Mixed Algorithm for Contact Problem between Rock Blocks under Static and Dynamic Conditions

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SUMMARY
A mixed algorithm is proposed for nonlinear contact problem between rock blocks under static and dynamic conditions. Based on this method, the system of forces acting on the contactor was divided into two parts: external forces and contact forces. The displacements of contactor were chosen as the basic variables and the nodal contact forces on possible contact region and displacements for rigid bodies of possibly moving blocks were chosen as iterate variables, so that the nonlinear iteration process was only limited on the possible contact surfaces. The relation between displacements of contactor and contact forces on possible contact regions are solved by general finite element method while the displacements for rigid bodies are fixed, The contact forces and displacements for rigid bodies of possibly moving rock blocks are solved through equations of global force equilibrium for each possibly moving block and contact equations which are modified according to contact conditions (such as opening, bonding and sliding). Thus the iterative procedure became easily to be carried out and much more economical. For dynamic problems, the velocity and acceleration are considered in the contact equations. By the way, the velocities and accelerations for rigid body moving are also considered in the global force equilibrium equations under dynamic condition.
As an example, the dam abutment stability of a double curvature concrete arch dam is analyzed by the algorithm presented in this paper. The failure mode and its safety factor under seismic action gives reasonable explanation.

Keywords: contact problem, rock blocks, dynamic FEM

1. Introduction

In slope stability analysis under static or dynamic actions, nonlinear contact between rock blocks is the main nonlinear source. It belongs to the local discontinuous scope. For general discontinuous medium, Serrano and Olalla (1996) give the conditions that the equivalent continuous medium method can be used. For this kind of problem, the corresponding solving methods can be thin layer element method (TLEM) (Desai, 1984), discrete element method (DEM)(Cundall,1979), discontinuous deformation analysis (DDA)(Shi, G. H.,1984). If it is taken as contact problem, the method suggested by Haug(1980) and Peric(1992) can be used.

When the number of contact surfaces exists only one or two, the nonlinear happens on a local field. Zhao and Li (2006) proposes a mixed method using contact force and contact displacement as unknown variables. This method takes advantage of FEM to solve the contact boundary to get the flexibility matrix according to the possible contact body. The advantage of this method is that the overall matrix is symmetrical and the convergence is relatively fast. If the contact body has the rigid motions, Chen(1979) gives the solving method of plane problem,. Two points are chosen from the contact body to consider the rigid motion. The disadvantage of the method is that the stiffness matrix of the coupling equation between unknown contact forces and rigid displacements is asymmetric.

The combined solution by FEM and DEM is suggested in this paper. The relation between displacements of contactor and contact forces on possible contact regions are solved by general finite element method while the displacements for rigid bodies are fixed, The contact forces and displacements for rigid bodies of possibly moving rock blocks are solved through equations of global force equilibrium for possible moving block and contact equations which are modified according to contact conditions (such as opening, bonding and sliding). Thus the iterative procedure became easily to be carried out and much more economical. For dynamic problems, the velocity and acceleration are considered in the contact equations. By the way, the velocities and accelerations for rigid body moving are also considered in the global force equilibrium equations under dynamic condition.

2. STATIC CONTACT EQUATIONS FOR ROCK BLOCKS

If the contact body is rock block, the rigid motion should be considered. Assuming the rigid displacement at centroid point is γ , the rigid displacement at arbitrary point with relative coordinate of $(\Delta x, \Delta y, \Delta z)$ to centroid point can be expressed as:

$$u_g = \begin{bmatrix} 1 & 0 & 0 & 0 & \Delta z & -\Delta y \\ 0 & 1 & 0 & -\Delta z & 0 & \Delta x \\ 0 & 0 & 1 & \Delta y & -\Delta x & 0 \end{bmatrix} \begin{Bmatrix} \overline{u} \\ \overline{v} \\ \overline{w} \\ \theta_x \\ \theta_y \\ \theta_z \end{Bmatrix} = \omega\gamma \tag{2.1}$$

For contact bodies of Ω_1 、 Ω_2 , the equilibrium equation for FEM at step (n+1) without considering rigid motion is：

$$K\Delta u_n = R_{n+1} + \Delta f_n \tag{2.2}$$

where

$$R_{n+1} = F_{n+1} + f_n - \int B^T \sigma_n d\Omega$$

K is the global stiffness matrix; Δu_n is displacement increment matrix,, F_{n+1} is load vector, f_n contact internal force vector at step n, Δf_n contact internal force increment at step (n+1), σ_n stress vector at step n.

