible dam, the shape has been changed to another and the peak value of the hydrodynamic pressure moves towards to the upper of the dam with the frequency increasing.

4. CONCLUSIONS

A novel approach has been presented to solve the dam-reservoir dynamic interaction in the frequency domain. It is based on the couple with FEM and SBFEM. For the reservoir domain, the SBFEM can only discretize the interface of the dam-reservoir which can greatly decrease the computational efforts, and the results have the high precision. For the dam body, the finite element method is employed, and this can couple with the hydrodynamic effects from the fluid domain in the frequency domain.

The compressibility of the water and the absorption of the reservoir bottom affect the hydrodynamic pressures as the previously investigations. The flexibility of the dam has been considered into calculating the hydrodynamic pressures, the results show that the effect is depend on the frequency of the exciting. In the range of low frequency, it can increase the response of the reservoir but it is opposite in the domain of high frequency. Besides, the flexibility of the dam can change the distribution of the hydrodynamic pressures on the dam surface.

This paper only gives the results of the two dimensional gravity dam-reservoir system in the frequency domain, it is possible to model the three dimensional arch dams both in the frequency domain and in the time domain in the future.

REFERENCES

[1] Westergaard H.M. (1933). Water pressures on dam during earthquake. *Transactions* (ASCE) **98**:418-433.
[2] Chopra A.K. (1967) Hydrodynamic pressures on dams during earthquakes. *Journal of the Engineering Mechanics Division* (ASCE) **EM-6:93**,205-223.
[3] Chopra A.K. and Chakrabarti P. (1981). Earthquake analysis of concrete gravity dams including dam-fluid-foundation rock interaction. *Earthquake Engineering and Structural Dynamics* **9**,363-383.
[4] Hall J.F. and Chopra A.K. (1983).Dynamic analysis of arch dams including hydrodynamic effects. *Journal of Engineering Mechanics* (ASCE) **109**,149-167.
[5] Fenves G. and Chopra A.K. (1984). Earthquake analysis of concrete gravity dams including reservoir bottom absorption and dam-water-foundation reservoir interaction. *Earthquake Engineering and Structural Dynamics* **12**,663-680.
[6] Fok K.L. and Chopra A.K. (1986). Earthquake analysis of arch dams including dam-water interaction, reservoir boundary absorption and foundation flexibility . *Earthquake Engineering and Structural Dynamics* **14**,155-184.

[7] Fok K.L. and Chopra A.K. (1986). Frequency response functions for arch dams: hydrodynamic and foundation flexibility effects. *Earthquake Engineering and Structural Dynamics* **14**,769-794.
[8] Sharan S.K. (1987). Time domain analysis of infinite fluid vibration. *International Journal for Numerical Method in Engineering* **24**,945-958.
[9] Hanna Y.G. and Humar J.L. (1982). Boundary element analysis of fluid domain. *Journal of Engineering Mechanics* (ASCE) **108**,436-450.
[10] Bouaanani N., Paultre P. and Proulx J. (2003). A closed-form formulation for earthquake-induced hydrodynamic pressure on gravity dams. *Journal of Sound and Vibration* **261**,573-582.
[11] Bouaanani N. and Miquel B. (2010). A new formulation and error analysis for vibrating dam-reservoir systems with upstream transmitting boundary conditions. *Journal of Sound and Vibration* **329**,1924-1953.
[12] Wolf J.P. and Song C. (2000). The scaled boundary finite-element method- a premier: derivation. *Computers and Structures* **28**,191-210.
[13] Lin G., Du J. and Hu Z. (2007). Dynamic dam-reservoir interaction analysis including effect of reservoir boundary absorption. *Science in China Series E. Tech. Sci.* **50 Supp.1**,1-10.
[14] Wang X., Jin F., Prempramote S. and Song C. (2011). Time-domain analysis of gravity dam-reservoir interaction using high-order doubly asymptotic open boundary. *Computers and Structures* **89**,668-683.
[15] Gao Lin, Yi Wang, Zhiqiang Hu. An efficient approach for frequency-domain and time-domain hydrodynamic analysis of dam-reservoir systems, *Earthquake Engineering and Structure Dynamics* 2012: DOI:10.1002/eqe.2154.