

# Research on the Transient Response and Measure Method of Engineering Vibration Sensors



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## **SUMMARY: (10 pt)**

This paper introduces a kind of measure methods of the transient response of engineering vibration sensor. Perform simulating calculation for transient response of the system, in the different duration time for single half sine wave and single square wave inputting condition, and make a comparison with the result of actual measurement of transient response by actual measure methods when other things being equal, get an anastomotic result.

*Keywords: vibration sensor, transient response, simulation*

## **1. INSTRUCTIONS**

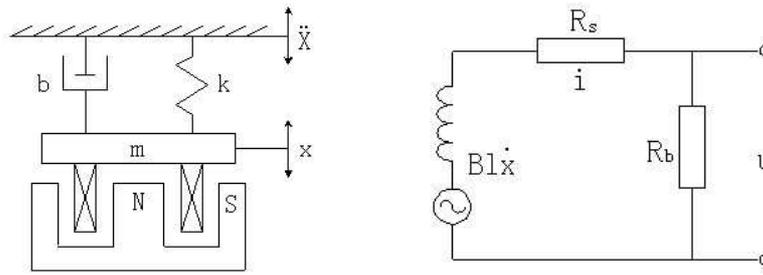
In the sinusoidal vibration, random vibration and steady state vibration measurement of the vibration, amplitude measurement error depends mainly on the amplitude frequency characteristic of vibration sensors. For the vibration signal of the natural earthquake, blasting, shock and earthquake simulation etc, the vibration amplitude (especially the amplitude of the first few waveform ) measurement error depends mainly on the transient response of vibration sensor. As the transient vibration signal operates the transient response of the sensor itself is involved, forming a superimposed signal, so the real vibration waveform becomes distortion. Therefore in measurement of vibration, the vibration sensor must be carefully chosen, otherwise it will bring great error to the measurement results.

If its transient response can be analysed conveniently and accurately, the application selection and measurement error judgment of the engineering vibration sensors will be great convenience. Then for the design research of the engineering vibration sensors, a reasonable simulation model should be established. By optimizing the parameters of the model to obtain the best matched parameter, so as to make the transient performance index of the sensors meet the demand of the measurement better. Further research each link, design a feasible correction measures (to improve sensor performance index ), in order to obtain effective compensation method.

## **2. BASIC PRINCIPLE OF THE VIBRATION SENSOR**

Principle diagram of moving-coil sensor as shown in Figure 2.1.

In the Figure 2.1,  $m$  is the mass of the motion part,  $k$  is the spring coefficient,  $b$  is the air damping constant,  $\ddot{X}$  is the acceleration of the ground, and  $x$  is the displacement of the mass relative to the base.  $Bl$  is the electromechanical coupling content of the coil,  $i$  is the electric current,  $R_s$  is the resistance of coil,  $R_b$  is the resistance, and  $u$  is the output voltage.



**Figure 2.1.** Principle diagram of moving-coil sensor

The equations of motion is :

$$\left. \begin{aligned} m\ddot{x} + b\dot{x} + kx &= -m\ddot{X} - Bli \\ i &= \frac{Bl\dot{x}}{R_s + R_b} \end{aligned} \right\} \quad (2.1)$$

$$m\ddot{x} + \left( b + \frac{(Bl)^2}{R_s + R_b} \right) \dot{x} + kx = -m\ddot{X} \quad (2.2)$$

The solutions for Equation(2.2) in the Laplace transform is :

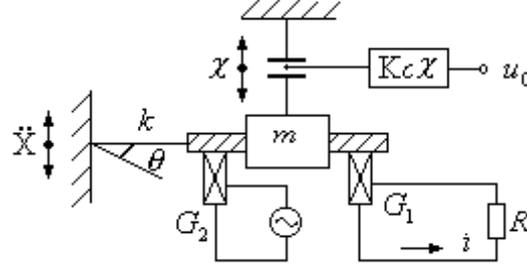
$$\frac{x(s)}{X(s)} = -A \frac{s^2}{s^2 + 2Dns + n^2} \quad (2.3)$$

In Equation(2.3),  $A$  is the amplitude sensitivity,  $n = \sqrt{\frac{k}{m}}$ , Where the air damping can be neglected, the damping constant  $D = \frac{(Bl)^2}{2mn(R_s + R_b)}$ . When the vibration frequency and damping constants are different values, inertia type moving-coil sensor can be derived from different forms.