Remembering $\Delta \overline{u}_n = K^{-1} R_{n+1}$, and using $C = K^{-1}$ to represent flexibility matrix，formula (2.2) can be written as,

$$\Delta u_n = \Delta \overline{u}_n + C\Delta f_n \tag{2.3}$$

When the rigid displacements are considered for contact bodies, the total displacements for points on contact boundaries of Ω_1 and Ω_2 are

$$\Delta u_{n_n}^1 = \Delta \overline{u}_n^1 + C^1 \Delta f_n^1 + \omega_1 \gamma^1 \tag{2.4}$$

$$\Delta u_{n_n}^2 = \Delta \overline{u}_n^2 + C^2 \Delta f_n^2 + \omega_2 \gamma^2 \tag{2.5}$$

Because of $-\Delta f_n^1 = \Delta f_n^2 = \Delta f_n$, and taking account to $C = C^1 + C^2$, the following formula can be obtained from formula of (2.4) and (2.5),

$$C\Delta f_n + \Delta u_g^2 - \Delta u_g^1 = (\Delta u_n^2 - \Delta u_n^1) - (\Delta \bar{u}_n^2 - \Delta \bar{u}_n^1) \tag{2.6}$$

For each contact body, the static equilibrium equation related to centroid point for all forces acted on it could be written as follows.

$$\omega_1^T \Delta f_n^1 + \omega_1^T \Delta F_n^1 = 0 \tag{2.7}$$

$$\omega_2^T \Delta f_n^2 + \omega_2^T \Delta F_n^2 = 0 \tag{2.8}$$

Finally, for two contacted bodies, the equations included the contact forces and rigid displacements of each contact body are:

$$\begin{bmatrix} C & -\omega^1 & \omega^2 \\ -\omega_1^T & 0 & 0 \\ \omega_2^T & 0 & 0 \end{bmatrix} \begin{Bmatrix} \Delta f_n \\ \Delta \gamma^1 \\ \Delta \gamma^2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega_1^T & 0 \\ 0 & 0 & -\omega_2^T \end{bmatrix} \begin{Bmatrix} \Delta \bar{u}_n^1 - \Delta \bar{u}_n^2 \\ \Delta F_n^1 \\ \Delta F_n^2 \end{Bmatrix} \tag{2.9}$$

3. DYNAMIC CONTACT EQUATIONS FOR ROCK BLOCKS

For each contact body without considering rigid motions, the incremental dynamic equilibrium equation for n+1th step at time can be expressed as:

$$M\ddot{u}_{n+1} + D\dot{u}_{n+1} + Ku_{n+1} = F_{n+1} + f_n + \Delta f_n \tag{3.1}$$

where M is the global mass matrix; D is the global damping matrix, here the proportional Rayleigh damping assumption is adopted $D = \alpha M + \beta K$; F_{n+1} is the vector of total external load at time t+ △t; f_n is the vector of total contact force at time t; and Δf_n , vector of incremental contact force at time t which are greatly concerned about. u_{n+1} , $\dot{u}_{n+1}$, $\ddot{u}_{n+1}$ is the vector of displacement increment, velocity increment, acceleration increment at time t+△t, respectively, the superscript (•) refers to time differential.

Following the generalized Newmark time integration scheme, the equation with the variables of the acceleration increment is obtained as:

$$\bar{K}\Delta \ddot{u}_n = \Delta \bar{F}_n + \Delta f_n \tag{3.2}$$

Where $\bar{K}$, $\Delta \bar{F}_n$ is the effective stiffness matrix and effective external load increment and can be expressed as following, respectively:

$$\bar{K} = (1 + \alpha\beta_1 \Delta t)M + (\beta\beta_1 \Delta t + \beta_2 \Delta t^2)K \tag{3.3}$$

$$\Delta \bar{F}_n = F_{n+1} + f_n - (\ddot{u}_n + \alpha \dot{u}_n^p)M - (u_n^p + \beta \dot{u}_n^p)K \tag{3.4}$$

Here the superscript p refers to the predicted quantities, β_1 , β_2 are Newmark parameters for time integration.

