Development of slice method based residual deformation analysis of slopes against earthquake

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SUMMARY
Estimation for residual deformation of earth structure against earthquake is an acute task in Japan since the feasibility of massive earthquake is higher in late years. The Newmark's method employing the limit equilibrium method with slices is widely used in design. It is simple and superior to other methods from a viewpoint of limitation in ground survey. It affords the estimation result with the use of limited soil parameters on strength parameters. Since the Newmark's method computes the residual displacement by direct time integration of acceleration the magnitude of which is over the limit acceleration obtained by quasi-static limit equilibrium method, its application is basically limited to the time duration of earthquake motion. This paper newly develops the simplified residual deformation analysis method based on the slice method, by introducing the rigid plastic constitutive equation into the slip line between the sliding soil block and the fixed ground. This method analyses the equation of motion in moment and it is applicable to not only the earthquake motion but also pre and post processes of the earthquake. When the strength of soil is reduced by earthquake, soil structures happen to deform after the earthquake. This behavior is triggered by reduction in soil strength from the peak to the residual strength due to shear deformation by earthquake. The similar phenomenon is observed when the higher excess pore water pressure is generated by earthquake motion. It is well known as the soil liquefaction induced collapses. In this paper, the reduction of soil strength from the peak to the residual states is newly modeled with the shear displacement along the slip line. It makes possible to assess the residual deformation of earth structure for the cases of post-earthquake gravity collapses. Through case studies, the applicability of developed method is examined in comparison with the Newmark's method.

Keywords: Residual deformation，Disintegration，Newmark's method

1. INTRODUCTION

The seismic design of earth structures has been conducted with the factor of safety which is determined by the seismic coefficient method. The design method tends to be modified to permit the finite deformation of earth structures against the level 2 ground motion since the Hyogo-ken Nanbu earthquake of 1995. From the viewpoint of rationalization of design, the seismic design of earth structures is checked by the residual deformation of structures caused by earthquake. The seismic coefficient method is difficult to assess the residual deformation of earth structure against the earthquake. Simple method to estimate the deformation of earth structure is required in design practice. Newmark's method is the commonly employed method to assess the residual deformation of earth structures against earthquake. It introduces the assumption of rigid plastic behavior into earth structures. Since it gives the residual deformation of earth structures based on simple assumptions, it is widely employed as a valid and practical method, the results of which can be multilaterally examined. Newmark's method adopts a limit horizontal acceleration of 1.0 in safety factor as a threshold value where an earth structure deforms. However, the limit horizontal acceleration is set to be constant during earthquake. Shear strength of soil sometimes declines by shear deformation. It is a great problem that Newmark's method can't consider the decline of shear strength (angle of shear resistance, cohesion) at slip line by large deformation. Earth structures often collapse after earthquake due to the decline of shear strength by shear deformation caused by earthquake. They deform by gravity and sometimes attain to collapse after earthquake. It is one of important aspects for deformation mechanism, but it is difficult for Newmark's method to estimate the residual deformation of earth structures in case of post-earthquake collapse. To make the practical applicability of Newmark's method higher, it is necessary to extend its applicability to post-earthquake collapse problem.

In this study, the authors developed a rigid plastic constitutive equation for the slip line between the sliding soil block and the fixed ground. This equation is expressed by the correlation between stress and velocity at slip line. Slice method is applied to residual deformation analysis since it is familiar and easy understanding for practical engineers. Rigid plastic constitutive equation is introduced into the slice method. Residual deformation of earth structures is computed by time integration of equation of motion in moment based on the framework of slice method. This method is possible to analyze the whole process of time deformation relationship of earth structure against earthquake.

In this paper, the formulation of slice method with the use of rigid plastic constitutive equation at slip line will be shown. Residual deformation is computed by time integration of equation of motion in moment. Through the case studies in comparison with the Newmark's method, the applicability of proposed method will be examined. Especially, the applicability to post-earthquake collapse will be closely examined.

2. FORMULATIOM OF RESIDUAL DEFORMATION ANALYSIS

This study developed the simplified residual deformation analysis based on the slice method. It expands the Newmark's method to analyze the whole process of time deformation relationship of earth structure. The formulation of residual deformation analysis based on the slice method will be expressed in the following.