### 3. SIMULATION TEST

It introduces several typical engineering vibration sensor principle and deduced its transfer function, the following will be the system simulation test by Simulink module in the MatLab software. Based on the angle type zero frequency acceleration sensor, that is used on the bridge deflection measurement in the typical project, its simulation model was set up.

Basic principles is shown in Figure 3.1,  $k$  is spring stiffness,  $b$  is damping ratio that includes air damping and damping coefficient,  $G_1 = Bl_1L_k$  is to meet the set coil electric constant,  $BL_1$  is electromechanical coupling coefficient,  $L_k$  is to indicate the length of pendulum,  $G_2$  is electrical constant of itself calibration coil,  $\theta$  is the angle,  $K_c$  is capacitive sensing sensitivity,  $x$  is displacement of the capacitor plate.



**Figure 3.1.** The model of zero frequency accelerometer for measure obliquity

The equations of motion is:

$$K_1\theta + b\dot{\theta} + k\theta + G_1 i = -\frac{K_1}{L_k} X \quad (3.1)$$

Neglecting the air damping, solutions for Equation(3.1) in the Laplace transform is:

$$\theta = \frac{s^2 X}{n^2 L_k} \cdot \frac{1}{\left(\frac{s^2}{n^2} + \frac{2D}{n} s + 1\right)} \quad (3.2)$$

Where the  $s$  is operator,  $n$  is natural circular frequency,  $n^2 = \frac{k}{K_1}$ ,  $D$  is the damping constant,

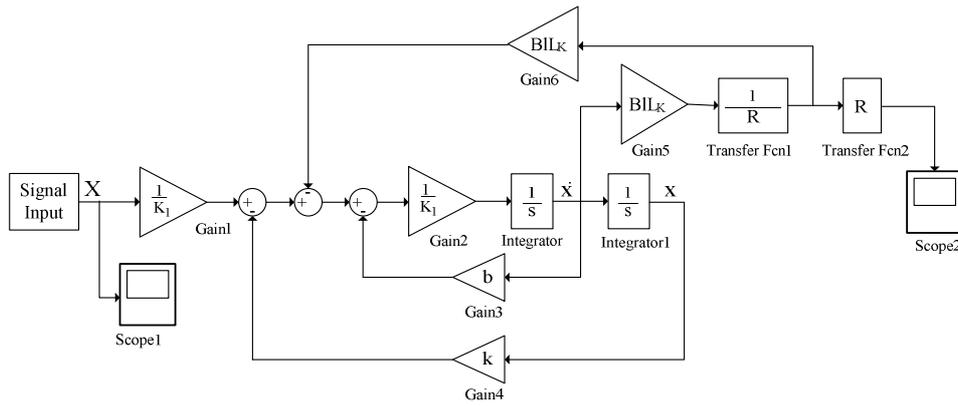
$D = \frac{G_1^2}{2K_1 n R}$ ,  $R$  is the coil loop resistance,  $u_0$  is output voltage:

$$u_0 = K_c x = K_c L_k \theta = \frac{K_c V_0 s^2 X}{n^2} \frac{1}{\left(\frac{s^2}{n^2} + \frac{2D}{n} s + 1\right)} \quad (3.3)$$

$$\frac{u_0}{s^2 X} = \frac{K_c V_0}{n^2} \frac{1}{\left(\frac{s^2}{n^2} + \frac{2D}{n} s + 1\right)} \quad (3.4)$$

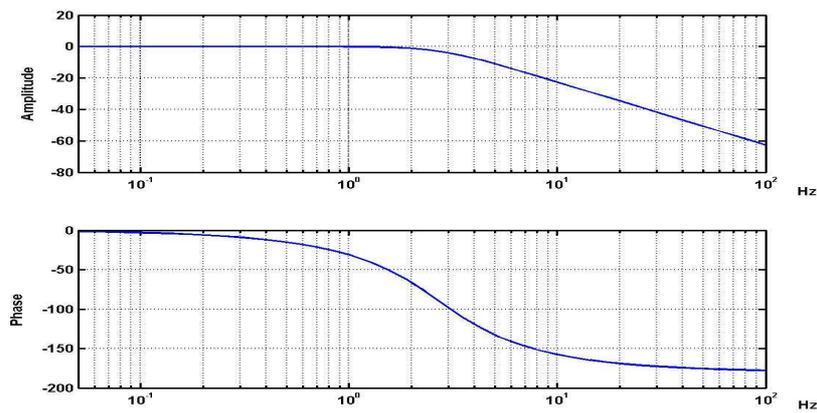
Where  $\frac{K_c V_0}{n^2} = 1$ ,  $D = 0.707$  and  $n = 2 \times \pi \times 2.7$ .

