Yield Surface of Each Story for Inelastic Analysis of Frames under Horizontal Force and Torsion

R. Minatogawa
Department of Built Environment, Tokyo Institute of Technology, Japan

Y. Sakai
Nippon Steel Engineering Co., Ltd., Japan

D. Ehara, M. Yamanari & K. Ogawa
Graduate School of Science and Technology, Kumamoto University, Japan

SUMMARY:
This paper is concerned with yield surface of each story under combined forces that are a couple of bending moment due to horizontal translation as well as torsional effect for inelastic analysis of moment frames. In dealing with three-dimensional frame with eccentricity, torsional effect at a story of the frame must be considered. However, this torsional effect makes the problem concerning seismic response analysis more complicated. For the purpose of simplification of the problem, the authors propose a couple of new expressions of yield surfaces for each story that consist of yield surface of beams and columns. They are used for the two kinds of cases that beams at the floor could yield and columns at the floor could yield respectively. Adopting the yield surface which was obtained through this study for seismic response analysis, it is sure that a higher accuracy of results can be acquired.

Keywords: Yield surface, Torsion, Inelastic analysis, Three-dimensional frame, Seismic response

1. INTRODUCTION

The process of seismic design is typically dealt as a plane frame. However, practical frames are three-dimensional object, some problems which should be considered as a space frame have been left. One of the important problems is torsional vibration caused by eccentricity. Most of the studies on effects of eccentricity in the story were conducted in one-story and one-bay frames. Although there still has been the unknown problems regarding that eccentricity affects seismic response of multi-story frames, it makes necessity of developing the tool for examining these problems. Taking the above state into consideration, the authors explored the shape of yield surface concerning the frames which has multiple structural planes and propose a couple of yield surfaces for generic analysis results.
Frames with eccentricity in the story are added extra bending moment due to torsional effect under seismic motion. Yield surface, namely, correlating function which consists of a couple of bending moment due to horizontal translation as well as torsional effect is indispensable to seismic design for space frames. These yield surfaces are determined through simple plastic analysis, and this analysis is supposed that the force which affects the frame with in elastic range is regarded as bending moment only. Here, the yield surface is concerning each story of the frame, moreover, it consists of both yield surfaces of beams and yield surface of columns. They are used for the two kinds of cases that beams at the floor could yield and columns at the floor could yield respectively. The yield surface of beams was obtained through the theoretical analyses which were conducted with changing the number of structural planes. On the other hand, it is too difficult to derive the yield surface of columns from theoretical analysis. Therefore, numerical calculations by setting up the centers of torsional rotation with continuously varying were carried out to obtain it. In this paper, a couple of pertinent shape on yield surface for each story of the frames was clarified through the examination by theoretical analysis and numerical calculations.

2. YIELD SURFACE OF BEAMS

On the yield surface of beams, two kinds of collapse mechanisms of the frame should be considered for seeking the least collapse load; one is due to torsional effects, the other one is due to sway in two horizontal directions. The examined frame is adopted the assumptions that the beams could yield and the columns should keep elastic while external force increases. When the beams in orthogonal structural planes are considered individually, the considerable collapse mechanism is sway collapse mechanism and torsional collapse mechanism which has the center of rotation at each structural plane of beams. This sway collapse mechanism can be considered as torsional collapse mechanism which has the center of rotation at infinity. It equally implies to carry out simple plastic analysis for torsional collapse mechanism and investigate all collapse mechanism. Provided that, the structural planes of beams are evenly spaced and the beams have equal yield strength. Because of simple plastic analysis, the forces acting the frames are considered as bending moment due to horizontal translation and additional bending moment due to torsional effects. Yield surface of beams can be obtained through the theoretical analyses that were conducted with changing the number of structural planes. Additionally, each structural plane of beams is named in alphabetical order from left side.

