Choice of in-structure Damping Model: Do we have an Answer?

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SUMMARY:
Recent researches have shown that the optimal distribution of dampers is sensitive to the choice of the in-structure damping models. Common practice is to use the classical viscous damping model originated by Rayleigh, through his famous 'Rayleigh dissipation function'. The main advantage of this model is that the orthogonality of the modes is preserved; thereby rendering the classical modal analysis for undamped vibration readily applicable to damped vibration as well. In a controlled frame, addition of external dampers makes the damping non-classical and the orthogonality of modes no longer exists. So use of the classical in-structure damping model (Rayleigh model) for controlled frames is not convincing and no justification is provided in the literature for the choice of this damping model. In this paper, the effect of choice of the damping models on the optimal distribution of dampers is investigated. It is observed that the optimal distribution of dampers could change based on the choice of the damping models. The results raise a huge concern regarding the realism of the optimality criterion achieved in terms of response reduction when a particular damping model is assumed with no specific justification.

Keywords: In-structure Damping, Non-classical Viscous, Classical viscous, Dampers

1. INTRODUCTION

Conventional capacity design strategy relies on the "evasion" of seismic forces by enduring inelastic deformations. This philosophy could also be observed as "dissipation with degradation" as seismic energy is dissipated by inelastic deformation. Due to the reliance of this philosophy on inelastic deformations, this incurs heavy damages to the parent structure making it non-functional mostly after a major seismic event. So in order to reduce damage, from a dynamic perspective, a more rational approach would be to rely on "dissipation without degradation" rather than "evasion/dissipation" of seismic forces by degradation. One way to achieve this is by increasing the amount of damping in the system by adding dissipation dampers. So the resultant net damping in the system would be a combination of the inherent in-structure damping in the system (mainly due to the material or structural damping) and damping due to added dampers. This net damping would be responsible for the reduction of unwanted response during a seismic event.

Earlier studies have shown that in order to achieve a reliable performance an optimal distribution of this added damping devices is required (Takewaki 2009). More recently, Takewaki (2009) showed that the optimal distribution process of this added devices also depends on the inherent in-structure damping in the system. The sensitivity study presented illustrated the fact that the distribution of the capacities of these added dampers change with the extent of in-structure damping. In analytical terms this means, if the in-structure damping model fails to capture the realistic damping in the system, then what seems optimal in analysis might not be optimal in reality. This aspect calls for the most significant question "what is the most realistic in-structure damping model?" There is no single answer for this particular question as there is no single universally accepted model for damping (Woodhouse 1998). This non-acceptance could be due to the fact that, to date the *state variables* controlling the damping force is only known in an ad-hoc phenomenological manner (Adhikari 2000). Common practice is to use the classical viscous damping model originated by Rayleigh, through his famous 'Rayleigh dissipation function', in which only the instantaneous velocities are considered as the relevant *state variables* and on employing Taylor's expansion results in a model which captures the damping through the formation of a 'dissipation matrix' (Adhikari 2000). Other than mathematical convenience, the adoption of the choice of viscous damping model has no relevant explanation in the literature.

Focusing on the uncertainty prevalent in the choice of the in-structure damping model, this paper mainly illustrates the effect of different choice of in-structure damping models on the optimal distribution of damping devices in terms of response. Two numerical studies are used for the illustration purpose. Though no specific conclusions are drawn, our main intention here is to qualitatively highlight the issues associated with certain prevalent assumptions regarding the in-structure damping and its effect on the optimal distribution of dampers. The paper also presents a brief overview of the in-structure damping models existing in literature.

2. BRIEF OVERVIEW OF THE MODELS OF DAMPING

This section mainly gives a brief overview of classical and non-classical viscous damping models which are used in the numerical studies presented in this paper. To get a detail review on all other models of damping interested readers should refer to Banks and Inman (1991), Woodhouse (1998), Adhikari (2000), Muravski (2004), Puthanpurayil et al (2011), Smyrou et al (2011). Banks and Inman (1991) deals with damping mainly in continuous system whereas all the above referred other papers deal with damping in discrete systems. A full state of the art description of the damping is given in DeSilva (2007).