2.1. Rigid Plastic Constitutive Equation

Rigid plastic assumption is applied to the behavior the elastic component of which is much smaller than the plastic component and is negligible. It is expressed in terms of stress and strain rate. The rigid plastic finite element method is proposed as calculation technique using rigid-plastic constitutive equation. It has mainly applied in the field of metal processing. Application in geotechnical engineering has been conducted to stability problem by Tamura and the authors. In this study, the rigid plastic constitutive equation for slip line is developed in the same way with that for soil proposed by Tamura and the authors.

The authors proposed the rigid plastic constitutive equation for soil as Eq.(2.2) by using the Drucker-Prager yield function of Eq.(2.1) in case of the friction materials. In the equations, I_1, J_2 are the first invariant of stress tensor and the second invariant of deviatoric stress tensor. α and k are coefficients corresponding to shear resistance angle and cohesion. $\dot{\boldsymbol{\varepsilon}}$, $\dot{e}$ are strain rate tensor and equivalent strain rate (Norm of the strain rate tensor). β, $\mathbf{I}$ are indefinite multiplier and unit tensor. Tensile stress is defined as positive. Details of derivation for rigid-plastic constitutive equation Eq.(2.2) are shown in the literature.

$$f(\boldsymbol{\sigma}) = \alpha I_1 + \sqrt{J_2} - k = 0 \qquad I_1 = tr(\boldsymbol{\sigma}) \;,\; J_2 = \frac{1}{2}\mathbf{s}:\mathbf{s} \tag{2.1}$$

$$\boldsymbol{\sigma} = \frac{k}{\sqrt{3\alpha^2 + 1/2}} \frac{\dot{\boldsymbol{\varepsilon}}}{\dot{e}} + \beta \left\{ \mathbf{I} - \frac{3\alpha}{\sqrt{3\alpha^2 + 1/2}} \frac{\dot{\boldsymbol{\varepsilon}}}{\dot{e}} \right\} \tag{2.2}$$

In this study, the rigid plastic constitutive equation of slip line will be developed with the Mohr Coulomb yield function. It is expressed in terms of stress and velocity at slip line. Stress on yield state is divided into two components of friction and adhesion. In the circular slip method, relative velocity of slip line between slices is related with the geometric condition and the angular rotation rate around the center. The rigid plastic constitutive equation of slip line is proposed as Eq.(2.3). It is expressed in terms of stress and velocity at slip lines. In the equation, τ, σ are shear stress and normal stress. $\dot{\theta}$, $|\dot{\theta}|$ are the angular rotation rate and its norm.

$$\tau = (c + \sigma \tan\phi)\frac{\dot{\theta}}{|\dot{\theta}|} \tag{2.3}$$

The above equation is expressed by the angular rotation rate. The equation is applicable when the angular rotation rate is not zero. It is difficult to apply in case of the angular rotation angle is very small as the earth structure is safe against external forces. It is necessary to define the constitutive equation even though the earth structure is safe. The authors introduce a threshold parameter, $|\dot{\theta}_0|$ into the equation. $|\dot{\theta}_0|$ is set small enough to express the rigid state. When the norm of angular rotation rate is less than the threshold value, the following equation is employed as the constitutive equation instead of Eq.(2.3).

$$\tau = (c + \sigma \tan\phi)\frac{\dot{\theta}}{|\dot{\theta}|}\frac{|\dot{\theta}|}{|\dot{\theta}|_0} \qquad \textit{in case of } \dot{\theta}/|\dot{\theta}|<1 \tag{2.4}$$

The above equation prevents zero division by the denominator in case of the angular rotation rate is very small and the earth structure is undeformed. Furthermore, introduction of threshold value reduces the shear stress below the yield function by $\dot{\theta}/|\dot{\theta}|<1$. It is possible to determine the stress within the yield function by permitting infinitesimal angular rotation rate around the center. It is, however, a problem how to set the threshold value in Eq.(2.4). In this study, trial computation is made by varying threshold value to examine the effect of threshold value on the computational result. By making the threshold value smaller, the computational result of residual deformation converges to specific magnitude. The threshold value is set as smaller enough magnitude to afford the converged estimation.