2.1. Two-structural Plane System

First of all, the yield surface in the case that beams of Y direction are constituted with two structural planes is examined as shown Figure 2.1. Then, each plastic bending strength of a beam is equally set to M_p and each distance between two structural planes is set to $2l$. When the center of torsional rotation is set at structural plane A, elastic bending moment (M) and plastic bending moment (M_p) act on the beam at structural plane A and structural plane B respectively. Consequently, bending stress due to translation of Y direction (M_Y), and due to torsional deformation of the floor (S), each full-plastic strength ($M_{p,b,Y}$) and ($S_{p,b,Y}$) can be denoted by Eqns.2.1, 2.2, 2.3, and 2.4.

$$M_Y = M_p + M \tag{2.1}$$

$$S = l\left(M_p - M\right) \tag{2.2}$$

$$M_{p,b,Y} = 2M_p \tag{2.3}$$

$$S_{p,b,Y} = 2lM_p \tag{2.4}$$

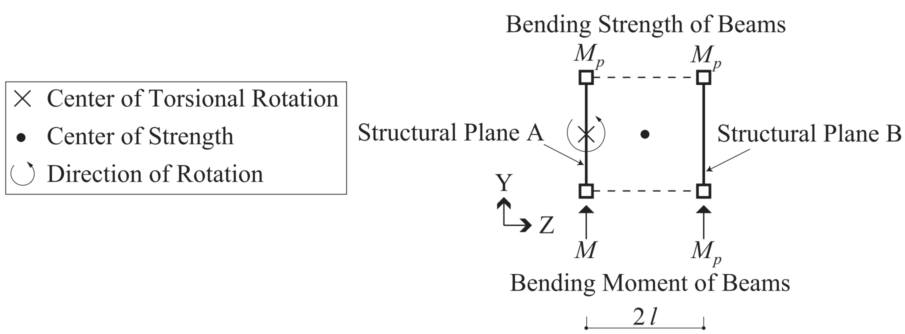


Figure 2.1. Beam model in two-structural plane system

Using the above four equations, the yield surface in the case that beams of Y direction are constituted with two structural planes can be expressed by Eqn.2.5.

$$F = \frac{S}{S_{p,b,Y}} + \frac{M_Y}{M_{p,b,Y}} - 1 = 0 \tag{2.5}$$

2.2. Three-structural Plane System

The yield surface in the case that beams of Y direction are constituted with three structural planes is examined as shown Figure 2.2. Then, bending stress and full-plastic strength are denoted likewise the case of two structural planes. To obtain the yield surface in the case of three structural planes, however, the center of torsional rotation should be set at not only structural plane A but also structural plane B. These cases can be denoted respectively by Eqn.2.6.a or Eqn.2.6.b.

$$F = \frac{2}{3} \cdot \frac{S}{S_{p,b,Y}} + \frac{M_Y}{M_{p,b,Y}} - 1 = 0 \tag{2.6.a}$$

$$F = \frac{S}{S_{p,b,Y}} - 1 = 0 \tag{2.6.b}$$

The yield surface in the case that beams of Y direction are constituted with three structural planes can be expressed by Eqn.2.6.a or Eqn.2.6.b.