2.1. Classical viscous damping

Viscous damping is mainly achieved by the incorporation of Rayleigh's Dissipation function given as

$$F = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\dot{q}_i\dot{q}_j = \frac{1}{2}\dot{q}_j C\dot{q}' \quad (1)$$

where 'C**'** represents a non-negative definite symmetric matrix. Rayleigh further demonstrated that one way of obtaining the **'**C**'** matrix is by a linear combination of the Mass and Stiffness, which is given as,

$$C = \alpha M + \beta K \quad (2)$$

where α and β are calculated as functions of frequency using a preconceived damping ratio.

This model is commonly used to model damping in MDOF (Multi-Degree of Freedom) systems and its popularity is mainly due to the fact that it uses the already computed mass and stiffness matrices and demands only the calculation of the constants α and β (Carr 2007). The main advantage of this theory is that the proportionality of the modes is preserved; thereby facilitating the classical modal analysis to be performed more or less similar to the un-damped vibration. But there is no explicit justification for the preservation of this proportionality phenomenon; and in reality the test results indicate complex nature of the eigen modes. This implies non-proportionality of the mode shapes and indicates the presence of non-classical damping (Adhikari 2000).

2.2. Non-classical viscous damping

A model in which the damping force is a function of past history of motion via convolution integrals over a suitable causal Kernel function constitutes non-classical viscous damping. They are called non-classical viscous because the force depends on *state variables* other than just the instantaneous velocity (Adhikari et al 2003). The most generic form of linear non-classical viscous damping given in the form of modified dissipation function is as follows (Woodhouse 1998, Adhikari 2000):

$$F(q) == \frac{1}{2}\dot{q}'\int_0^t g(t-\tau)\dot{q}(\tau)d\tau \qquad (3)$$

where $g(t)$ represents the Kernel function and $\dot{q}(\tau)$ represents system velocity. This could also be looked as a time hysteresis model applied to discrete systems. The generality of this model is evident from the aspect that the Kernel function $g(t)$ could adopt any causal model where the energy functional is non-negative (Adhikari et al 2003). In literature this is commonly referred to as non-viscous damping model (Woodhouse 1998), considering the fact that integration by parts of eq. (3) would result in the damping force being expressed as a function of displacement. But considering the fact that damping force in its form as given in eq. (3) is a function of velocity, the authors prefer to address the formulation as non-classical viscous damping. Incorporating this model, the equation of motion of the system can be expressed as:

$$M\ddot{u} + \int_0^t g(t-\tau)\dot{u}(\tau)d\tau + Ku = f(t) \qquad (4)$$

where *M* is the mass, *K* the stiffness, $f(t)$ the applied force, $\ddot{u}$ the acceleration, $\dot{u}$ the velocity and u the displacement of the system.

3. NUMERICAL STUDY

Two optimally controlled shear frames in which the damper capacities are derived using two different optimization schemes are used to illustrate the effect of in-structure damping models on the optimal distribution of dampers. Displacement and acceleration responses of these frames predicted using different in-structure damping models are compared to qualitatively highlight the effect of different damping models. Damping models used for the study are the classical Rayleigh model and the non-classical viscous model given by equation (3) as it represents the most general damping model within the scope of a linear analysis (Woodhouse 1998).

3.1. Description of the frames

3.1.1. Frame 1 (Frame with uniform Stiffness)

The optimal distribution of dampers derived by Takewaki (1997) in a six storey shear building model is used as frame 1 for the study. The shear building model is shown in Figure 1. All masses are assumed to be lumped at storey levels with $m_1=m_2\ldots..=m_6= 0.8\text{x}10^5$ kg. A uniform storey stiffness is assumed with $k_1=k_2=\ldots\ldots\ldots=k_6= 4.0\text{x}10^7$ N/m. The optimal damper locations are indicated in Figure 1. The value of the optimal damper coefficients as calculated by Takewaki is as follows: $c_1= 4.8\text{x}10^6$ N-s/m and $c_2= 4.2\text{x}10^6$ N-s/m. One fact to be noted is that Takewaki neglected the contribution of the in-structure damping while calculating these coefficients. From our sensitivity analysis point of view, this is ideal because the damper coefficient values obtained do not have any contribution from in-structure damping; thereby providing us with a flexibility of incorporating different in-structure damping models with apparently no significant error. The undamped frequencies of the uncontrolled frame are recorded in Table 1.