2.2. Formulation of Governing Equation Based on Slices

Formulation of governing equation will be displayed based on slice method with the use of the rigid plastic constitutive equation Eq.(2.3). The slice method uses the circular arc slip line and the slices of moving soil mass. Figure 2.1 shows the forces acting on slices. *W* is the weight of slice. *P*, *S* express normal and shear forces at slip line, respectively. *E*, *X* indicate normal and shear forces at both sides of slice. Basic items are expressed with forces defined in Figure 2.1 to derive the governing equation. The assumption of Eq.(2.5) is employed in the same way with the Fellenius's method to simplify the boundary value problem. Index number expresses the slice number. Equilibrium equations of forces in vertical and horizontal directions are displayed. Moments around the center are also shown. In the equations, *R* is radius of circular arc slip line. *b* and *l* indicate width and length of slice. τ_f and τ_m show shear strength and shear resistance, respectively. M_R and M_D express resistance and overturning moments. x_g and y_g denote the coordinate of gravity center of slice. k_v, k_h express the seismic coefficients of vertical and horizontal directions.

$$\sum(X_n - X_{n+1}) = \sum(E_n - E_{n+1}) = 0 \tag{2.5}$$

1: Equilibrium equation in vertical direction

$$\gamma A(1 + k_v) = P\cos\alpha + S\sin\alpha \tag{2.6}$$

2: Equilibrium equation in horizontal direction

$$\gamma A k_h = S\cos\alpha - P\sin\alpha \tag{2.7}$$

3: Resistance moment around the center

$$M_R = \sum(\tau_m l)R \tag{2.8}$$

4: Overturning moment around the center

$$M_D = \sum\left(x_g\gamma A(1+k_v)\right) + \sum\left(y_g\gamma A k_h\right) \tag{2.9}$$

Eq.(2.10) is derived from resistance and overturning moments. R_g is distance from gravity center of slice to center of circular arc slip line, $\ddot{\theta}$ is angular acceleration.

$$\begin{aligned}&\sum R_g^2\left(\frac{\gamma A}{g}\right)\ddot{\theta} = \\ &\sum\left(\gamma A\left[x_g(1+k_v)+\left(y_g k_h\right)\right]\right) - \sum(\tau_m l)R\end{aligned} \tag{2.10}$$

By using Eqs.(2.6), (2.7) and (2.3), Eq.(2.10) becomes the following equation. It is arranged by angular rotation acceleration.

$$\ddot{\theta} = \frac{\sum\left(\gamma A\left[x_g(1+k_v)+\left(y_g k_h\right)\right]\right) - \sum R\left(cl + \left[\gamma A\left[(1+k_v)\cos\alpha - k_h\sin\alpha\right]\right]\tan\phi\right)\dfrac{\dot{\theta}}{\left|\dot{\theta}\right|}}{\sum R_g^2\left(\dfrac{\gamma A}{g}\right)} \tag{2.11}$$

The above equation includes the angular rotation rate and acceleration. Using the linear acceleration method, it can be analyzed by explicit integration method. However, the equation contains the norm of angular rotation rate and the equation becomes a non-linear function of angular rotation acceleration. It should be computed by iterative computation method to update the norm of angular rotation rate in the equation. In the Newmark's method, the limit horizontal seismic coefficient is employed as a threshold magnitude of sliding.

However, the proposed method of Eq.(2.11) does not need the limit horizontal seismic coefficient. It is valid for the whole process of the behavior of earth structure against earthquake. It matches with the Newmark's method fundamentally in case of assessing the residual deformation against the external load. From this viewpoint, the developed method seems superior to the Newmark's method.

$$\ddot{\theta} = \frac{\left(k_h - k_y\right)\left(\sum\gamma A y_g + \sum R\gamma A\sin\alpha\tan\phi\right)}{\sum R_g^2\left(\dfrac{\gamma A}{g}\right)} \tag{2.12}$$

$$k_y = \frac{\sum Rcl + \sum R\gamma A\cos\alpha\tan\phi - \sum\gamma A x_g}{\sum y_g\gamma A + \sum R\gamma A\sin\alpha\tan\phi} \tag{2.13}$$