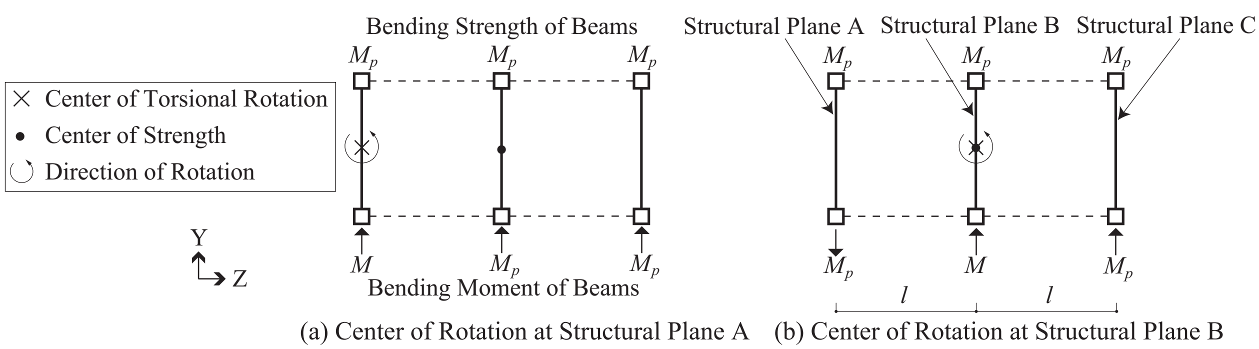


Figure 2.2. Beam model in three-structural plane system

2.3. Four and Five Structural Plane System

The yield surface in the case that beams of Y direction are constituted with four structural planes can be denoted by the following equation likewise the case of three structural planes.

$$F = \frac{2}{3} \cdot \frac{S}{S_{p,b,Y}} + \frac{M_Y}{M_{p,b,Y}} - 1 = 0 \tag{2.7.a}$$

$$F = \frac{S}{S_{p,b,Y}} + \frac{1}{2} \cdot \frac{M_Y}{M_{p,b,Y}} - 1 = 0 \tag{2.7.b}$$

The yield surface in the case that beams of Y direction are constituted with four structural planes can be expressed by Eqn.2.7.a or Eqn.2.7.b.

The yield surface in the case that beams of Y direction are constituted with five structural planes is examined. Then, bending stress and full-plastic strength are denoted likewise the above mentioned cases. To obtain the yield surface in the case of five structural planes, however, the center of torsional rotation should be set at between A-structural plane and C-structural plane. These cases can be denoted respectively by the following equations.

$$F = \frac{3}{5} \cdot \frac{S}{S_{p,b,Y}} + \frac{M_Y}{M_{p,b,Y}} - 1 = 0 \tag{2.8.a}$$

$$F = \frac{6}{7} \cdot \frac{S}{S_{p,b,Y}} + \frac{5}{7} \cdot \frac{M_Y}{M_{p,b,Y}} - 1 = 0 \tag{2.8.b}$$

$$F = \frac{S}{S_{p,b,Y}} - 1 = 0 \tag{2.8.c}$$

The yield surface in the case that beams of Y direction are constituted with five structural planes can be expressed by the equation that falls under any of Eqn.2.8.a through Eqn.2.8.c.

2.4. Infinite Structural Plane System

Finally, the yield surface in the case that beams of Y direction are constituted with infinite structural planes is examined as a limiting case that beams of Y direction are constituted with numerous structural planes as shown Figure 2.3. Then, the beam per unit width has the constant sum of bending strength, distance between the origin and the outermost structural plane is set to l, bending stress and full-plastic strength are denoted likewise the other cases. Additionally, the center of torsional rotation is considered to be located at $\alpha(-l \le \alpha \le 0)$ which is set as distance from the origin that located at the center of strength. Adopting the above conditions, bending stress due to translation of Y direction (M_Y), and due to torsional deformation of the floor (S), each full-plastic strength ($M_{p,b,Y}$) and ($S_{p,b,Y}$) can be denoted by the following equations.

$$M_Y = 2\int_0^{-\alpha} M_p \; dx = -2\alpha M_p \tag{2.9}$$

$$S = 2\int_{-\alpha}^{l} M_p \cdot x \; dx = \left(l^2 - \alpha^2\right) M_p \tag{2.10}$$

$$M_{p,b,Y} = \int_{-l}^{l} M_p \; dx = 2l M_p \tag{2.11}$$

$$S_{p,b,Y} = \int_0^{l} M_p \cdot x \; dx = l^2 M_p \tag{2.12}$$

Using the above four equations, yield surface in the case that beams of Y direction are constituted with infinite structural planes can be expressed by Eqn.2.13.

$$F = \left| \frac{S}{S_{p,b,Y}} \right| + \left(\frac{M_Y}{M_{p,b,Y}} \right)^2 - 1 = 0 \tag{2.13}$$