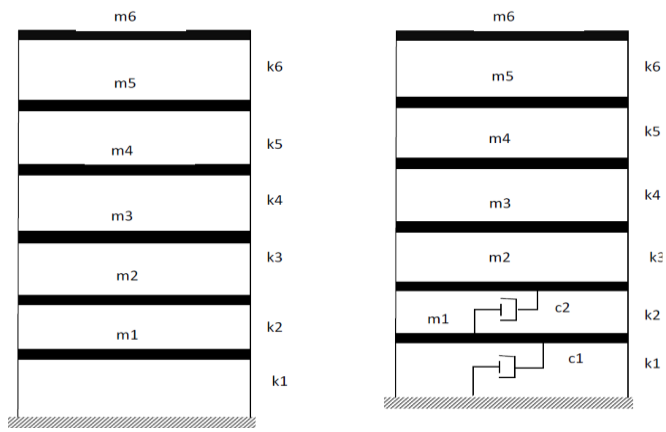


Figure 1 Uncontrolled shear frame building (left) and optimally controlled shear frame building (right)

Table 1 Modal frequencies of frame 1

Modes	1	2	3	4	5	6
Frequency (Hz)	0.86	2.52	4.04	5.32	6.30	6.91

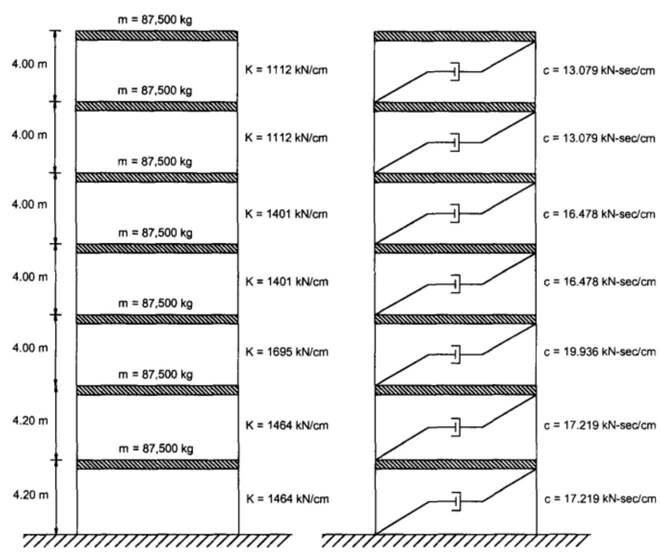


Figure 2 Uncontrolled shear frame building (left) and optimally controlled shear frame building (right) (fig. adopted from Garcia (2001))

Table 2 Modal frequencies of frame 2

Modes	1	2	3	4	5	6	7
Frequency (Hz)	1.36	3.80	6.20	8.45	9.88	11.34	12.81

3.1.2. Frame 2 (Frame with non uniform stiffness)

Gluck et al. (1996) used a seven story shear frame to illustrate their technique of adaptation of optimal control theory using the linear quadratic regulator to design linear viscous dampers. The same frame is adopted as frame 2 for the present study; it is illustrated in Figure 2. The mass, stiffness and external damper properties are described in the figure. A uniform inherent damping ratio of 1% is assumed in the original study. Table 2 gives its undamped modal frequencies.

3.2. Description of the ground motions

In order to assess the sensitivity of the optimally controlled frames to different in-structure damping models, the controlled frames are subjected to two different ground motions; identified hereafter as the Chi-Chi ground motion (from the 1999 Chi-Chi, Taiwan earthquake) and the Sakaria ground motion (from the 1999 Izmit Earthquake). The time history plot and Fourier amplitude spectra of both ground motions are presented in Figure 3.