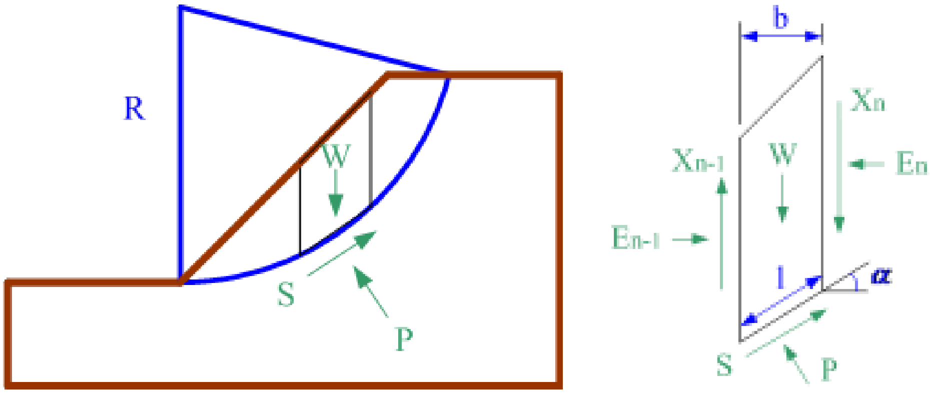


Figure 2.1. The slice method

2.3. Reduction of Shear Strength

Since the proposed method employs the rigid plastic constitutive equation for the whole process of earthquake, it is possible to analyze the residual deformation of earth structure against post-earthquake collapse. However, it is necessary to introduce the strength softening model into the proposed method in order to analyze the post-earthquake collapse. The strength softening model is newly developed to analyze the post-earthquake collapse of earth structure. It estimates the shear strength dependent of shear displacement of slip line, from the peak to the residual strengths. This model was determined by the results of direct shear box test as mentioned later.

3. CASE STUDIES OF RESIDUAL DEFORMATION ANALYSES

Applicability of proposed method is examined through case studies of slope the inclination of which is 1:1.5. Obtained results are discussed in comparison with those by the Newmark's method. Figure 3.1 shows the obtained slip line (blue line) by stability analysis, where width of slice is set as 0.1m. And Table 3.1 shows analysis properties. Stability analysis employs the Fellenius's method. Two case studies are performed to show the superiority of the proposed method over the Newmark's method. In Case 1, the slope is excited by horizontal vibration. The factor of safety is set greater than 1.0. In Case 2, the factor of safety is less than 1.0 and the slope causes the gravity collapse.