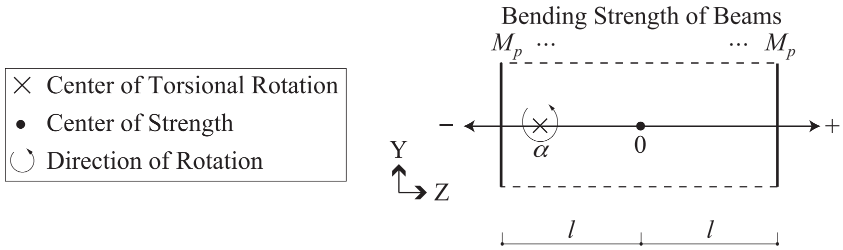


Figure 2.3. Beam model in infinite structural plane system

2.5. Yield Surface of Beams in Each Structural Plane System

Figure 2.4 shows the results of yield surface in the case of each structural plane. Both vertical axis and horizontal axis in this figure take non-dimensional values which are expressed as bending stress due to translation of Y direction divided by its full-plastic strength and bending stress due to torsional deformation of the floor divided by its full-plastic strength respectively. Excepting the case of two structural planes, the yield surface in the case of infinite structural planes can mostly approximate the all cases of structural planes. Obtaining a generalized analysis result concerning the frame which has multiple structural planes, the yield surface which has inclusive shape should be set. Thus the yield surface in the case of infinite structural planes shown as Eqn.2.13 is proposed as the yield surface of beams. Additionally, the same result is acquired on the yield surface in the case that beams of Z direction.

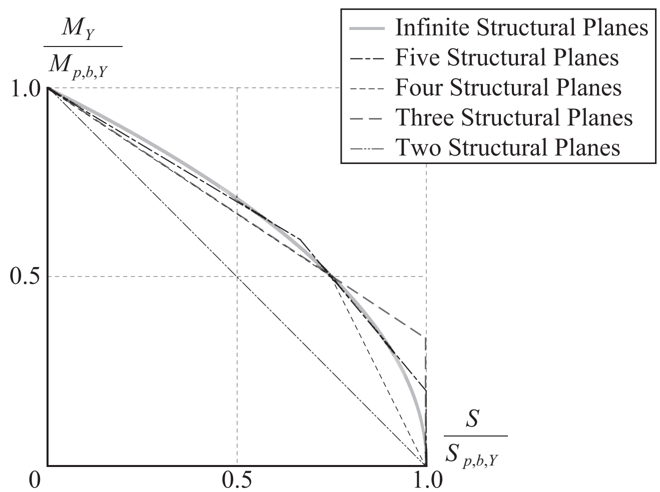


Figure2.4. Yield surface of beams in each structural plane system

3. YIELD SURFACE OF COLUMNS

In this section, the yield surface of columns is explained. In contrast to the yield surface of beams, it is too difficult to derive the yield surface of columns from theoretical analysis. Accordingly, numerical calculations by setting up the centers of torsional rotation with continuously varying were carried out to obtain it. Provided that torsional strength of columns is ignored because it was clarified that torsional strength of columns scarcely affects torsional strength of each story.

3.1. Two-structural Plane System

The yield surface in the case of two structural planes is examined as shown Figure 3.1. Then, each plastic bending strength of a column is equally set to M_p , distance between two structural planes is set to $2l$, and column number is set as shown Figure 3.1. Additionally, the origin is set at the center of strength own frame. On the yield surface of columns, all columns could yield and bending moment set to M_p acts on the all columns respectively because the centers of torsional rotation are set up with continuously varying. Distance of Y direction between the centers of torsional rotation and each columns (Y_k) and distance of Z direction between the centers of torsional rotation and each columns (Z_k) can be expressed by Eqns.3.1, 3.2, 3.3, and 3.4 on each column number when the centers of torsional rotation is set to (Z_R , Y_R).

$$Y_1 = Y_2 = Y_R + l \tag{3.1}$$
$$Y_3 = Y_4 = Y_R - l \tag{3.2}$$
$$Z_1 = Z_3 = Z_R + l \tag{3.3}$$
$$Z_2 = Z_4 = Z_R - l \tag{3.4}$$