Introducing the matrix C into above equation, equation (3.2) can be rewritten as

$$\Delta \ddot{u}_n = \overline{K}^{-1}\Delta \overline{F}_n + C\Delta f_n \tag{3.5}$$

Considering equation (3.5) and $\Delta u_n = u_{n+1} - u_n$, if $\Delta \overline{u}_n$ is defined as

$$\Delta \overline{u}_n = u_n^p + \beta_2 \Delta t^2 \overline{K}^{-1}\Delta \overline{F}_n - u_n \tag{3.6}$$

Then, the displacement including the rigid motion is

$$\Delta u_n = \Delta \overline{u}_n + \beta_2 \Delta t^2 C\Delta f_n + \omega \Delta \gamma \tag{3.7}$$

Moreover,

$$\beta_2 \Delta t^2 C\Delta f_n + (-\omega_1 \Delta \gamma^1 + \omega_2 \Delta \gamma^2) = G_u \tag{3.8}$$

Where

$$G_u = ((\Delta u_n^2 - \Delta u_n^1) - (\Delta \overline{u}_n^2 - \Delta \overline{u}_n^1))$$

For each block， the motion balance equation of discrete element method is

$$W_i \ddot{\gamma}_{n+1}^i + \alpha W \dot{\gamma}_{n+1}^i = \omega_i^T f_{n+1}^i + \omega_i^T F_{n+1}^i \tag{3.9}$$

Where

$$W_i = \begin{bmatrix} m_i & 0 \\ 0 & I_i \end{bmatrix}$$

m_i、 I_i are mass and moment of inertia related to the centroid of ith block, and α is the mass damp coefficient. For small displacement problem, the mass damp coefficient can be taken as zero. In this situation, the equation (3.9) can be discreted as

$$\frac{1}{\beta_2 \Delta t^2} W_i \Delta \gamma_n^i - \omega_i^T \Delta f_n^i = G_\gamma \tag{3.10}$$

where

$$G_\gamma = (\omega_i^T f_n^i + \omega_i^T F_{n+1}^i) - W_i \ddot{\gamma}_n^i - \frac{1}{\beta_2 \Delta t^2}(\gamma_n - \gamma_{n+1}^p)$$

Combined equation (3.8) and (3.10), the coupled equations of contact forces and block motions for two blocks are

$$\begin{bmatrix} \beta_2 \Delta t^2 C & -\omega^1 & \omega^2 \\ -\omega_1^T & W_1 & 0 \\ \omega_2^T & 0 & -W_2 \end{bmatrix} \begin{Bmatrix} \Delta f_n \\ \Delta \gamma^1 \\ \Delta \gamma^2 \end{Bmatrix} = \begin{Bmatrix} G_u \\ G_\gamma^1 \\ -G_\gamma^2 \end{Bmatrix} \tag{3.11}$$

4. ABUTMENT STABILITY OF AN ARCH DAM UNDER SEISMIC ACTION

Take a double curvature concrete arch dam as an example (See Figure 4.1). Normal storage water level is 825m, crest elevation is 834.00m and the maximum dam height is 289m. The material properties of concrete are: the dynamic elastic modulus is 31.2 GPa, the density is 2400 kg/m3, and the Poisson's ratio is 0.17. The material constants in the foundation region are: the dynamic elastic modulus is 23.4 GPa, the density is 2700 kg/m3, and the Poisson's ratio is 0.24.

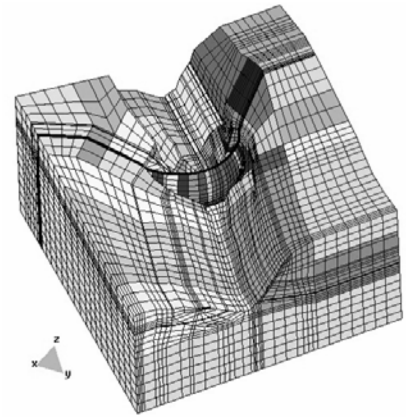


Figure 4.1. Finite element model

The dam located in an extremely strong earthquake region with 8° design seismic intensity. The design horizontal peak acceleration is 325gal, while the vertical peak acceleration is 217gal. Three artificial ground motions, as shown in Figure 4.2~4.5, are applied from the streetwise, cross-stream and vertical direction.

