Effects of Co-efficient of Restitution, Structural Yielding and Gap Ratios on the Impact Mechanics of Building Pounding

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SUMMARY:
A linear and non-linear model are developed to analyze the structural impact and response of two single degree of freedom structures, representing adjacent buildings or bridge sections. Different impact coefficients of restitution, normalized distances between structures and a range of different structural periods are considered. The probability of impact and the displacement changes that can result from these collisions are computed. The likelihood of an increase in displacement is quantified in a probabilistic sense. A full matrix of response simulations are performed to individually investigate and delineate the effects of inter-structure gap-ratio, period ratios, structural non-linearity and impact elasticity. Column inelasticity is incorporated through the use of a Ramberg-Osgood type hysteresis rule.

The minimum normalized distance, or gap-ratio, required between two structures to ensure that the likelihood of increased displacement of more than 10% for either structure for 90% of the given earthquake ground motions is assessed as one of many possible design risk bounds. Increased gap ratio, defined as a percentage of spectral displacement, is shown to reduce the likelihood of impact, as well as close structural periods. Larger differences in the relative periods of the two structures were seen to significantly increase the likelihood of impact. Inclusion of column inelasticity and higher plasticity of impact reduce displacement increases from impact and thus possible further damage to the structures. Such information can be used as a guideline to manage undesirable effects of impact in design - a factor that has been observed to be very important during the recent Canterbury, New Zealand Earthquakes.

Keywords: Building Pounding, Impact Mechanics, Spectral Analyses,

1. INTRODUCTION

In highly urbanized areas, distances between adjacent buildings can be relatively small due to a lack of space. When an earthquake occurs, this proximity can lead to impacts between structures, causing damage not expected in design. This phenomenon has appeared previously, after events in Mexico in 1985 and Kobe in 1995, resulting in significant damage beyond the damage from the event alone.

The same issue has been observed on bridges. Impact occurs because expansion joints between sections of the bridge allow relative displacement that can lead to collision and then induce increase in demand and damage. Hence, bridge structures at the deck level are another area where seismic impact can commonly occur.

There are no significant guidelines for impact. Typically, design guidelines are used to avoid impact, and thus do not consider it explicitly. Therefore, there is a need to assess risk and possible outcomes by creating probabilistic design guidelines. The aim of this paper is to rigorously analyze structural impact and response in a generic format to generate these guidelines and manage this issue. A number of studies have been conducted to look at pounding effects as summarised by (Cole, Dhakal et al. 2011). These have not usually been performed in a probabilistic sense and information is generally not available to answer the following questions:

1) For oscillators a certain distance apart, what is the probability of impact over suites of probabilistically scaled ground motions? What is the influence of the oscillator period?
2) What is the probability that displacements will increase as a result of impact?
3) How far apart should structures be placed to minimize or avoid increased displacement?

This paper describes an initial study used to investigate these questions

2. METHODS

2.1. Models

Two single-degree-of-freedom oscillators are used to model single degree-of-freedom (SDOF) decoupled structures, as shown in Figure 1, which can represent adjacent bridge sections or structures. When no impact occurs (x1 < x2 + λ) the oscillators are decoupled and a model defined:

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1 x_1 = -m_1\ddot{x}_{gr}$$
$$m_2\ddot{x}_2 + c_2\dot{x}_2 + k_2 x_2 = -m_2\ddot{x}_{gr}$$

(1.1)

where x1 and x2 are displacements, k_1 and k_2 are stiffness, m_1 and m_2 are mass, c_1 and c_2 are damping, and λ is the gap, where all these variables are shown in Figure 1.
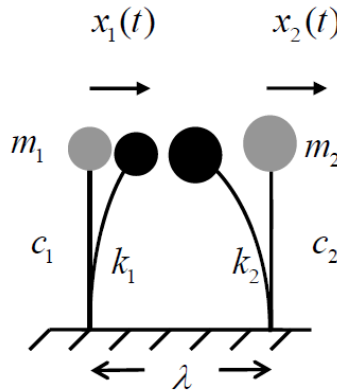

Figure 1. Model used for the impact study

Impact occurs when the two masses touch and $x_1 = x_2 + \lambda$. The amount of dissipated energy is quantified within a coefficient of restitution, e, which is equal to 1.0 when the collision is fully elastic (no energy dissipated), and 0.0 when the collision is fully plastic (all the energy is dissipated). In reality, the coefficient of restitution is sensitive to the prior-impact velocity and tends to a range between 0.6 and 0.8 (Jankowski 2007; Jankowski 2010).

