Evaluation of the Relationship between Maximum Responses and Input Energies in Long-Period Ground Motions for Design Earthquake Motions

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SUMMARY:
The object in this study is to propose a method that evaluates design earthquake motions by using a velocity response spectrum (S_V-spectrum) which represents the maximum response of structures, an energy spectrum (V_E-spectrum) which reflects the input energy into structures, and an f-value (Akiyama and Kitamura, 2006) which depends on the ratio of the average V_E-spectrum to the average S_V-spectrum. The proposed method has the advantage of being able to adjust the input energy for every site easily by the V_E-spectrum without changing the S_V-spectrum. This study evaluates the f-value for every divided period range by using the observed earthquake records, and distinguishes the f-value in the short-period range from that in the long-period range. The relationships among the f-value, the hypocentral distance, the magnitude, and the site characteristics have been revealed through the above analyses.

Keywords: Long-Period Ground Motion, Design Earthquake Motion, Maximum Response, Input Energy, The 2011 off the Pacific Coast of Tohoku Earthquake

1. INTRODUCTION

Long-period, long-duration waves in earthquake ground motions are known to have serious effects on high-rise buildings. Some high-rise buildings continued to be shaken for more than ten minutes by the 2011 off the Pacific coast of Tohoku, Japan, earthquake. In the case that the Tokai earthquake and the Tonankai earthquake occur simultaneously as predicted, the earthquake motions may induce long-period waves, and increase the earthquake input energy owing to its long duration, compared to the case that each earthquake occurs separately. Therefore, besides the amplitude characteristics, the envelope and the duration characteristics should be taken into consideration in simulating the earthquake ground motions, and the earthquake input energy to structures should be evaluated in the design of high-rise buildings and seismic isolated structures.

In this study, we propose a method that evaluates design earthquake motions by using the velocity spectrum (S_V-spectrum) which represents the maximum response of structures, and the energy spectrum (V_E-spectrum) which reflects the input energy into structures, the f-value (Akiyama and Kitamura, 2006) which depends on the ratio of the average V_E-spectrum to the average S_V-spectrum. Since the f-value is known to be proportional to the input energy and the duration, it is considered to be an effective index for evaluating the envelope characteristics. Design earthquake motions can be evaluated by one of the following two methods: (1) Give the target S_V-spectrum and the f-value in order to set the target V_E-spectrum and the envelope characteristics, (2) Give the target S_V-spectrum and the target V_E-spectrum in order to set the f-value and the envelope characteristics. These proposed methods have advantages of being able to adjust the target spectrum easily and compensate the lack of the duration without changing the S_V-spectrum at each site.

While the previous study (Akiyama and Kitamura, 2006) deals with the *f*-value in the entire period range by mainly considering the predicted earthquake motions using the fault models, this study evaluates the *f*-value in every divided period range (0.1-1s, 1-2s, 2-3s, and 3-10s) by using the observed earthquake records including the 2011 off the Pacific coast of Tohoku earthquake. In this paper, the relationships among the *f*-value, the hypocentral distance, the magnitude, and the site characteristics have been revealed.

2. NUMBER OF REPETITION OF ELEMENT EARTHQUAKES

2.1. Definition of *f*-value

Under the assumption that the long-duration earthquake ground motions consist of the repetition of elemental earthquakes, Akiyama and Kitamura (2006) defined the *f*-value by the following equations:

$$F(h) = \bar{S}_{V,h} \Big/ \bar{V}_{E,h=0.1}\ , \tag{1}$$

$${}_0F(h) = {}_0\bar{S}_{V,h} \Big/ {}_0\bar{V}_{E,h=0.1}\ , \tag{2}$$

$$f = \left(\frac{{}_0F(h)}{F(h)}\right)^2 = \left(\frac{{}_0\bar{S}_{V,h} \big/ {}_0\bar{V}_{E,h=0.1}}{\bar{S}_{V,h} \big/ \bar{V}_{E,h=0.1}}\right)^2 = \left(\frac{\bar{V}_{E,h=0.1} \big/ \bar{S}_{V,h}}{{}_0\bar{V}_{E,h=0.1} \big/ {}_0\bar{S}_{V,h}}\right)^2, \tag{3}$$