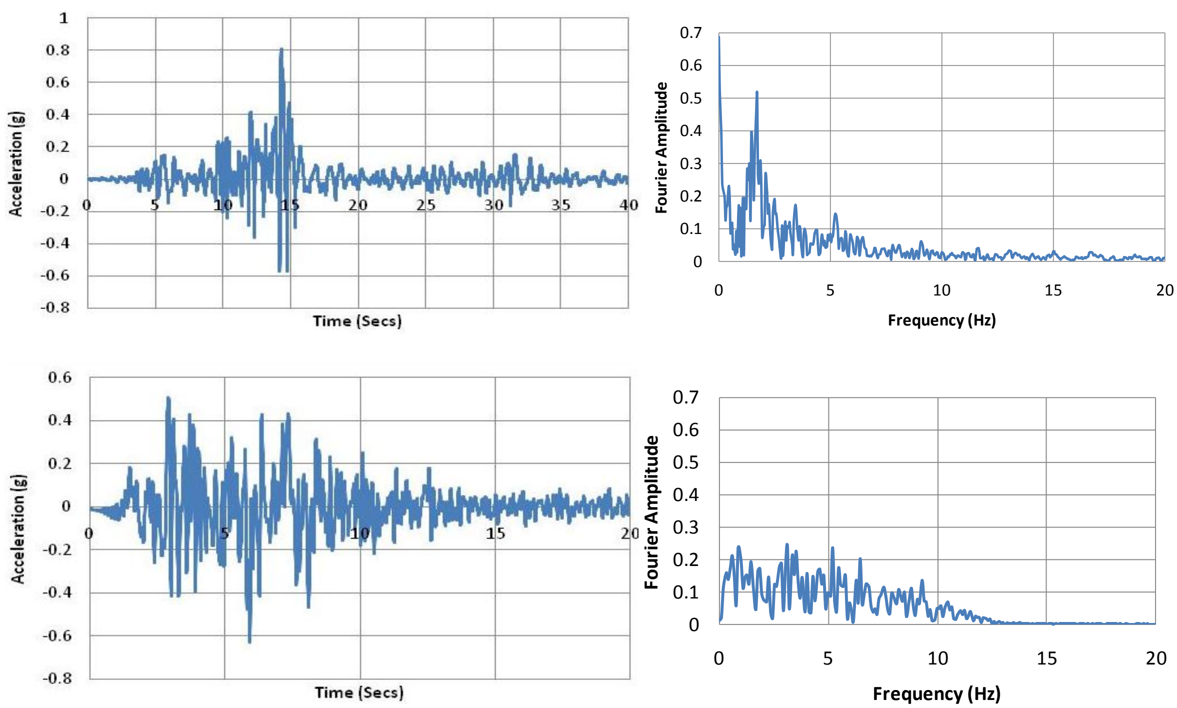


Figure 3. Acceleration time histories and Fourier amplitude spectra of the Chi-Chi (top) and the Sakaria (bottom) ground motion records

Figure 3 (top) shows that the Chi-Chi record has a narrow band spectrum with a predominant frequency content of 1.7Hz. On the other hand, the Sakaria record has a broad band spectrum with Fourier peaks occurring between 0.5 and 10 Hz as is evident in Figure 3 (bottom).

The choice of the ground motion records is made with a focus to excite as many modes as possible. For example, reviewing the modal frequencies given in Table 1 and Table 2, it becomes evident that in the case of the Chi-Chi record the predominant excitation is expected to happen in the first 2 modes for both frames, whereas the Sakaria record is expected to excite several higher order modes to varying degrees for both frames. As our main intention is only to qualitatively highlight the possible uncertainties arising in the response due to the interaction between the inherent in-structure damping of the system and the ‘added damping’ supplied by the mechanical dampers, the choice of these two ground motions may be deemed to be appropriate.