Conservation of momentum and kinetic energy enables post-impact velocities to be calculated:

$$\dot{x}_1' = \dot{x}_1 - (1+e)\frac{m_2}{m_1+m_2}(\dot{x}_1 - \dot{x}_2)$$

$$\dot{x}_2' = \dot{x}_2 + (1+e)\frac{m_2}{m_1+m_2}(\dot{x}_1 - \dot{x}_2)$$

(1.2)

where $\dot{x}_1$ and $\dot{x}_2$ are the velocities before impact, and $\dot{x}_1'$ and $\dot{x}_2'$ are the velocities just after impact.

To model impact, an equivalent impulse force is used in Equation (1) as one time step external load:

$$F = \frac{m(\dot{x}' - \dot{x})}{dt} \qquad (1.3)$$

Hence, the equations of motion are coupled only when impact occurs, and are defined:

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 = -m_1\ddot{x}_{gr} + F$$
$$m_2\ddot{x}_2 + c_2\dot{x}_2 + k_2x_2 = -m_2\ddot{x}_{gr} + F \qquad (1.4)$$

The gap ratio, GR, normalizes the gap between masses to their spectral displacements, generalizing the analysis to a much wider range of structures.

$$GR = \frac{\lambda}{Sd_1 + Sd_2} \qquad (1.5)$$

Spectral displacements, Sd_1 and Sd_2, are the maximum absolute displacements of the uncoupled structures for a given ground motion input without impact. Thus, impact may only occur if GR < 1.0, and the two systems are independent otherwise. Therefore, the probability of impact increases as the gap ratio decreases toward GR → 0.0 when the structures are touching at rest.

To include column yielding the Ramberg-Osgood nonlinear model is used. This value modifies column forces and stiffness each time step. In this study, an elasto-plastic curve is used with ~5-10% post-yield stiffness.

The models are solved using numerical integration. Inner iteration is required each time step with the Ramberg-Osgood model as the column force is a function of displacement. Due to the speed of impact and use of Equation (1.3) a time step of $dt = 0.001$s is chosen based on a convergence analysis.

2.2. Earthquakes

The earthquake records utilized are a subset from the three suites from the SAC Steel Project in Los Angeles (Sommerville et al. 1997). In this study, only the medium suite of design level events (10% in 50 years probability of occurrence) is used to ensure yielding of columns without having ground motions of significantly different magnitudes that may skew results. Each record was run in both directions to remove directional effects, thus eliminating any effect of which order the two structures of Figure 1 are placed, and resulting in 40 simulations for the entire suite.

2.3. Analyses and Metrics

For each ground motion the following periods were analysed: T = 0.2, 0.5, 0.8, 1.0, 3.0 and 5.0 sec. The following gap ratios were used A worst case coefficient of restitution of e = 1.0 was used, as well as a high dissipation value of e = 0.4 to show a range across least to greatest dissipation. The mass of each oscillator was M = 1000kg and periods are varied by changing the linear stiffness. Inelastic cases use a column yielding force of 10% of oscillator weight Structural damping of the columns was 5%. The following two main metrics were assessed:

1) Likelihood of impact for GR = 0.2, 0.5, 0.8 and 0.9 for both elastic and inelastic columns (1)
2) Minimum gap ratio (GR) that ensures < 10% increase in displacement for 90% of events (2)

The first assesses the likelihood of impact, the second is a design guideline to minimise the likelihood of significant damage, where both metrics present the results in a fashion directly useful for use within a probabilistic design framework.