where $F(h)$ indicates the damping function of structures in the earthquake response (h is the damping factor), $\bar{S}_{V,h}$ indicates the average S_V-spectrum in the period range of 0-10s, $\bar{V}_{E,h=0.1}$ indicates the average V_E-spectrum in the period range of 0-10s, ${}_0F(h)$ indicates the damping function of structures for the elemental earthquake, ${}_0\bar{S}_{V,h}$ indicates the average S_V-spectrum of the elemental earthquake, and

Table 1. The average S_V-spectra, the average V_E-spectra, the values of the damping function of structures, and the *f*-values in the divided period ranges.

	Period (sec)	$\bar{S}_V(h=0.1)$ (cm/s)	$\bar{V}_E(h=0.1)$ (cm/s)	$F(h=0.1)$	${}_0F(h=0.1)$	f-value
El Centro record (The 1940 Imperial Valley earthquake)	0.1-10	27.25	60.60	0.450		1.04
	0.1-1	40.17	97.69	0.411		1.24
	1-2	42.23	96.72	0.437	0.458	1.10
	2-3	50.12	89.67	0.559		0.67
	3-10	20.04	46.09	0.435		1.11
Hachinohe record (The 1968 Tokachi-Oki earthquake)	0.1-10	30.97	70.22	0.441		1.08
	0.1-1	31.89	76.72	0.416		1.21
	1-2	50.41	115.40	0.437		1.10
	2-3	63.27	133.51	0.474		0.93
	3-10	23.45	53.82	0.436		1.10
JMA KOBE record (The 1995 Hyogoken-Nanbu earthquake)	0.1-10	52.38	108.09	0.485		0.89
	0.1-1	114.50	231.59	0.494		0.86
	1-2	136.33	244.94	0.557		0.68
	2-3	80.57	133.73	0.603		0.58
	3-10	27.66	67.59	0.409		1.25
Tomakomai (HKD129) record (The 2003 Tokachi-Oki earthquake)	0.1-10	40.76	109.25	0.373		1.51
	0.1-1	9.81	34.96	0.281		2.66
	1-2	23.46	74.02	0.317		2.09
	2-3	36.07	107.10	0.337		1.85
	3-10	48.24	124.99	0.386		1.41
Shinjuku (TKY007) record (The 2011 off the Pacific coast of Tohoku earthquake)	0.1-10	22.51	79.94	0.282		2.64
	0.1-1	16.94	70.74	0.239		3.66
	1-2	22.49	85.49	0.263		3.03
	2-3	25.20	91.32	0.276		2.75
	3-10	22.91	78.81	0.291		2.48

derived by equation (1) [$F(h=0.1)$]
derived by equation (4) [${}_0F(h=0.1)$]
derived by equation (3) and equation (4) [f-value]

${}_{0}\bar{V}_{E,h=0.1}$ indicates the average V_E-spectrum of the elemental earthquake. Equation (3) implies that the *f*-value increases in the case of the earthquake that has a larger V_E-spectrum. Since the V_E-spectra of the long-duration ground motions are larger than those of other ground motions at the same period, the *f*-values increases relatively.
In the case that the El Centro record was selected as the elemental earthquake, the damping function of the elemental earthquake was approximated as follows by Akiyama *et al.* (2006):

$$ {}_{0}F(h) = \frac{1}{\sqrt{1+12\pi h}}. \tag{4} $$

The gray columns in Table 1 show the *f*-values of the five observed earthquake ground motions in the period range of 0.1-10s derived by equation (3) and equation (4), where the El Centro record is adopted as the elemental earthquake.
As shown in Table 1, the *f*-value of the El Centro record in the period range of 0.1-10s, as well as the Hachinohe record, is nearly equal to 1.0. The *f*-value of the JMA KOBE record that has short duration waves is less than 1.0, and those of the Tomakomai and Shinjuku records that have long-duration waves are greater than 1.0. These results imply that the duration characteristics of the earthquake records can be represented by the *f*-value.