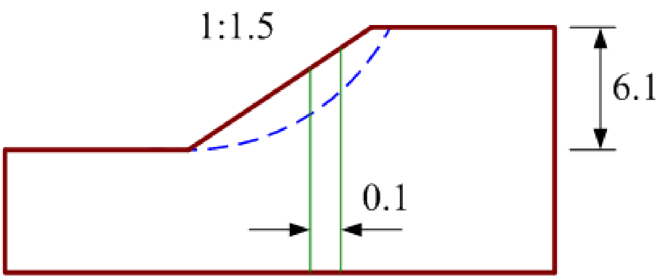


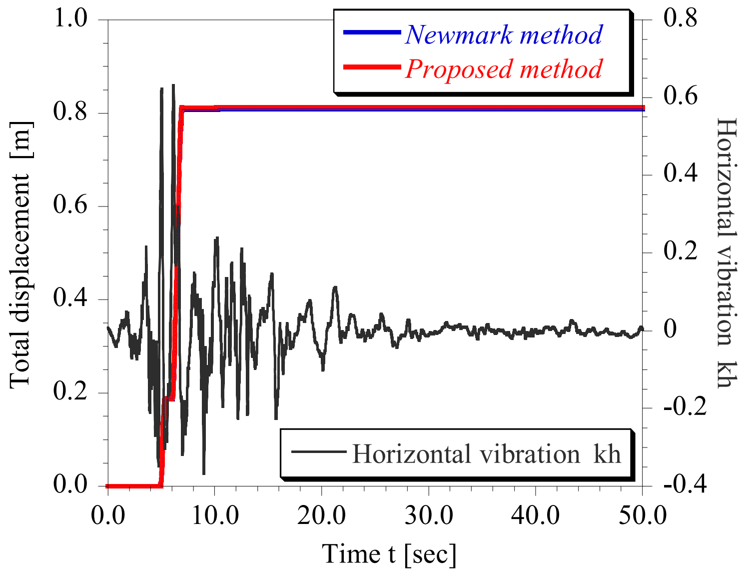
Figure 3.1. Analysis object

Table 3.1. Analysis condition

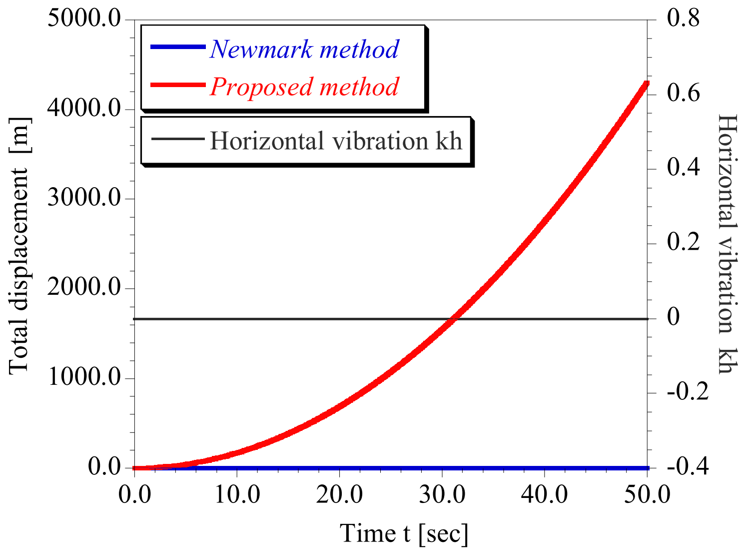
	Case 1	Case 2
Angle of shear resistance ϕ [°]	32.0	10.0
Cohesion c [kPa]	5.0	1.0
Unit volume weight γ [kN/m^3]	20.0	
Safety factor Fs	1.472	0.382
Split width b [m]	0.1	
Time interval Δt [sec]	0.01	

Figure 3.2 expresses the obtained results of Cases 1 and 2. In Case 1, the computed residual deformations of two methods increased at 6.13 sec. The Newmark's method assessed the residual deformation as 0.809 m in horizontal displacement, on the other hand, the proposed method computed as 0.813 m. The obtained results are almost coincident, but small discrepancy was observed. It is because the proposed method assessed the horizontal displacement even though the horizontal vibration is within the limit if the generation condition of residual deformation is satisfied.

In Case 2, the proposed method analyzed the residual displacement increased rapidly even though the Newmark's method gave no displacement. This result clearly shows the applicability of the proposed method to gravity collapse of earth structure.



Case 1 Fs=1.472



Case 2 Fs=0.382

Figure 3.2. Relationship of time and horizontal vibration, and cumulative displacement

4. MODEL TEST AND ITS SIMULATION

The applicability of proposed method is discussed in comparison with shaking table model tests. Slope model is constituted by two steel blocks as shown in Photo 4.1 Slope inclination is set as 1:1. Slip line of two blocks are shaped as a circular arc line. Sandpaper is glued to bring out the friction along the slip line. Table 4.1 shows basic properties of sandpaper obtained by simple static friction tester. Cohesion is set zero since the sandpaper is basically cohesionless. Two case studies of maximum horizontal seismic coefficients are performed as kh_max = 0.362 (Case A) and kh_max = 0.513 (Case B). Figure 4.1 shows the obtained results for Cases A and B.
The comparison of obtained results in model test and computation is performed in regards to the followings: [1] time of displacement generation, [2] trend of relationship between time and displacement, and [3] residual displacement.

In Case A, time of displacement generation almost coincided, but trend of relationship between time and displacement and resultant residual displacement differed in model test and computation. This is because displacement of model test increased with repeats of vibration. It seems to reflect the reduction in friction property of sandpaper due to shear deformation (Figure 4.2). On the other hand, with the increase in displacement, displacement increment reduced against vibration loading. It is due to geometrical change of model by large displacement. In computation, material property of sandpaper is set constant and geometrical change is not taken into account. It causes the discrepancy of obtained results between model test and computation. Figure 4.1 indicates the simulation result in case that 10% of shear resistance was reduced by generation of shear displacement. The figure shows the trend of relationship between time and displacement becomes to comparatively match with that of model test. It is clear that the proper modeling of shear strength is important to assess the residual deformation of earth structures.