The first Figure 3.1 and the Equation (3.4) to set up the basic simulation model diagram as follows:

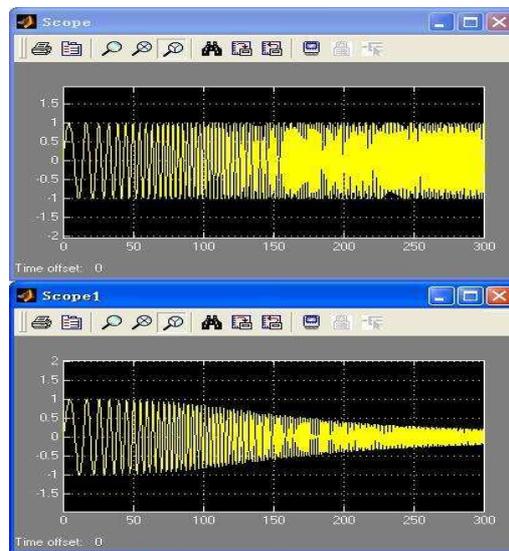


**Figure3.2.** The simulating block diagram

In the simulation model diagram Figure3.2, each link of the Equation(3.4) and the parameters were the most simplified decomposition, so that these different parameters can be optimized conveniently in the simulation test. In the Figure 3.2, "Signal Input" is written using Matlab language program running on the "workspace" on the formation of the input waveform data. This paper uses the frequency sweep wave input; "Scope, Scope1" is the observation point to observe the input waveform and response of the system output in the simulation test.



**Figure3.3.** Frequency characteristic curves of the system



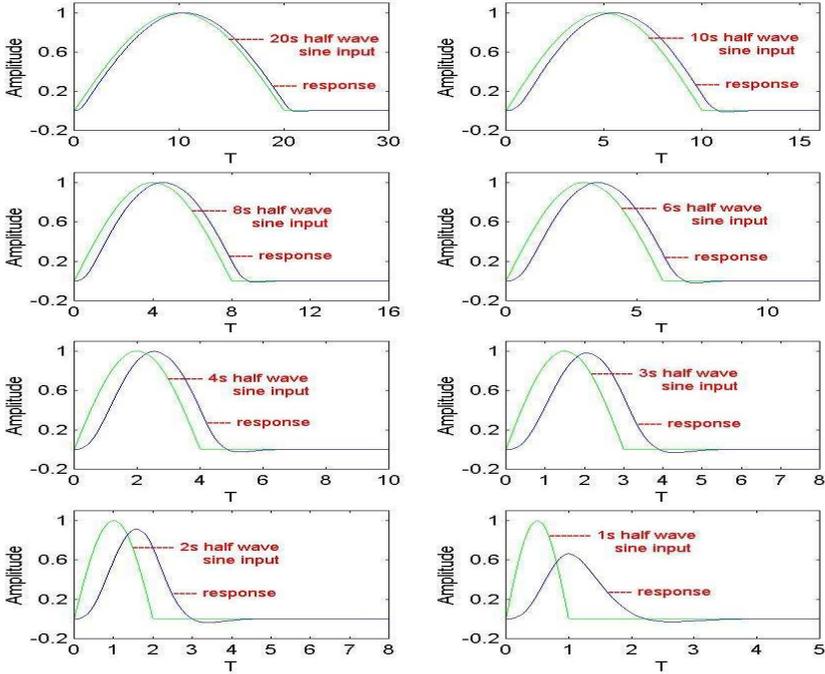
**Figure3.4.** Characteristic of the chirp signal

The simulation test inputs single half sine wave and single square wave , the waveform widths are 20s, 10s, 8s, 6s, 4s, 3s, 2s and 1s, the amplitude is 1. The simulation results that the sensor inputs single half sine wave and single square wave are shown in Figure3.5 and Figure3.6. Table3.1 gives the response amplitude and time difference of the system on single half sine wave and single square wave .