2.2. Evaluation of *f*-value in Each Divided Period Range

Since equation (4) is for the *f*-value in the period range of 0-10s, it is not clear whether the same equation is able to be applied to the *f*-values for narrow period bands.
The *f*-value which was calculated by substituting equation (4) into equation (3) was evaluated in each divided period range of 0.1-1s, 1-2s, 2-3s, and 3-10s. Table 1 shows the average S_V-spectra, the average V_E-spectra, the values of the damping function of structures, and the *f*-values in each divided period range. As for the JMA KOBE and Tomakomai records, the *f*-values in the short-period ranges are different from those in the long-period ranges. The *f*-values of the El Centro and Hachinohe records have similar values regardless of the period range. In Table 1, the *f*-values of the El Centro record are not 1.0 exactly because equation (4) rather than equation (2) was used as the ${}_{0}F(h)$. This result implies that equation (4) can be applied to the El Centro and Hachinohe records in every divided period range.
Hence, in the following chapters, equation (4) is used when evaluating the *f*-values in each divided period range, and the El Centro record is adopted as the elemental earthquake.

3. ANALYSES OF *f*-VALUE FOR OBSERVED EARTHQUAKE RECORDS

The *f*-values, the average S_V-spectra, and the average V_E-spectra are analyzed by using the observed earthquake records of the 2011 off the Pacific coast of Tohoku, Japan, earthquake.
Figure 1(a)-(d) show the distribution maps of the average S_V-spectra, and Figure 2(a)-(d) show the distribution maps of the average V_E-spectra. In the period range of 0.1-1s, the average S_V-spectrum and the average V_E-spectrum become smaller according to the hypocentral distances as shown in Figure 1(a) and Figure 2(a). In the period range of 3-10s, the differences among the sites become remarkable, as shown in Figure 1(d) and Figure 2(d). Especially, the average S_V-spectra and the average V_E-spectra at the sites on the sedimentary basins and plains seem to be amplified.
Figure 3(a)-(d) show the distribution maps of the *f*-values. Figure 3(a) implies that the *f*-values in the short-period ranges depend on the hypocentral distances and directions of the observed points from the hypocenter. The site characteristics of the *f*-values are not identified clearly in short-period ranges as shown in Figure 3(a) and (b), while the *f*-values in the long-period ranges seem to be affected by the site effects as shown in Figure 3(d). The *f*-values at the sites on the sedimentary basins and plains seem to be larger than those at the other sites.