3.3. Analysis of the controlled frames

Direct time integration is performed using the Newmark total equilibrium method (Carr 2007). MATLAB codes were developed for the time domain analysis incorporating both classical Rayleigh viscous and non-classical viscous damping models. Linear elastic time domain analysis is performed as the optimization schemes employed in deriving the damper capacities is based on the assumption of linear elastic behavior of the parent frames. In the case of non-classical viscous damping, a single exponential model called Biot’s relaxation function is used as the Kernel function. The Biot’s relaxation function is of the form

$$g(t) = \mu e^{-\mu t} \tag{5}$$

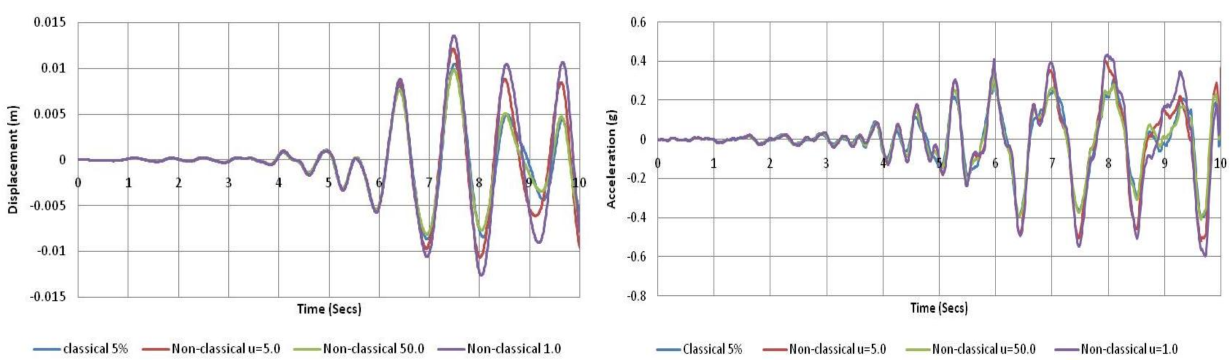
where μ is a dissipation constant. A very low value of μ indicates strong non-viscous characteristics and a high value of μ indicates close to viscous characteristics (Adhikari 2000). Now the interesting question is what μ values would reflect reality? At this point of time unfortunately this question remains unanswered and demands further research. In this sensitivity study we use μ =1.0, 5.0 and 50.0, based on past research evidence (Adhikari, 2000).

4. RESULTS AND DISCUSSION

This section presents the plots of responses in terms of displacement and acceleration of the roof for both the frames. The comparisons plotted in the figures can be used to qualitatively investigate the effect of different damping models on the responses.

4.1 Chi-Chi ground motion

4.1.1. Frame 1 (Uniform Stiffness)



(a) (b)

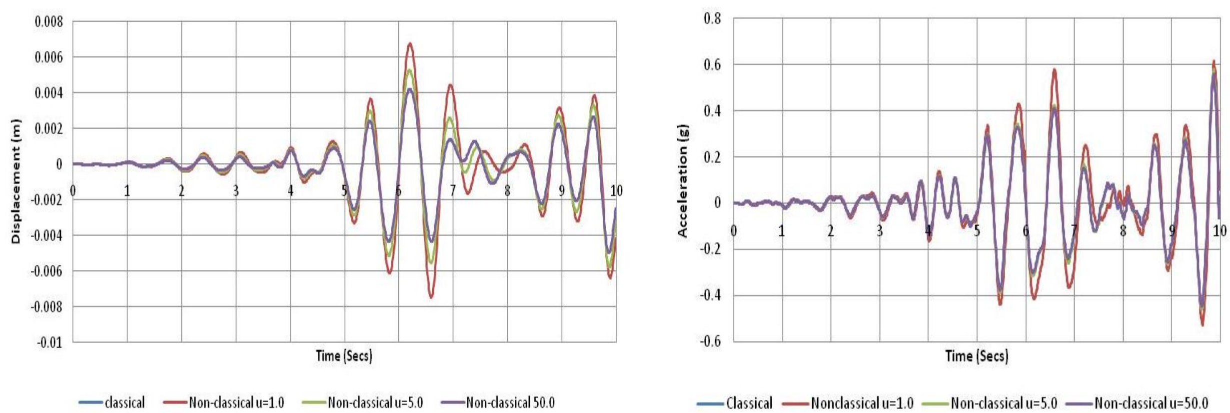
Figure 4 Roof displacement histories (a) and Roof acceleration histories (b) of frame 1 due to the Chi-Chi record