3. RESULTS AND DISCUSSION

3.1. Likelihood of Impact

Table 3.1 shows the likelihood of impact results for all gap ratios for the elastic (left) and inelastic columns (right). The diagonal elements are 0.0 as two identical oscillators will never impact for ground motions in either direction. The matrices are symmetric given that ground motions are run in both directions. Note that the likelihood or probability of impact is independent of the coefficient of restitution used. In this study, e = 1.0 and 0.4 are used to span a range of possibilities.

In general, the probability of impact increases as T_1 becomes more different from T_2. The probability of impact also increases as the gap ratio decreases, as expected because the lower the gap ratio, the closer the oscillators. Note that for GR= 0.2, in the linear case, impact always occurs because the two structures are so close in this configuration.

Table 3.1. Probability of at least one impact (elastic columns (left), inelastic columns (right), gap ratios = (0.2, 0.5, 0.8, 0.9), for all 40 ground motions. Note GR = 1.0 has 0.0 probability of impact.

GR=0.2

T1 / T2	0.2	0.5	0.8	1.0	3.0	5.0
0.2	0.00	1.00	1.00	1.00	1.00	1.00
0.5	1.00	0.00	1.00	1.00	1.00	1.00
0.8	1.00	1.00	0.00	1.00	1.00	1.00
1.0	1.00	1.00	1.00	0.00	1.00	1.00
3.0	1.00	1.00	1.00	1.00	0.00	1.00
5.0	1.00	1.00	1.00	1.00	1.00	0.00

GR=0.2

T1 / T2	0.2	0.5	0.8	1.0	3.0	5.0
0.2	0.00	0.65	0.70	0.73	0.95	1.00
0.5	0.65	0.00	0.40	0.55	0.93	0.98
0.8	0.70	0.40	0.00	0.28	0.85	0.95
1.0	0.73	0.55	0.28	0.00	0.88	0.95
3.0	0.95	0.93	0.85	0.88	0.00	0.95
5.0	1.00	0.98	0.95	0.95	0.95	0.00

GR=0.5

T1 / T2	0.2	0.5	0.8	1.0	3.0	5.0
0.2	0.00	1.00	1.00	1.00	1.00	1.00
0.5	1.00	0.00	0.98	0.98	1.00	1.00
0.8	1.00	0.98	0.00	0.95	1.00	1.00
1.0	1.00	0.98	0.95	0.00	0.98	0.98
3.0	1.00	1.00	1.00	0.98	0.00	0.95
5.0	1.00	1.00	1.00	0.98	0.95	0.00

GR=0.5

T1 / T2	0.2	0.5	0.8	1.0	3.0	5.0
0.2	0.00	0.08	0.25	0.38	0.63	0.88
0.5	0.08	0.00	0.10	0.20	0.48	0.78
0.8	0.25	0.10	0.00	0.00	0.48	0.68
1.0	0.38	0.20	0.00	0.00	0.43	0.63
3.0	0.63	0.48	0.48	0.43	0.00	0.50
5.0	0.88	0.78	0.68	0.63	0.50	0.00

GR=0.8

T1 / T2	0.2	0.5	0.8	1.0	3.0	5.0
0.2	0.00	0.68	0.85	0.80	0.98	0.85
0.5	0.68	0.00	0.20	0.35	0.80	0.83
0.8	0.85	0.20	0.00	0.00	0.53	0.73
1.0	0.80	0.35	0.00	0.00	0.43	0.50
3.0	0.98	0.80	0.53	0.43	0.00	0.25
5.0	0.85	0.83	0.73	0.50	0.25	0.00

GR=0.8

T1 / T2	0.2	0.5	0.8	1.0	3.0	5.0
0.2	0.00	0.00	0.00	0.03	0.35	0.40
0.5	0.00	0.00	0.00	0.00	0.18	0.30
0.8	0.00	0.00	0.00	0.00	0.20	0.30
1.0	0.03	0.00	0.00	0.00	0.13	0.23
3.0	0.35	0.18	0.20	0.13	0.00	0.05
5.0	0.40	0.30	0.30	0.23	0.05	0.00