3.1. Directional Dependency of *f*-value

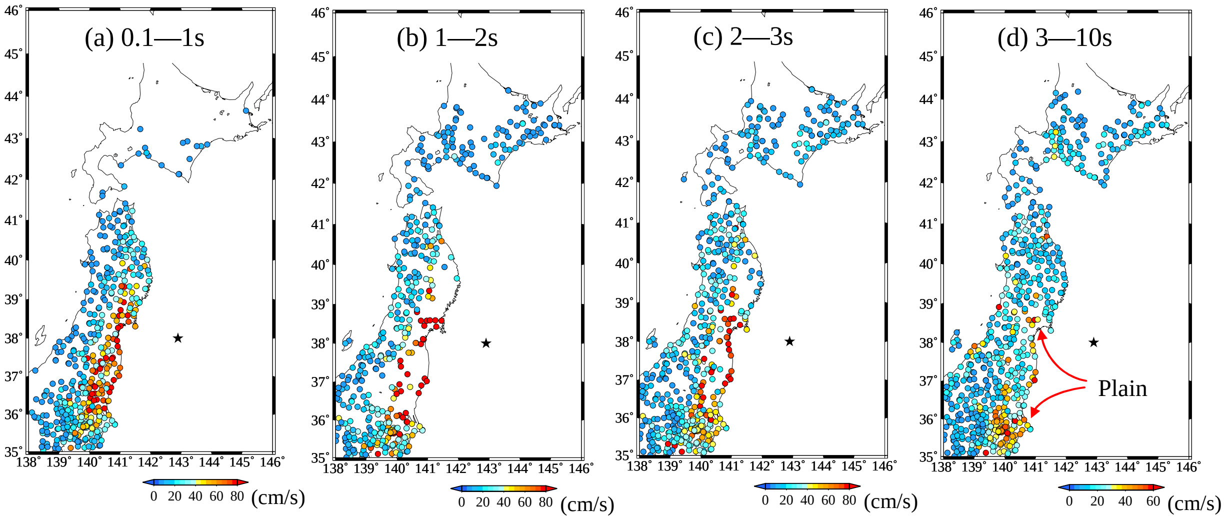


Figure 1. Distribution maps of the average S_V-spectra (h=0.05).

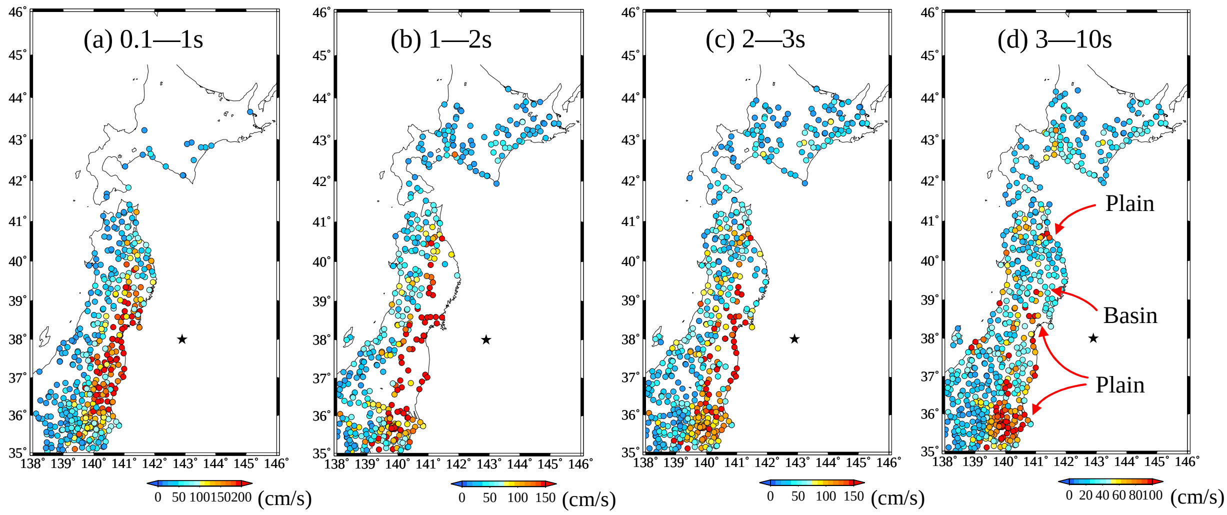


Figure 2. Distribution maps of the average V_E-spectra (h=0.1).

