Structural damage detection based on imperfect static responses by means of pattern search algorithm

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SUMMARY:
Damage detection and estimation in structures using incomplete static responses are presented. In the proposed approach, damage location and severity is determined by solving an optimization problem using pattern search algorithm. Also, the objective function is formulated using static responses. Because of limitation in using sensors and difficulties in sensing rotational degree of freedoms, the effect of using incomplete responses has been evaluated. The performance of the proposed method is evaluated using a numerically example consist of simply supported beam. The results indicated that the proposed method is effective and robust in detection and estimation of damage.

Keywords: damage detection, static data, optimization, pattern search algorithm

1. INTRODUCTION

Structural damage detection in civil and mechanical engineering structures during their service life has drawn wide attention during last few decades. Structural damage can be identified as weakening of the structure that causes negative changes in its performance. Damage may also be considered as any change in property of material and original geometry of structure that make undesirable stress or displacement and vibration in structure. So, most of the damage detection methods are on the basis of the changes of dynamic characteristics and static responses (He et al., 2007).
Static responses are more sensitive to damage than dynamic responses (Li et al, 1999 & Hjelmstad et al., 1997) and the equipments of static testing, and precise static displacements of structures could be obtained rapidly and economically (He et al., 2007). However, there are two main drawbacks in the static damage identification methods: (1) Static testing provides less information as compared to dynamic testing; (2) The effect of damages on static responses for damage detection may be cryptic due to limited load paths (He et al., 2007).
Some researchers used static responses for damage detection of structures. Force error estimator and displacement error estimator for static parameter grouping scheme to identify the damage error by least squares minimization was presented by Banan et al (1994). Hjelmstad and Shin (1997) proposed a data perturbation scheme for the baseline structure, to establish the damage threshold between noise and the damaged structure to compare the damage indices. Hwu and Liang (2001) used static strain measurement from multiple loading models for identification of the hole and cracks in linear anisotropy elastic materials with nonlinear optimization. Hajela and Soeiro (1990) presented a damage detection algorithm based on static displacements, mode shapes and frequencies. To solve an unconstrained optimization problem, an iterative non-linear programming method was developed. Paola and Bilello (2004) proposed a damage identification procedure based on a least-square constrained nonlinear minimization problem for Euler-Bernoulli beams under static loads. Yam et al.

(2002) proposed sensitivities analysis in static and dynamic parameters damage indices quantification for their identification capabilities over plate-like structures. Hua et al. (2009) proposed a new damage detection procedure for cable-stayed bridges by changes in cable forces. Also, Lee et al. (2010) developed a method used continuous strain data from fiber optic sensor and neural network model. Recently, Cao et al. (2011) presented the sensitivity of fundamental mode shape and static deflection for damage identification in cantilever beams, wherein these features are extremely similar in configurations.
In this research, a new method for localizing and estimating the severity of structural damage is introduced. The damage identification is carried out through pattern search algorithm to minimize an objective function derived from incomplete static characteristics of damaged structure. Numerical example shows that the proposed method can be considered as a flexible and robust approach in damage identification of structures.

2. Proposed method

The static equilibrium equation of a structure in a displacement based finite element frame work can be expressed as follows:

$$\left[\mathrm{K}^{ud}\right]\{\mathrm{x}\} = \{\mathrm{F}\} \tag{2.1}$$

Where, $\mathbf{K}^{\mathrm{ud}}$ is the stiffness matrix of structure for health condition, and $\mathbf{F}$ and $\mathbf{x}$ are the force and displacement vectors; respectively.
One of the simplest techniques to determine damage-induced alteration stiffness is the degradation in Young's modulus of an element as follows:

$$E_j^d = E_j^{ud}(1-d_j) \tag{2.2}$$

Where, $E_{\mathrm{j}}^{\mathrm{d}}$ and $E_{\mathrm{j}}^{\mathrm{ud}}$ are the damaged and undamaged Young's modulus of the *j*th element in the finite element model, respectively; and d_{j} indicates the damage severity at the *j*th element in the finite element whose values are between 0 for an element without damage and 1 for a ruptured element.
Moreover, it is assumed that no change would occur after damage in the mass matrix, which seems to be reasonable in most real problems.
From Eq. (2.1), the static equilibrium equation of a damaged structure can be obtained as:

$$\left[\mathbf{K}^{d}\right]\{\mathbf{x}^{d}\} = \{\mathbf{F}\} \tag{2.3}$$

Where, superscript *d* is noted as the damage state. In fact, not all displacements in $\mathbf{x}^d$ can be measured. Therefore, Eq. (2.3) is partitioned into the master and slave coordinates as bellow:

$$\begin{bmatrix} \mathbf{K}_{mm}^{d} & \mathbf{K}_{ms}^{d} \\ \mathbf{K}_{sm}^{d} & \mathbf{K}_{ss}^{d} \end{bmatrix} \begin{Bmatrix} \mathbf{x}_{m}^{d} \\ \mathbf{x}_{s}^{d} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_{m} \\ \mathbf{F}_{s} \end{Bmatrix} \tag{2.4}$$

Which, the subscripts *m* and *s* are the master and slave coordinates, respectively. The vector of slaved displacements $\mathbf{x}^{\mathrm{d}}_{\mathrm{s}}$ is condensed out, following static condensation and Eq. (2.4) reduces to the following:

$$\left[\mathbf{K}_{r}^{d}\right]\{\mathbf{x}_{m}^{d}\} = \{\mathbf{F}_{r}\} \tag{2.5}$$

Where:

$$\left[\mathbf{K}_r^d\right]=\left(\left[\mathbf{K}_{mm}^d\right]-\left[\mathbf{K}_{ms}^d\right]\left[\mathbf{K}_{ss}^d\right]^{-1}\left[\mathbf{K}_{sm}^d\right]\right)\left\{\mathbf{x}_m^d\right\} \tag{2.6}$$

$$\left\{\mathbf{F}_r\right\}=\left\{\mathbf{F}_m\right\}-\left[\mathbf{K}_{ms}^d\right]\left[\mathbf{K}_{ss}^d\right]^{-1}\left\{\mathbf{F}_m\right\} \tag{2.7}$$

In which, $\mathbf{K}^d_r$ and F_r are the condensed stiffness matrix and the condensed load vector of damaged structure; respectively.
From Eq. (2.5), the measured displacement of damaged structure can be obtained as:

$$\left\{\mathbf{x}_m^d\right\}=\left[\mathbf{K}_r^d\right]^{-1}\left\{\mathbf{F}_r\right\} \tag{2.8}$$

Finally, the objective function is defined in terms of output errors between computed and measured displacements as follow:

$$f(d)=\sum_{i=1}^{p}\left(\left(\mathbf{x}_{m,i}^d-\mathbf{x}_{t,i}^d\right)^2\right) \tag{2.9}$$

Where, $\mathbf{x}^d_{m,i}$, $\mathbf{x}^d_{t,i}$ are the measured and theoretically computed displacement of the *i*th point of a damaged structure, respectively; *p* is the number of a considered displacement point.

3. Optimization using pattern search method

Pattern search method is a subclass of direct search methods which was first introduced in 1950s (Box, 1957); however, in 1991, there was a growth in interest of direct search method. Since then two things have become increasingly clear (Kolda et al., 2003):

1. Direct search methods stay an effective option, and sometimes the only choice, for several varieties of difficult optimization problems.
2. For a large number of direct search methods, it is possible to provide thorough guarantee of convergence.

Pattern search method is a derivative-free method for solving a variety of optimization problems where typical optimization methods are not so effective. The main idea of this procedure is to generate a sequence of iterate which consider the behavior of objective function at a pattern of points, all of which lies on a logical lattice without utilizing any information about derivatives including, gradient and second-order derivatives of objective function.
The pattern search method can be briefly explained in a way that starts by establishing set of points called mesh around the given point which could be computed from previous step of iteration or from the initial starting point provided by the user. The mesh is created by adding scalar multiple set of vectors called pattern to the current point, then it searches a set of points (mesh) around the current point of parameters to find a point where the objective function has a lower value. After a point with lower objective function value is detected, the algorithm sets the point as its current point and iteration can be considered successful. Then, the algorithm steps to the next iteration with extended mesh size which is induced by expansion factor. If algorithm does not find a point that improves objective function, the iteration is called unsuccessful. The current points stay the same in the next iteration and the mesh size decreases due to the contraction factor (Lewis et al., 2002). The pattern search optimization algorithm stops when any of the following situations occurs (Coelho et al., 2006):

- The number of iteration or evaluation of objective function reaches the max value.
- The mesh size becomes less than mesh tolerance.
- The distance between two successful points obtained in two consecutive iterations is less that

the given tolerance.