Figure 4 shows close views of the displacement and acceleration time history responses (for the first 10 seconds) of the roof due to the Chi-Chi ground motion with classical viscous damping model and non-classical viscous damping models with different values of μ. The plots show that there is a clear distinction between the displacement and acceleration responses obtained using classical viscous and non-classical viscous models.

Figure 4(a) shows that the peak displacement response for frame with highly non-classical model (μ =1.0) is approximately 40-45% more than the frame with classical viscous model. The responses of other models lie in between the classical and non-classical with μ =1.0. Figure 4(b) also exhibits a similar trend with the peak acceleration response showing an increase of approximately 70% for highly non-classical model with μ =1.0 in comparison to the classical viscous model.

4.1.2. Frame 2 (Variable Stiffness)

Figure 5 illustrates the displacement and acceleration responses of the roof of frame 2 for the Chi-Chi earthquake ground motion. It is evident that the peak displacement and acceleration responses are considerably different for the different damping models. The largest difference is exhibited between the highly non-classical viscous model (μ =1.0) and the classical viscous model; which is in the order of 50-55% for the peak displacement and 40-50% for the peak acceleration.



(a) (b)

Figure 5 Roof displacement histories (a) and Roof acceleration histories (b) of frame 1 due to the Chi-Chi record

4.2 Sakaria ground motion

4.2.1. Frame 1 (Uniform Stiffness)

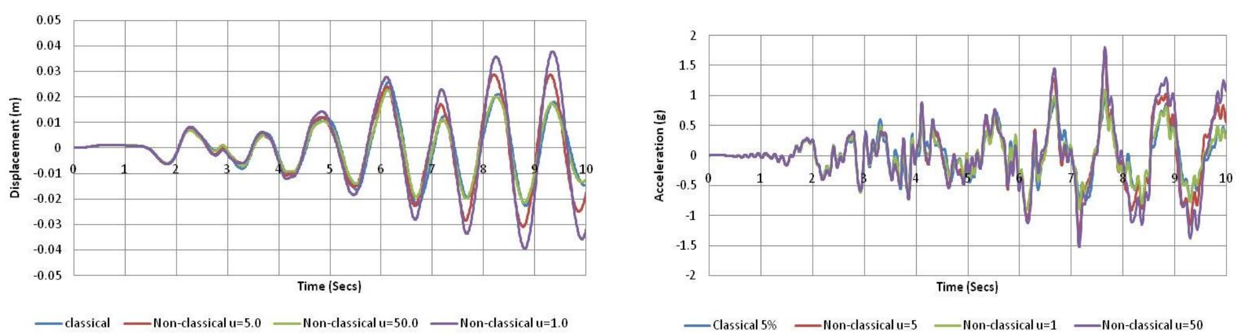


Figure 6 Displacement (left) and acceleration histories (right) of frame 1 roof due to the Sakaria ground motion

Figure 6 shows the displacement and acceleration time history responses of the roof of frame 1 for the Sakaria ground motion with classical viscous and non-classical viscous damping models. In comparison with the classical viscous model, the non-classical model with μ =1.0 shows approximately 80-90% increase in the peak displacement response and approximately 50% increase in the peak acceleration response.

4.2.2. Frame 2 (Variable Stiffness)

Figure 7 depicts the variations of the displacement and acceleration responses of the roof of frame 2 when subjected to the Sakaria ground motion. Compared to classical model, the highly non-classical model (μ =1.0) results in approximately 80% increase in the peak displacement and close to 50% increase in the peak acceleration.