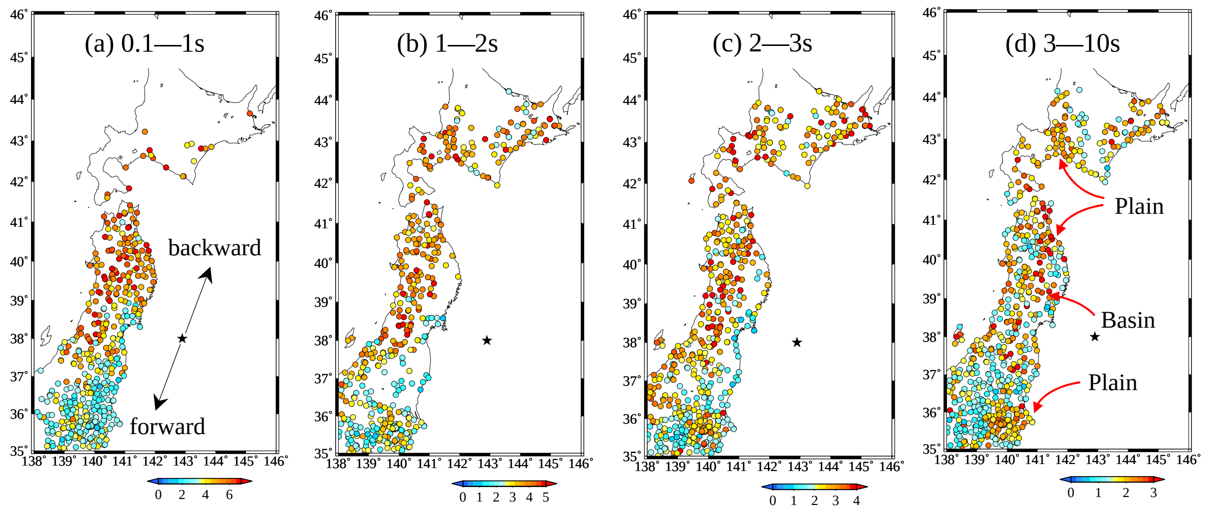


Figure 3. Distribution maps of the f-values.

In the short-period range, seismic waves that own short duration and sharpness are generated in the direction of the rupture propagating. Since the maximum amplitude of the earthquake ground motion causes the maximum response of structures in general, the S_V-spectrum in the forward rupture direction becomes much larger. In the backward rupture direction, seismic waves that own long duration and obtuseness are generated and the energy of the seismic waves are accumulated, which increases the V_E-spectrum in this direction. Therefore, a directional dependency in north-south direction, based on the average S_V-spectra and the average V_E-spectra, appears in Figure 3(a).

3.2. Hypocentral Distance Dependency of *f*-value

Akiyama and Kitamura (2006) revealed that the correlation between the duration of the earthquake ground motions and the *f*-value. Satoh *et al*. (2010) clarified that the correlation between the duration of the earthquake ground motions and the hypocentral distance in the period range of 2s or less. Therefore, the *f*-value is supposed to be proportional to the hypocentral distance in the period range of 2s or less.

3.3. Site Dependency of *f*-value

Figure 4 shows the interpretations of the S_V-spectrum and the V_E-spectrum, and indicates that the V_E-spectrum depends on the duration more than the S_V-spectrum.

$$S_V \Leftrightarrow \dot{x}$$

$$V_E = \sqrt{\frac{2E}{m}} \Leftrightarrow E = -\ m \int_0^t \ddot{z}\,\dot{x}\ dt$$

earthquake power　　duration

response amplitude

E : input energy to structures during the earthquake
m : mass of structures
$\ddot{z}$: earthquake acceleration
$\dot{x}$: relative response velocity

Figure 4. Interpretations of the S_V-spectrum and the V_E-spectrum.

Therefore, in the long-period range, the long duration of the earthquake on the sedimentary basin affects the *f*-value. The duration characteristics of the earthquake can be revealed by the *f*-value.

4. QUALITATIVE EVALUATION OF *f*-VALUE

4.1. Relationship between Hypocentral Distance and *f*-value

In this section, we analyzed the records of the 2011 off the Pacific coast of Tohoku earthquake (the 2011 earthquake, hereafter), the 2003 Tokachi-Oki earthquake (the 2003 earthquake, hereafter), and the largest aftershock of the 2003 earthquake (the 2003 aftershock, hereafter).
Figure 5(a)-(d) show the relationship between the hypocentral distances and the *f*-values with the line by the least-squares approximation method. The *f*-values in the period range of 0.1-1s vary widely from the approximation line. However, the dispersion seems to become smaller as the period range becomes longer. The slopes of the approximation lines get gentle in the long-period ranges, and the slope in the period range of 3-10s is almost flat.
Table 2 shows the correlation coefficients between the hypocentral distances and the *f*-values in each earthquake. These correlation coefficients are evaluated under the assumption that the *f*-value depends on the following three factors: the directional dependency which is one of the source characteristics, the hypocentral distance which is the index that evaluates the path characteristics, and the site characteristics.
In the short-period ranges, the correlation coefficient is almost zero or negative for the 2011 earthquake and the 2003 aftershock, while it is positive for the 2003 earthquake. This is because no obvious site characteristics of the *f*-value are found in the short-period ranges in the three earthquakes