GR=0.9

T1 / T2	0.2	0.5	0.8	1.0	3.0	5.0
0.2	0.00	0.13	0.48	0.50	0.75	0.73
0.5	0.13	0.00	0.03	0.10	0.43	0.45
0.8	0.48	0.03	0.00	0.00	0.18	0.30
1.0	0.50	0.10	0.00	0.00	0.13	0.23
3.0	0.75	0.43	0.18	0.13	0.00	0.05
5.0	0.73	0.45	0.30	0.23	0.05	0.00

GR=0.9

T1 / T2	0.2	0.5	0.8	1.0	3.0	5.0
0.2	0.00	0.00	0.00	0.00	0.15	0.20
0.5	0.00	0.00	0.00	0.00	0.05	0.10
0.8	0.00	0.00	0.00	0.00	0.05	0.05
1.0	0.00	0.00	0.00	0.00	0.03	0.00
3.0	0.15	0.05	0.05	0.03	0.00	0.00
5.0	0.20	0.10	0.05	0.00	0.00	0.00

Compared to the linear case, the probability of impact is lower in the non-linear case. This result occurs because column yielding dissipates energy. The result is a generally decreased displacement for this suite of design level events. Even if the yielding force chosen is quite high, it is sufficient to introduce measurable differences between the elastic and inelastic models.

In summary, determining the likelihood of at least one impact is important in design, because it can lead to an increase of displacement and thus, potential damage. In general, a decrease of the gap ratio, GR, leads to an increasing likelihood of impact. Considering inelastic columns significantly decreases this probability compared to the linear case.

3.2. Displacement and Impact

Figure 2 presents the displacement of both oscillators and impacts over time with 2 different values of e, for the linear and non-linear case, and for one earthquake (LA21). It may be noted that the displacement magnitude for the oscillator with the higher period is greater than for the other oscillator, and for both of them it is larger for the higher value of e, where less energy is dissipated during impact. For the non-linear case, the displacements of both oscillators do not tend to 0, because column yielding induces plastic deformation. In contrast, more impacts occur for the linear case and larger values of e.
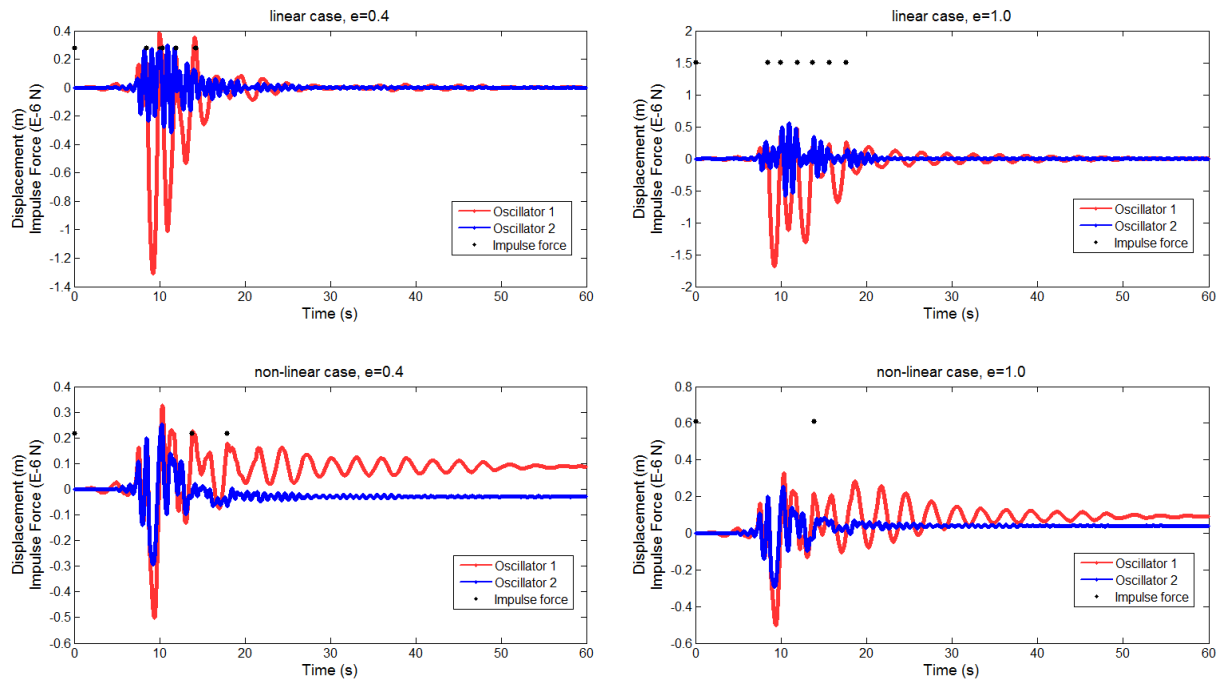