When bending moment which acts each column is divided with distance of Y direction (Y_k) and distance of Z direction (Z_k), bending moment of each directions can be expressed by Eqn.3.5 and Eqn.3.6.

$$M_{Y,k} = \frac{Z_k \cdot M_p}{\sqrt{{Y_k}^2 + {Z_k}^2}} \tag{3.5}$$
$$M_{Z,k} = \frac{Y_k \cdot M_p}{\sqrt{{Y_k}^2 + {Z_k}^2}} \tag{3.6}$$

Using the above equations, bending stress due to displacement of Y direction (M_Y), due to displacement of Z direction (M_Z), and due to torsional deformation of the floor (S), each full-plastic strength ($M_{p,c,Y}$), ($M_{p,c,Z}$) and ($S_{p,c}$) can be denoted by the following equations.

$$M_Y = \sum M_{Y,k} \tag{3.7}$$
$$M_Z = \sum M_{Z,k} \tag{3.8}$$
$$S = S_Y + S_Z \tag{3.9}$$
$$M_{p,c,Y} = M_{p,c,Y} = 4 M_p \tag{3.10}$$
$$S_{p,c} = 4\sqrt{2}\ l\ M_p \tag{3.11}$$

where,

$$S_Y = \left(M_{Y,1} - M_{Y,2} + M_{Y,3} - M_{Y,4}\right) l \tag{3.12}$$
$$S_Z = \left(M_{Z,1} + M_{Z,2} - M_{Z,3} - M_{Z,4}\right) l \tag{3.13}$$

Setting the centers of torsional rotation into the above equations between Eqn.3.7 and Eqn.3.13 with continuously varying, the yield surface in the case of two structural planes is obtained.
On the yield surface of columns, it is determined by numerical calculations. Therefore, the yield surface in the case that the frame has more than two structural planes is derived in the same system as the case of two structural planes.

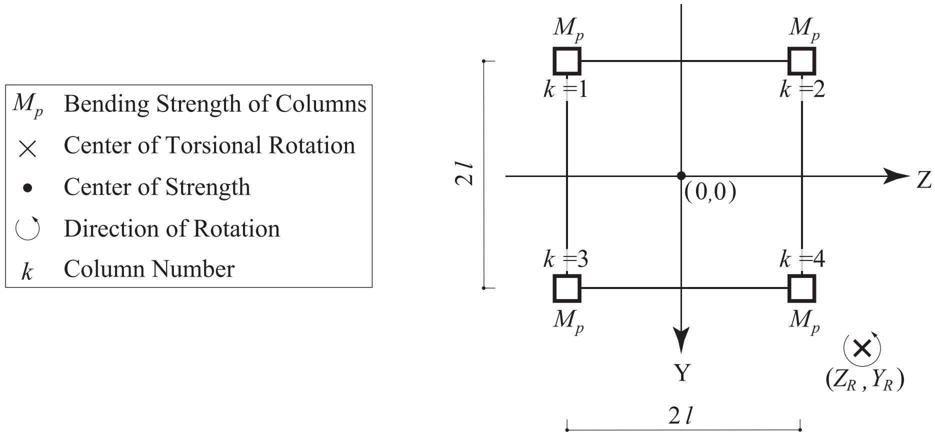


Figure3.1. Column model in two-structural plane system

3.2. Numerous Structural Plane System

The yield surface in the case of infinite structural planes is examined as a limiting case that columns at Z-Y structural plane are constituted with numerous structural planes as shown Figure 3.2. Then, all bending strength of a column is equally set to M_p , distances between the origin and the outermost structural plane of Y direction as well as Z direction are set to l respectively. Setting numerous values which can be considered as infinity to column number (k), the yield surface in the case of infinite structural planes is presumed. When the origin (0,0) is set at the center of strength and the column coordinate of k-th column is set to (z_k, y_k), bending stress due to displacement of Y direction (M_Y), due to displacement of Z direction (M_Z), and due to torsional deformation of the floor (S), each full-plastic strength ($M_{p,c,Y}$), ($M_{p,c,Z}$) and ($S_{p,c}$) can be denoted by the following equations.