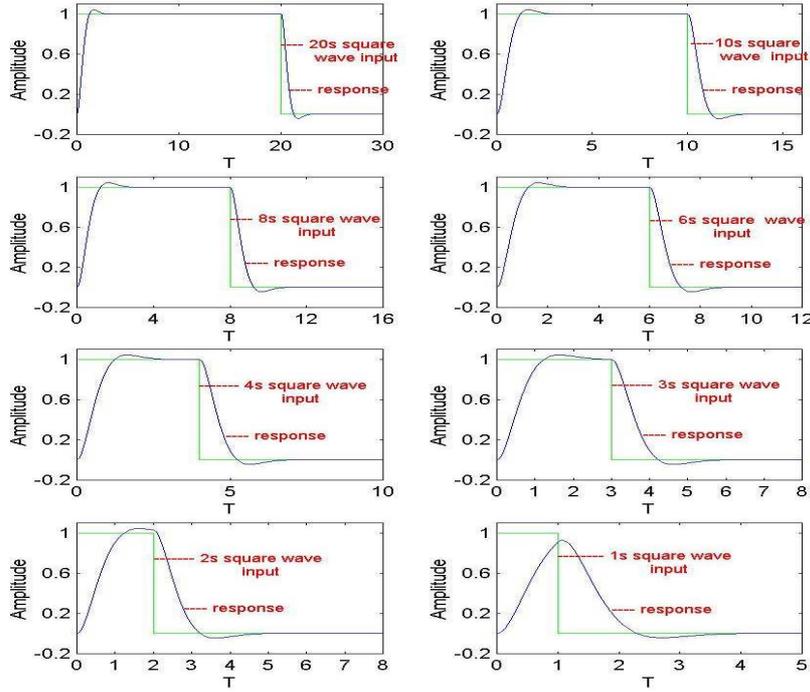
**Table 3.1.**

	Input waveform duration (s)	20	10	8	6	4	3	2	1
Single half sine wave	Peak response	1.000	1.000	1.000	1.000	0.997	0.980	0.913	0.662
	Time difference of the system response (s)	0.531	0.531	0.531	0.534	0.550	0.574	0.584	0.500
Single square wave	Peak response	1.043	1.043	1.043	1.043	1.043	1.043	1.043	0.928
	Time difference of the system response (s)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.890

From the simulation results we can see that, with the duration increase of the system input waveform, the response waveform is closer to the input waveform. The input waveform duration greater than 3 seconds, in response to the peak value and peak input relative error is less than 2%, response peak and peak input lag is almost a fixed value ( about 0.5 seconds).



**Figure3.5** The simulating results of the different duration time for single half sine wave



**Figure3.6** The simulating results of the different duration time for single square wave

#### 4. THE PRINCIPLE AND METHOD OF THE SENSOR TRANSIENT RESPONSE TEST

Angle type zero frequency acceleration sensors are shown in Figure3.5,  $G_2$  is measuring coil, used to measure the transient response, sensitivity, linearity and other parameters of the sensors. When the current  $I_{OC} \sin \frac{\pi\tau}{t_1}$  is inputed (half sinusoidal variation) in the measuring coil  $G_2$ , the moment of force is in the pendulum:

$$F_{OC} = G_2 I_{OC} \sin \frac{\pi\tau}{t_1} \quad (4.1)$$

The differential equation of motion is

$$K_1\theta + b_1\dot{\theta} + k\theta + G_2 i = G_2 I_{OC} \sin \frac{\pi\tau}{t_1} \quad (4.2)$$

The solutions for Equation(4.2)