In Case B, the time of displacement generation and the trend of relationship between time and displacement almost coincided, but residual displacement differed in model test and computation. This seems due to the geometric change by large deformation. Better fitting for trend of relationship between time and displacement is thought to be achieved by large displacement of Case B. Shear resistance of sandpaper quickly may converge to residual state by large shear deformation (Figure 4.2).

From the above, it is possible that the proposed method gives the more accurate estimation on residual displacement of earth structure by introducing a precise displacement dependent shear strength model. However, it is still difficult to take into account the geometrical change in computation. It is a further issue of this method.

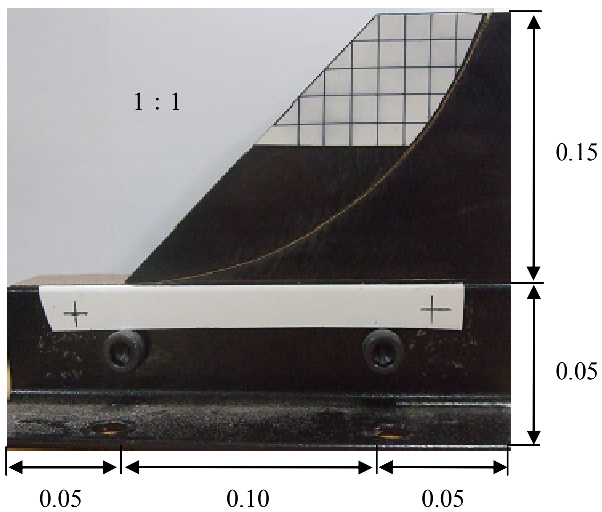
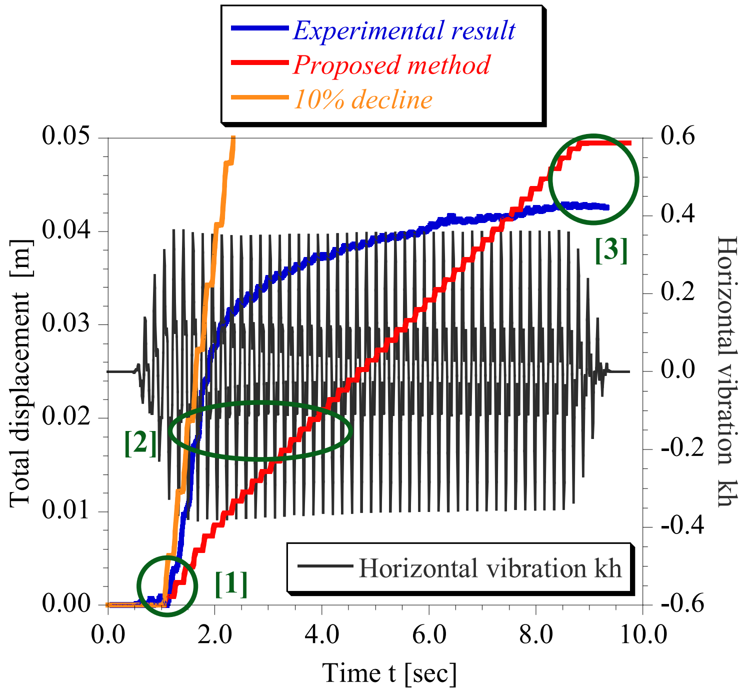