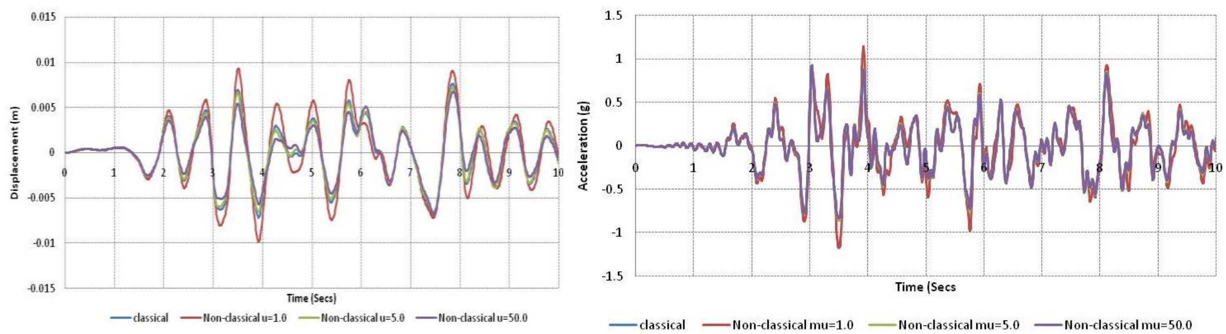


Figure 7 Displacement (left) and acceleration histories (right) of frame 2 roof due to the Sakaria ground motion

4.3 Discussions

From the results presented in the previous section it is important to note that the extent of effect the assumed damping model has on the responses of the two frames is different for the two different ground motions considered. Earthquakes are inherently uncertain phenomena with no human control, and so are the resulting ground motions at a site of interest. The aim of an analysis dealing with seismic actions should hence be to get a reliable prediction by reducing uncertainty, which renders the use of a more consistent and realistic damping model imperative. This raises the same question again; “What is the most realistic model of damping?”

Unfortunately, very little is known about realistic structural damping. Free vibration testing of real buildings indicates that the damping in the first mode, though not purely viscous, is very close to viscous and it could be said that μ =50.0 represents a realistic behavior, at least in the first mode. Our intention in plotting the highly non-classical viscous (μ=1.0) and close to viscous (μ=50.0) is to highlight this inherent variability existing in the modeling. There are other models such as the frequency independent damping model (Muravski 2004)) which would again give an entirely different set of responses.

The comparison of the two frame responses presented earlier provides a qualitative indication of the extent of effect different damping models can make on the predicted structural response. The interesting aspect to be noted in the above analytical results is that the differences exhibited in the predicted responses are rather large, and can have major implications on the adequacy of the structures designed based on these predictions. For example, if for argument sake non-classical damping with μ =1.0 is considered as representing the actual in-structure damping, from the results presented above it becomes evident that in some cases although the actual frame response would enter inelastic phase; but the analysis using classical model will not predict this. The analysis could suggest the structure in likely to remain elastic when in reality the structure should have been designed and detailed for inelastic response. Similar observations has been recorded in earlier studies (Val & Segal, 2005) in which it is found that there would be instances when the classical viscous damping assumption would underestimate the peak deflection and fail to capture the occurrence of nonlinearity in the parent frame. Similarly the underestimation of the floor acceleration by the classical damping model can result in underestimation of demands for acceleration sensitive non-structural components and contents.

All these observations raise a question on the so-called optimality obtained by ignoring the inherent variability in the in-structure damping models, because what is optimal in one analysis using a specific damping model might not be optimal in terms of response if we use a different damping model.

5. CONCLUSIONS

The significance of the correct choice of realistic in-structure damping model is qualitatively illustrated with the help of comparative sensitivity studies on controlled frames. Classical and non-classical viscous damping models are used for the study. It is observed that different damping models give different responses highlighting the need for a realistic representation of the in-structure damping to achieve more reliability in response prediction. It is also shown that an un-realistic damping model might underestimate the peak responses and thereby affect the so-called optimality achieved. Though no specific conclusion is drawn, the need for the use of a more generic damping model is emphasized. It has also been highlighted that the extent of the effect of these different models on the response of a system is an area which demands further research.

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