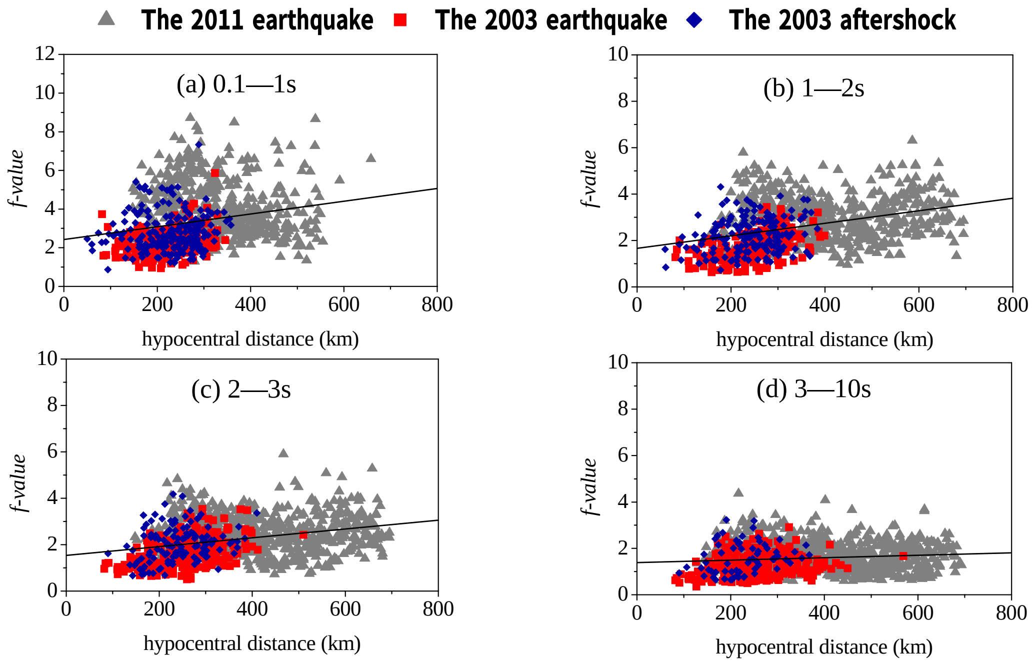


Figure 5. Relationship between the hypocentral distances and the *f*-values with the line by the least-squares approximation method.

Table 2. Correlation coefficients between the hypocentral distances and the *f*-values in the three earthquakes.

Earthquake	Period			
	short-period ranges		long-period ranges	
	0.1-1s	1-2s	2-3s	3-10s
The 2011 off the Pacific coast of Tohoku earthquake (The 2011 earthquake)	-0.157	-0.010	0.063	-0.108
The 2003 Tokachi-Oki earthquake (The 2003 earthquake)	0.339	0.505	0.478	0.204
The largest aftershock of the 2003 Tokachi-Oki earthquake (The 2003 aftershock)	0.050	0.221	0.238	0.246

and because the directional dependency of the 2003 earthquake is weak while that of the 2011 earthquake and that of the 2003 aftershock are much strong.