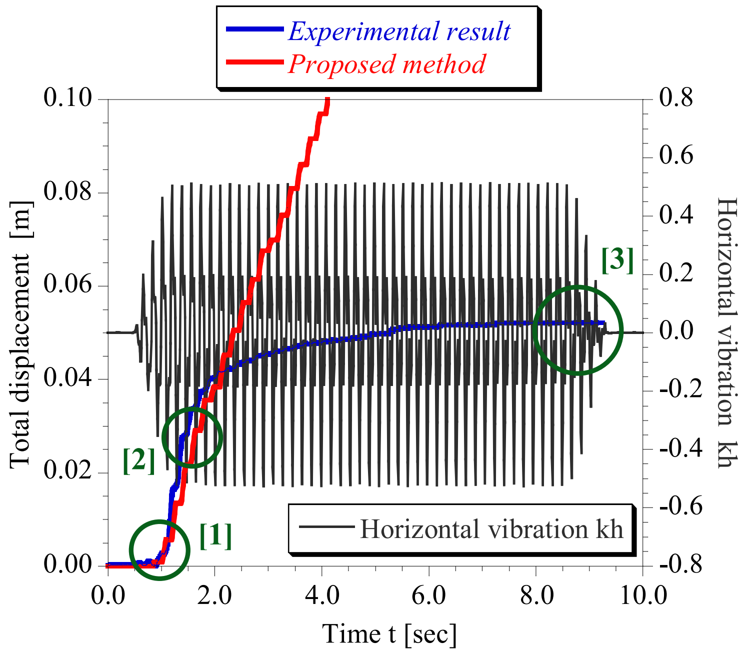
Photo 4.1. Model

Table 4.1. Analysis condition

Angle of shear resistance ϕ [°]	42.4
Cohesion c [kPa]	0.0
Unit volume weight γ [kN/m^3]	76.6
Safety factor Fs	1.424
Split width b [m]	0.01
Time interval Δt [sec]	0.01



Case A（maximum horizontal vibration k_{h_max} 0.362）



Case B（maximum horizontal vibration k_{h_max} 0.513）

Figure 4.1. Cumulative of time and total displacement and input vibration

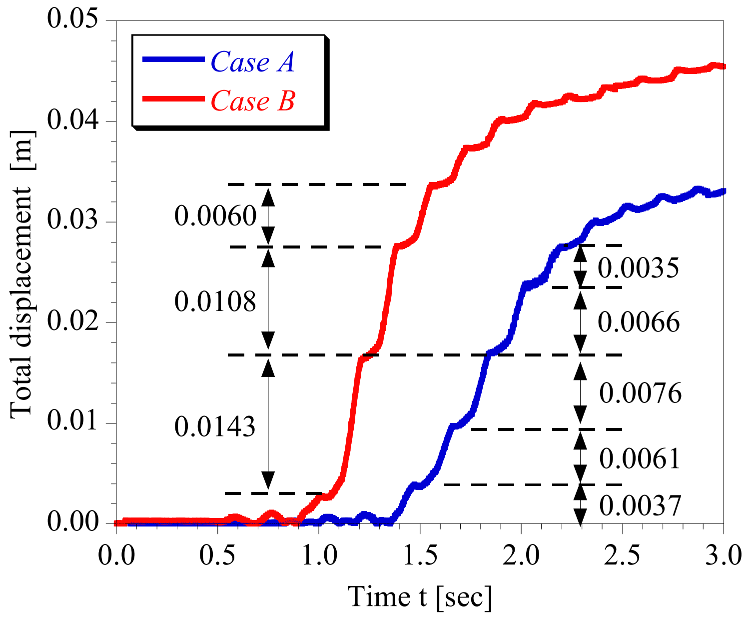


Figure 4.2. Cumulative of time and increase of total displacement

5. CONCLUSION

This study presents the rigid plastic constitutive equation of slip line and the simple residual deformation analysis method based on the slice method. This method expands the Newmark's method in order to analyze the whole process of displacement of earth structure against earthquake. It is possible to analyze the residual deformation even though the gravity collapse case. Through some case studies, the applicability of proposed method to residual deformation analysis was clarified. Since the proposed method gives the assessment of residual deformation of earth structure though the post-earthquake collapse case, it is concluded to be superior to the Newmark's method. However, the discrepancy in residual displacement between the computation and the model test are still observed in various conditions. Displacement dependent strength model is a current issue to be taken into account in simulation. Strength reduction model is necessary in order to assess the residual deformation of earth structure which shows the strain softening property such as the residual strength and the liquefaction. Large deformation analysis is a further issue for the slice method. It is yet desirable to predict the residual deformation with simple method.

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