Figure 2. Displacement response (LA21, $T_1 = 3s$, $T_2 = 0.8s$, $e = (0.4, 1.0)$, linear and non-linear cases)

3.3. Minimum GR to ensure an increase in displacement < 10% for 90% of ground motions

Tables 3.2 and 3.3 show the minimum GR for which displacement increases less than 10% for 90% of (40) earthquakes. Note that GR = 0 when $T_1 = T_2$ and are larger as periods differ more. In the linear case (Table 3.2), the extreme pair of periods (0.2, 5.0) always leads to a minimum gap ratio of 1.00 regardless of oscillator and value of e. Generally, an increase of the coefficient of restitution (e) increases the minimum GR. Finally, the two tables are symmetric, as with Table 3.1, as ground motions are run both directions. Equally, Table 3.2 shows that the pair of sub-tables are simply transposed sets as would be expected when periods of the two systems are reversed.

Table 3.2. Minimum gap ratio which insure less than 10% of increase for 90% of earthquakes, assuming elastic columns and e = (0.4 and 1.0), for oscillator 1 (left) and oscillator 2 (right)

e=0.4

T2 \ T1	0.2	0.5	0.8	1.0	3.0	5.0
0.2	0.00	0.85	0.95	0.95	1.00	1.00
0.5	0.00	0.00	0.35	0.70	0.80	0.70
0.8	0.00	0.00	0.00	0.15	0.75	0.60
1.0	0.00	0.50	0.65	0.00	0.65	0.60
3.0	0.85	0.90	0.90	0.75	0.00	0.00
5.0	1.00	0.95	0.90	0.95	0.75	0.00

T2 \ T1	0.2	0.5	0.8	1.0	3.0	5.0
0.2	0.00	0.00	0.00	0.00	0.85	1.00
0.5	0.85	0.00	0.00	0.50	0.90	0.95
0.8	0.95	0.35	0.00	0.65	0.90	0.90
1.0	0.95	0.70	0.15	0.00	0.75	0.95
3.0	1.00	0.80	0.75	0.65	0.00	0.75
5.0	1.00	0.70	0.60	0.60	0.00	0.00

e=1.0

T2 \ T1	0.2	0.5	0.8	1.0	3.0	5.0
0.2	0.00	0.85	0.95	0.95	1.00	1.00
0.5	0.00	0.00	0.60	0.75	0.90	0.85
0.8	0.00	0.65	0.00	0.30	0.80	0.80
1.0	0.60	0.80	0.70	0.00	0.70	0.80
3.0	0.95	0.95	0.90	0.85	0.00	0.50
5.0	1.00	0.95	0.95	0.95	0.85	0.00

T2 \ T1	0.2	0.5	0.8	1.0	3.0	5.0
0.2	0.00	0.00	0.00	0.60	0.95	1.00
0.5	0.85	0.00	0.65	0.80	0.95	0.95
0.8	0.95	0.60	0.00	0.70	0.90	0.95
1.0	0.95	0.75	0.30	0.00	0.85	0.95
3.0	1.00	0.90	0.80	0.70	0.00	0.85
5.0	1.00	0.85	0.80	0.80	0.50	0.00