$$M_Y = \sum M_{Y,k} \tag{3.14}$$

$$M_Z = \sum M_{Z,k} \tag{3.15}$$

$$S = \sum \left\{ M_{Y,k} \cdot (-z_k) + M_{Z,k} \cdot (-y_k) \right\} \tag{3.16}$$

$$M_{p,c,Y} = M_{p,c,Y} = \sum k \cdot M_p \tag{3.17}$$

$$S_{p,c} = \sum \sqrt{y_k^2 + Z_k^2} \cdot M_p \tag{3.18}$$

Setting the centers of torsional rotation into the above equations between Eqn.3.14 and Eqn.3.18 with continuously varying, the yield surface in the case of infinite structural planes is obtained.

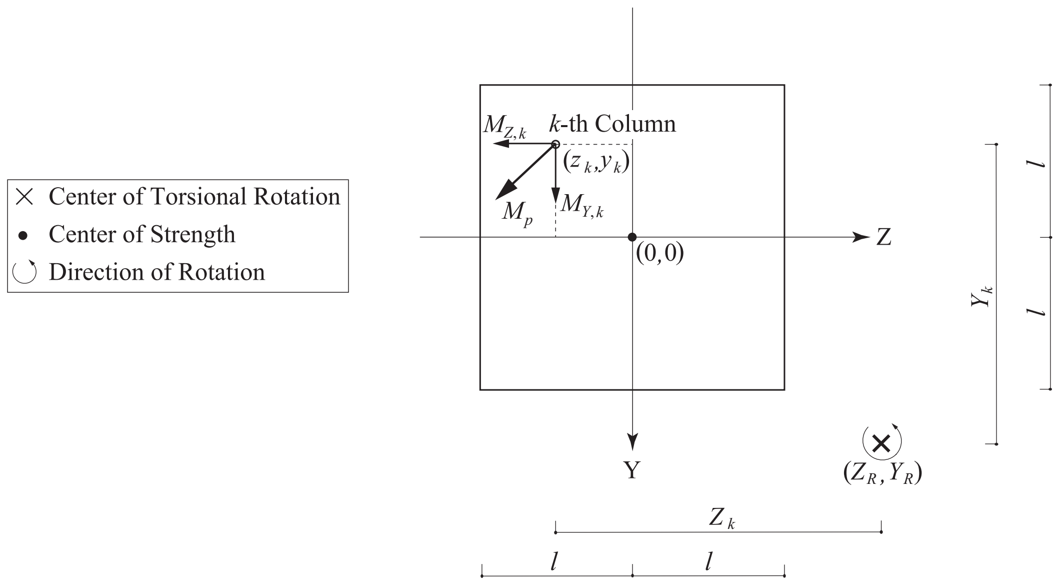


Figure3.2. Column model in infinite structural plane system

Unlike the yield surface of beams, the yield surface of columns is expressed as three-dimensional object which is composed of each bending stresses due to displacement in the two directions (M_Y) (M_Z) and bending stress due to torsional deformation of the floor (S). In this study, using the following equation, each bending stresses due to displacement in the two directions are combined to a single value. Consequently, the yield surface of columns in three dimensions can be expressed in two dimensions.

$$\frac{M}{M_{p,c}} = \sqrt{\left(\frac{M_Y}{M_{p,c,Y}}\right)^2 + \left(\frac{M_Z}{M_{p,c,Z}}\right)^2} \tag{3.19}$$