- Alteration in the improvement of objective function is less than the function tolerance.

The pattern search method applied to Eq. (2.9) to find optimal solution using incomplete static responses which leads to localizing and quantifying damage.

4. NUMERICAL STUDY

A Simply supported beam as illustrated in Fig. 4.1 with a finite-element model consisting of 10 beam elements and 11 nodes is considered. To formulate the objective function, two vertical point loads have been used. For the considered concrete beam, the material properties include Young's modulus of E=25 GPa, mass density of ρ=2500 kg/m3. The cross-sectional area and the moment of inertia of the beam are A=0.12 m^2 and I=0.0016 m^4, respectively.

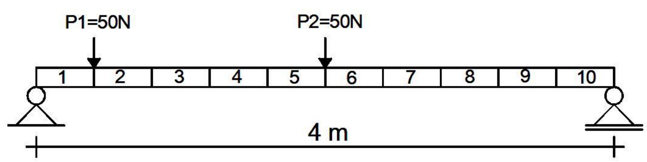

Figure 4.1. The simply supported beam with the finite element model

In this example, three damage scenarios are represented as the elements with reduction in Young's modulus. The damage severity in each element is given by the reduction factor listed in Table 4.1. In this case, only 9 translational DOFs are selected as measured DOFs.
Damage in the simply supported beam can be determined by using the proposed method. The pattern search method input parameters adopted for the following analyses are summarized in Table 4.2.

Table 4.1. Damage scenarios for simply supported beam

Scenario 1		Scenario 2	
Element 6	50%	Element 1	45%
		Element 6	50%
		Element 9	20%

Table 4.2. Input parameters for the pattern search method

Maximum iteration	200-2000
Maximum function evaluations	4000-10000
Bind tolerance	0.001
X tolerance	1.00E-10
Function tolerance	1.00E-10
Nonlinear constrain tolerance	1.00E-10
Expansion factor	2
Contraction factor	0.5
Mesh tolerance	1.00E-20

The obtained results of damage detection and quantification using the proposed objective function that are based on incomplete static responses of structure are shown in Fig. 4.2. The results show that the proposed method is robust and promising in localizing and quantifying of different damage scenarios.

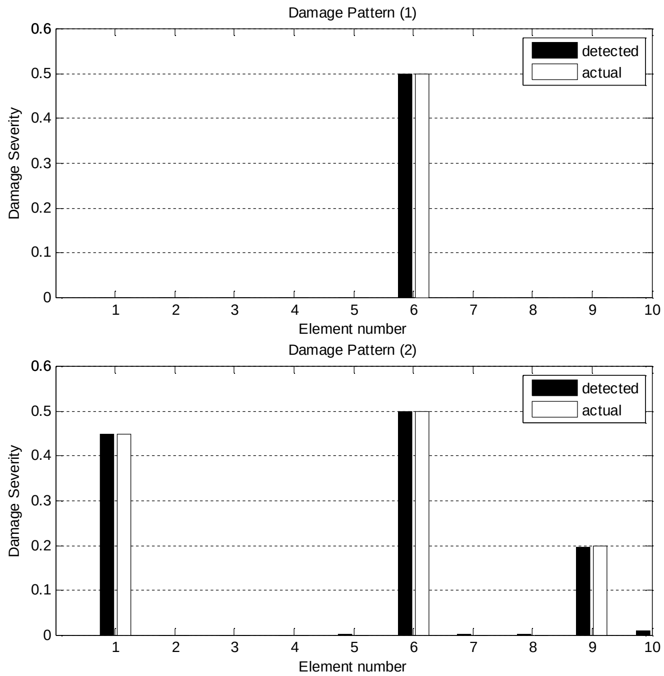

Figure 4.2. The obtained results for two damage patterns of the simply supported beam

5. CONCLUSIONS

In this paper, a method has been developed for detection and estimation of damage in structures based on the incomplete static responses of the damaged structure using an optimization problem. In this method, pattern search algorithm is used to determine the damage in structures by optimizing a cost function.

For damage detection and estimation, this proposed method was applied to a simply supported concrete beam with one or several damage patterns. The obtained results indicated that the proposed method is a strong and viable method to the problem of detection and estimation of damage in the structures. The results revealed high sensitivity of the proposed method to the damage in spite of incomplete measurements.

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