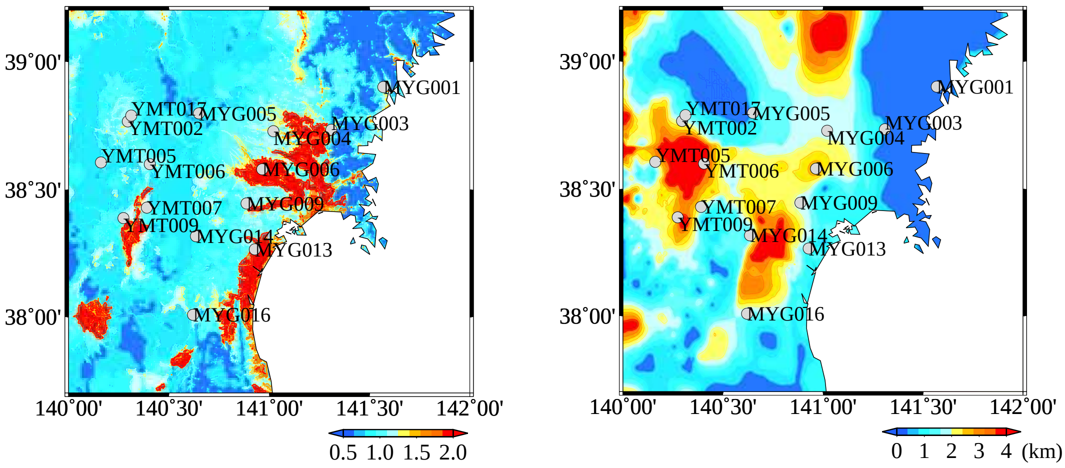
In the long-period ranges, the correlation coefficient is almost zero for the 2011 earthquake. It is a smaller positive value for the 2003 earthquake and the 2003 aftershock. This is because many sites have the thicker sedimentary layer and are also far from the hypocenter in the 2003 earthquake and the 2003 aftershock.

4.2. Relationship between Magnitude and *f*-value

The relationship between the moment magnitudes of the earthquakes and the *f*-values is evaluated. The observed earthquake records in 41 earthquakes including the aftershocks of the 2011 earthquake are used in this study. Figure 6(a) and (b) show the maps of the K-NET observation points around the Sendai Plain with the contours of the surface ground amplification ((a)) and the depth of the seismological basement ((b)).

Figure 7(a)-(d) show the relationship between the moment magnitudes M_W and the *f*-values with the line by the least-squares approximation method. The approximation line has a positive slope in the period ranges of 0.1-1s and 1-2s, has no slope in the period range of 2-3s, and has a negative slope in the period range of 3-10s.

Table 3 shows the correlation coefficient between the moment magnitudes and the *f*-values, which indicates that the f-value in the long-period ranges and the magnitude does not have positive correlation at least.



(a) Surface ground amplification

(b) Depth of the seismological basement (V_S=2700m/s)

Figure 6. Maps of the K-NET observation points around the Sendai Plain.