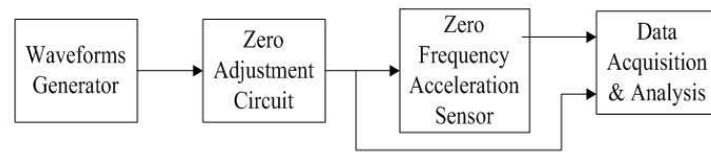
$$\begin{aligned} \theta &= \frac{1}{K_1 n^2} G_2 I_{OC} \sin \frac{\pi\tau}{t_1} \frac{1}{\left(\frac{s^2}{n^2} + \frac{2D}{n^2} s + 1\right)} \\ &= \frac{1}{k} G_2 I_{OC} \sin \frac{\pi\tau}{t_1} w \end{aligned} \quad (4.3)$$

In Equation(4.2),  $w = \frac{1}{(\frac{s^2}{n^2} + \frac{2D}{n^2}s + 1)}$ , The output voltage:

$$u_o = K_C L_K \theta = K_C L_K \frac{1}{k} G_2 I_{OC} \sin \frac{\pi \tau}{t_1} w \quad (4.4)$$

By the Equation(4.4) can be seen, the voltage output is proportional to the current  $I_{oc} \sin \frac{\pi \tau}{t_1}$  input in the measured coil.

The test block diagram of angle type zero frequency acceleration sensors transient response as shown in Figure4.1:

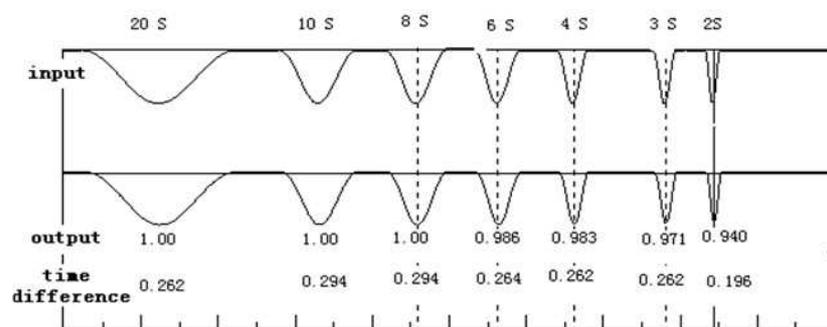


**Figure4.1** The composition diagram of transient response test system

Waveform generator can produce 1ms-1000s, cycle variable, amplitude adjustable single half sine wave, sawtooth wave and square wave. These wave is removed DC by the zero adjustment circuit, then a single half sine wave ( or a single square wave ) signal is input into the sensor measured coil, response waveform is also input to the data acquisition and analysis system in a single half sine wave excitation, and then analyzing and processing, the amplitude error is given in excitation response waveform of various cycle single half sine wave( or single square wave ).

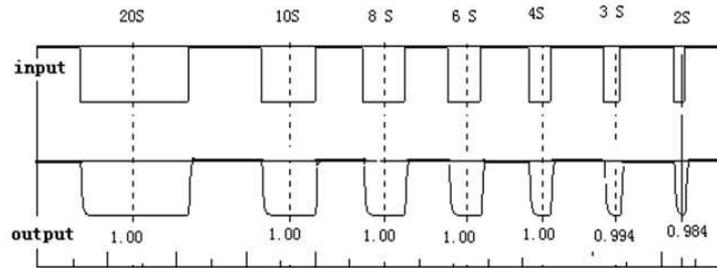
## 5. TEST RESULTS

For Angle type zero frequency acceleration sensor, the different duration of single half sine wave input waveform and response waveforms such as shown in Figure5.1, the different duration of single square wave input waveform and response waveforms are shown in Figure5.2.



**Figure5.1** The transient response of the different duration time for single half sine wave

From the test results can be seen that in a single half sine wave input case, the duration of greater than 3 seconds, amplitude response error is less than 3%, in a single square wave input case, the duration of greater than 3 seconds, the amplitude response error is less than 1%. The time differences between single sine wave response peak and input peak is less than 0.3 seconds.



**Figure5.2** The transient response of the different duration time for single square wave

## 6. THE CONCLUSION

From Table3.1, Figure5.1 and Figure5.2, the results of computer simulation and the test results are in good agreement. It proves that angle type zero frequency acceleration sensor for more than 3 seconds duration of single sine wave and square wave amplitude response has a higher precision. The waveform response peak and the input waveform peak have a certain time difference does not affect the actual application. Simulation waveform of 4% overshoot, we think that the damping ratio by using is slightly less than the actual value.

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