Using viscoplastic boundary to take the effect of foundation radiation damping into consideration, the seismic waves are inputted from the bottom of the model in three directions. Such earthquake lasts 24s by taking dynamic water pressure of reservoir into account.

The side surface of the abutment is the fault JX which lies in the left bank with the direction of $NE40^{\circ}/NW\angle 80^{\circ}$. The bottom surface of the abutment is the fault C3 which develops from left bank in the direction of $NE50^{\circ}/SE\angle 15^{\circ}$. And the upper stream side tension-crack surface is made by the upper stream vertical surface of the dam body. The material parameters of the structural surface is shown in Table 4.1 (where the modulus of elasticity is under static condition, and the modulus need to be increased by 30% in dynamic condition).

Table4.1 Mechanical material parameters of structural surface

Structural surface	Bulk density (kN/m3)	Elastic modulus (GPa)	Poisson ratio	f	C (MPa)
JX	2200	0.2	0.3	0.89	0.78
C3	2200	0.2	0.3	0.37	0.03
Tension-crack surface	2200	0.2	0.3	0.25	0.03

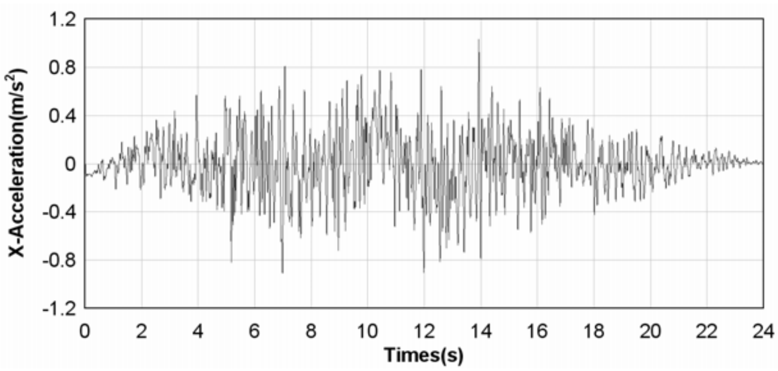


Figure 4.2. Time histories of artificial ground motion in cross-stream direction

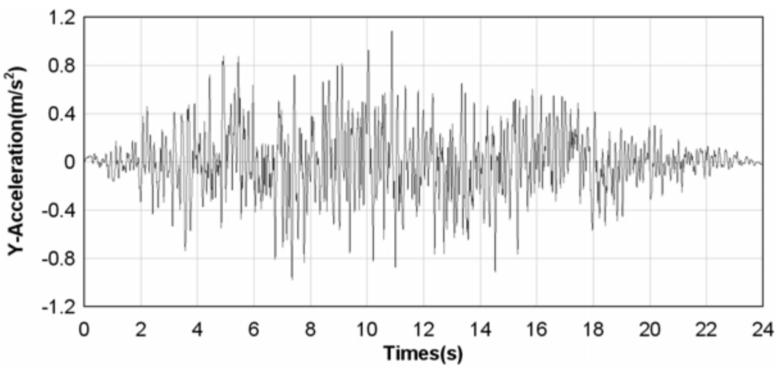


Figure 4.3. Time histories of artificial ground motion in streamwise direction

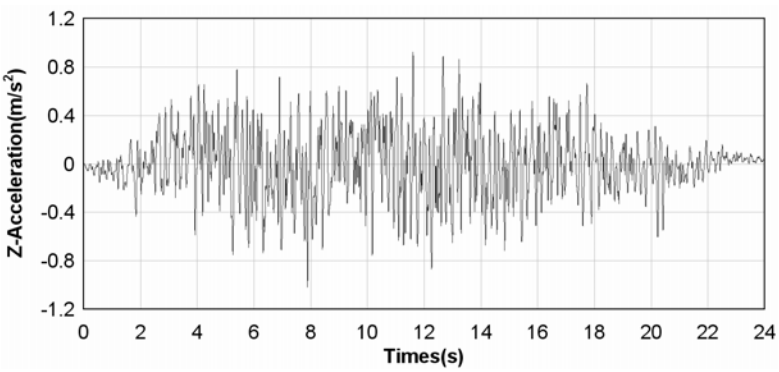
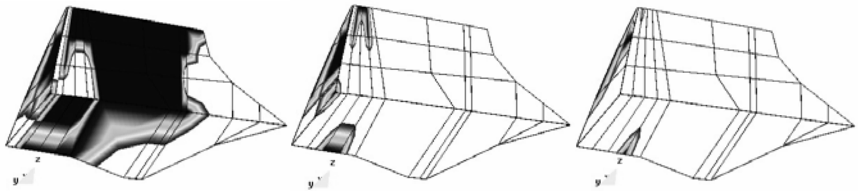


Figure 4.4. Time histories of artificial ground motion in vertical direction

The dynamic contact strength reduction method is used to analyze the abutment failure process and dynamic safety factor of the arch dam under seismic action.

By adopting the dynamic contact method, the yield condition of the structural surface around the abutment sliding blocks after the earthquake with the strength reduction as 1, 2, 3 respectively are shown in Figure 4.5. And the yield zone is represented as blanks.



K=1.0 K=2.0 K=3.0

Figure 4.5. Failue process of structural planes

Figure 4.5 shows that the yield of abutment sliding blocks begins from the end side. With the decrease of material strength, the yield gradually goes upstream until it all yield.