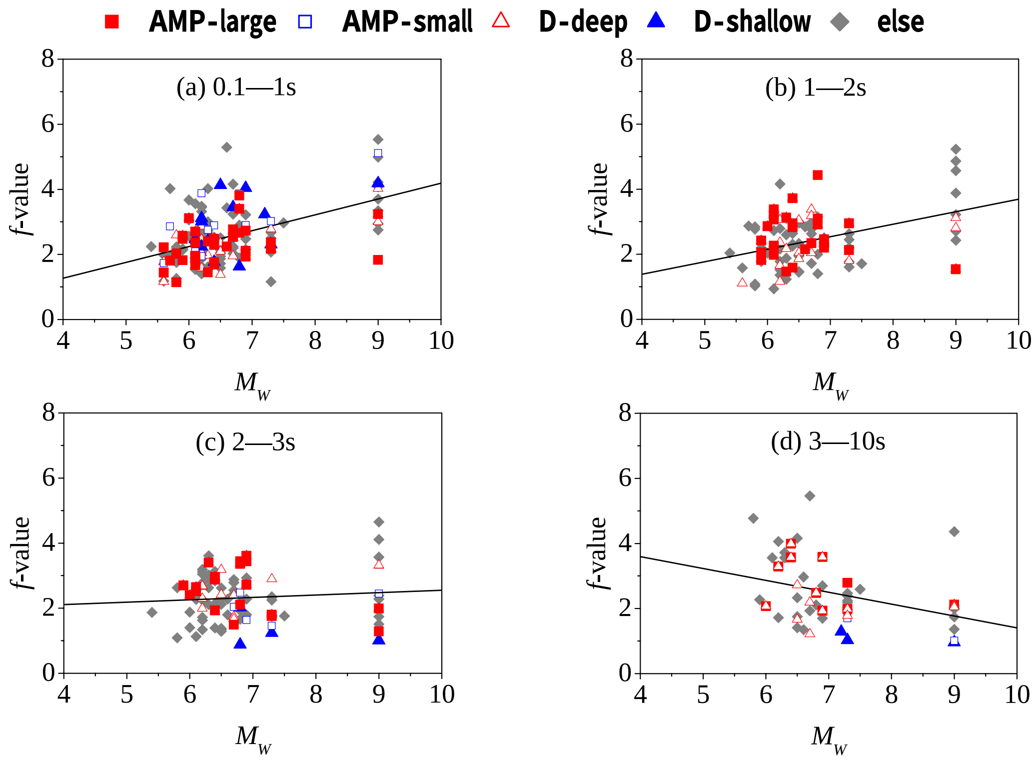


Figure 7. Relationship between the moment magnitudes M_W and the f-values with the line by the least-squares approximation method.

Table 3. Correlation coefficient between the moment magnitudes and the f-values.

Earthquakes	Period			
	short-period ranges		long-period ranges	
	0.1-1s	1-2s	2-3s	3-10s
The aftershocks of the 2011 off the Pacific coast of Tohoku earthquake *etc*.	0.502	0.412	0.094	-0.356

Table 4. Classification of the observation points based on the surface ground amplification (AMP) and the depth of the seismological basement (D).

Classification		Surface ground amplification (AMP)		
		small (AMP≦0.7)		large (AMP≧2.0)
Depth of the seismological basement (D)	shallow (D≦500m)		MYG001, MYG003	
		MYG014	MYG004, MYG005, MYG009, MYG016, YMT002, YMT005, YMT007, YMT017	MYG013
	deep (D≧3000m)		YMT006, YMT009	MYG006

4.3. Relationship between Site Amplification and *f*-value

The relationship between the site amplifications and the *f*-values is evaluated by using the same observed earthquake records in the previous section. Table 4 shows the classification of the observation points based on the surface ground amplification (AMP) and the depth of the seismological basement (D).

As shown in Figure 7, the *f*-values in the period ranges of 0.1-1s and 1-2s tend to independ on the classification of the observation points, while in the period ranges of 2-3s and 3-10s, the *f*-values tend to differ by the classification. The *f*-values of ‘AMP-small’ and ‘D-shallow’ are smaller than the approximation line. The *f*-values of ‘AMP-large’ are larger than the approximation line. The *f*-values of ‘D-deep’ have no distinctive tendency.

However, the *f*-values of ‘D-deep’ may become large because the thickness of the sedimentary layers corresponds to the depth of the seismological basement. In particular, in the case of the long-period ground motions, the seismic input energy to structures might increase by the interference of the seismic waves in the sedimentary basin, the *f*-values of ‘D-deep’ should be evaluated larger when we evaluate the envelope characteristics in the long-period range.

5. CONCLUSIONS

By using the observed earthquake records that include long-period ground motions, we qualitatively evaluated the relationships among the *f*-value, the hypocentral distance, the magnitude, and the site characteristics, and obtained the following conclusions:

1) In the El Centro and Hachinohe records, when evaluating the *f*-values in each divided period range, equation (4) by Akiyama and Kitamura (2006) is able to be used for evaluating the *f*-value in the entire period.

2) In the short-period range, the *f*-value has the directional dependency. The *f*-value is positively correlated with the hypocentral distance and the magnitude.

3) In the long-period range, the *f*-value is affected by the site effects mainly.

This study showed that the *f*-value is able to evaluate the characteristics of the earthquake ground motions in each period range, for each hypocentral distance, for each magnitude of the earthquakes, and at each site.

The *f*-value may be an effective index for evaluating the amplitude and the envelope characteristics for design earthquake motions.

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