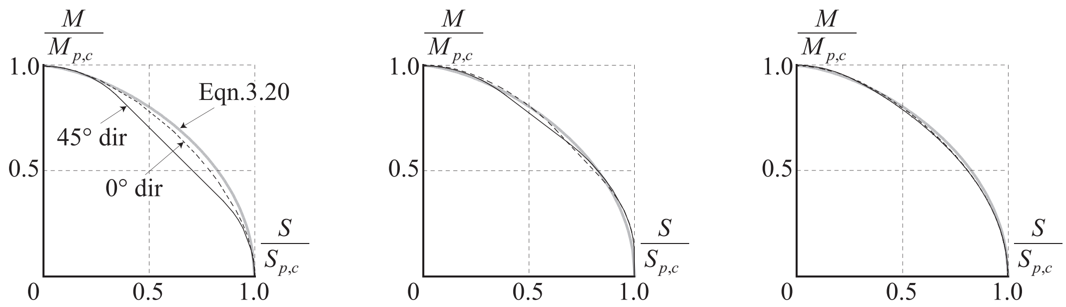
Moreover, the centers of torsional rotation are considered on two axes with continuously varying. These axes are expressed as two angles from the origin to each structural plane; one is zero degree direction and the other one is 45-degree direction. Consequently, shape characteristic on the yield surface of columns in three dimensions can be examined in two dimensions.

3.3. Yield Surface of Columns in Each Structural Plane System and Center of Torsional Rotation

Fig.3.3 and Fig.3.4 show the results on each number of structural planes and on the each center of torsional rotations respectively. Vertical axis in this figure is expressed as shown Eqn.3.19, and horizontal axis is expressed in the same manner as the yield surface of beams. Fig.3.3(a) shows that there is the difference of shape between two surfaces which consist of two different angles on the center of torsional rotation. It implies that the yield surface of columns undulates own surface. Fig.3.3(b) and (c) show that the difference of shape between two surfaces becomes gradually smaller with increase of structural planes, eventually, two surfaces can be expressed as mostly same shape. Additionally, the yield surface as shown the following equation is also denoted in Fig.3.3 and Fig.3.4.

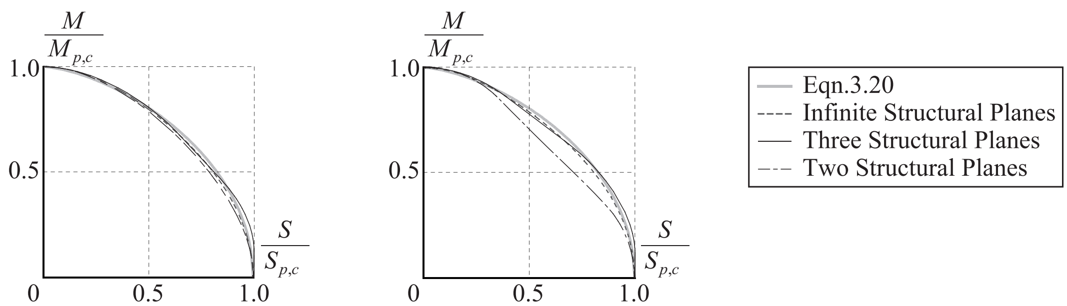
$$F = \left(\frac{S}{S_{p,c}}\right)^{1.5} + \left(\frac{M}{M_{p,c}}\right)^2 - 1 = 0 \tag{3.20}$$

The yield surface as shown Eqn.3.20 can approximate almost all the cases. Obtaining a generic analysis result concerning the frame which has multiple structural planes, yield surface of columns, which has inclusive shape, should be also set same as the case of beams. Consequently, yield surface as shown Eqn.3.20 is proposed as yield surface of columns.



(a) Two structural planes (b) Three structural planes (c) Infinite structural planes

Figure3.3. Yield surface of columns with respect to the number of structural planes



(a) 0° direction (b) 45° direction

Figure3.4. Yield surface of columns on each center of torsional rotations in each structural plane system

4. CONCLUSION

In this paper, the yield surfaces of each story, which has inclusive shape, were examined through simple plastic analysis for the frame which has multiple structural planes. As a result, the yield surface in the case of infinite structural planes shown as Eqn.2.13 and as shown Eqn.3.20 were anew proposed for the yield surface of beams and columns respectively.

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