When adopting dynamic contact strength reduction method, the abutment sliding body, foundation, arch dam body and reservoir water are taken as a system to conduct the dynamic analysis. The relative yielding area is proposed to evaluate whether the abutment is on unstable. For simplicity of computation convergence, the relative yield area of 95% the structural surface around the sliding blocks is taken as the safety limit. The yield area percentage process line of K=1.0 and the maximum yield area percentage –safety factor curve of left bank abutment sliding blocks are shown in Figure 4.6 and Figure 4.7.

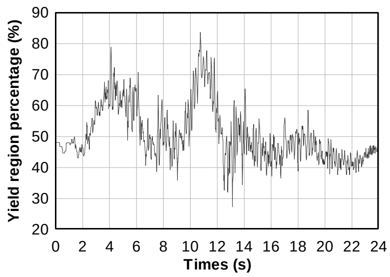


Figure 4.6. The yield area percentage process line

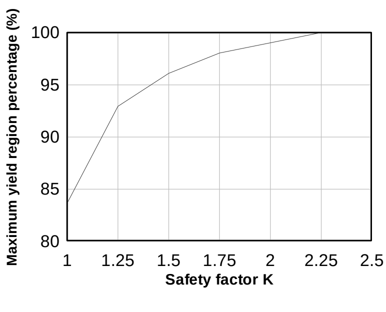


Figure 4.7. Maximum yield area percentage –safety factor curve

When the material strength reduced to 1.4, it is shown that the maximum yield area reached 95%. Therefore the dynamic safety factor of the left bank abutment sliding block can be taken as 1.40.

5. CONCLUSION

When the traditional finite element time-historic strength reduction method is used to make the dynamic stability evaluation of dam abutment, it dissevers the interaction between dam body and dam abutment, namely the solving of arch thrust and the descending calculation, into two independent processes, which can't truly reflect the state of the interaction between body and abutment under earthquake effect. On the other hand, the large finite element discretization causes low efficiency and poor convergence. Also, the safety evaluation that depends on the displacement mutation at the final ensure is not convincible due to time and human factor. The dynamic pressure algorithm that treats the sliding blocks of dam abutment, earth foundation, dam body and water of reservoir as a unified system can make the calculation more accurately with the consideration of dynamic interaction between dam abutment sliding blocks and body. On the other hand, according to the characteristics of contacting partial nonlinear, it makes the iterative calculation simple for reflecting the complicated nonlinear contacting friction on the change of contact force when it put the nonlinear contact iterative shrinkage on the possible contact face. Therefore, the result will be clearer by using the yielding area as safety evaluation. The calculation results show that, the simulation that uses contact descending method of considering the interaction between dam body and abutment on the interaction between body and abutment sliding blocks is more accurate, which makes the safety evaluation of abutment sliding blocks more reasonable.

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