Table 3.3. Minimum gap ratio which insure less than 10% of increase for 90% of earthquakes, assuming inelastic columns and e = (0.4 and 1.0), for oscillator 1 (left) and oscillator 2 (right)

e=0.4

T2 \ T1	0.2	0.5	0.8	1.0	3.0	5.0
0.2	0.00	0.30	0.55	0.55	0.80	0.80
0.5	0.25	0.00	0.20	0.30	0.65	0.70
0.8	0.20	0.00	0.00	0.10	0.50	0.55
1.0	0.30	0.00	0.00	0.00	0.40	0.55
3.0	0.50	0.35	0.25	0.25	0.00	0.30
5.0	0.70	0.65	0.60	0.60	0.50	0.00

T2 \ T1	0.2	0.5	0.8	1.0	3.0	5.0
0.2	0.00	0.25	0.20	0.30	0.50	0.70
0.5	0.30	0.00	0.00	0.00	0.35	0.65
0.8	0.55	0.20	0.00	0.00	0.25	0.60
1.0	0.55	0.30	0.10	0.00	0.25	0.60
3.0	0.80	0.65	0.50	0.40	0.00	0.50
5.0	0.80	0.70	0.55	0.55	0.30	0.00

e=1.0

T2 \ T1	0.2	0.5	0.8	1.0	3.0	5.0
0.2	0.00	0.35	0.55	0.60	0.80	0.85
0.5	0.30	0.00	0.25	0.30	0.65	0.80
0.8	0.25	0.00	0.00	0.10	0.55	0.70
1.0	0.30	0.10	0.00	0.00	0.45	0.65
3.0	0.65	0.40	0.30	0.30	0.00	0.40
5.0	0.70	0.65	0.65	0.60	0.60	0.00

T2 \ T1	0.2	0.5	0.8	1.0	3.0	5.0
0.2	0.00	0.30	0.25	0.30	0.65	0.70
0.5	0.35	0.00	0.00	0.10	0.40	0.65
0.8	0.55	0.25	0.00	0.00	0.30	0.65
1.0	0.60	0.30	0.10	0.00	0.30	0.60
3.0	0.80	0.65	0.55	0.45	0.00	0.60
5.0	0.85	0.80	0.70	0.65	0.40	0.00

Once again, the better way to respect this design risk bound for a relatively small gap ratio is to ensure close structural periods, non-linearity and high plasticity of impact. The interest and the significance of these results can be shown best by example.

Take two structures with a mass of 1000kg each that can be modeled as non-linear SDOF oscillators with a yielding force of 10% of their respective weight, and with a resonance period of 0.2s and 3.0s due to their respective stiffness. Considering the impact fully elastic (e = 1.0), the gap ratio required to ensure displacement increases less than 10% for 90% of the earthquakes for both oscillators is GR = 0.8. Now, consider that for the same design level, the maximum credible earthquake used for the design leads to spectral displacements of 0.15m and 1.00m, respectively. Then a gap of 0.92m is required between the two structures to ensure this level of confidence

4. CONCLUSIONS

This study was conducted to analyze structural impact and responses of two SDOF structures. Different values of impact coefficient of restitution were considered, together with linear and non-linear columns, and a range of gap ratios, thus covering a large number of real cases. It was found that:

- The probability of impact between structures increases as the gap ratio decreases and structural period differences increase.
- A non-linear model leads to a drop in probability of impact and the smaller the coefficient of restitution the smaller the likelihood of increased post-impact displacement.
- The gap ratio to ensure less than 10% increase in displacement for 90% or more of ground motions was assessed for linear and non-linear columns and differing coefficients of restitution. An example of the use of this value was presented, and its main value is it allows a gap ratio to be defined in design to meet an explicit risk requirement.

The overall results are created to provide performance based design framework for assessing risk directly in design, and the approach is completely general.

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