Period Elongation of Nonlinear Systems modeled with Degrading Hysteretic Rules

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ABSTRACT
This paper investigates the elongation of structural period of reinforced concrete (RC) buildings, with the objective of refining the process of selection and scaling earthquake records for seismic assessment. Use is made of five RC buildings designed according to Eurocode 8 as a representative of modern seismic design codes. The variable parameters of period elongation in this study are ground motion intensity and features of the hysteretic relationship (i.e., stiffness and strength degradation and pinching of the hysteresis loops). By means of a detailed parametric analysis involving the use of 20 earthquake ground motions records, a quantitative assessment is provided regarding the degree of elongation of the fundamental period. It is also shown that the Eurocode 8-proposed period elongation factor of 2.0 is a highly conservative estimate imposing several unfavorable implications in design and assessment. The latter observation is particularly true for buildings designed for low and moderate ductility and systems with low-to-medium stiffness degradation.

Keywords: period elongation, inelastic response history analysis, degrading hysteretic rules

1. INTRODUCTION

The impact that earthquake-induced strong ground motion has on response of structures is inherently related to the dynamic characteristics of the structural systems. The fundamental period of buildings is a key parameter in seismic codes for earthquake record selection and scaling necessary for response history analysis. Several researchers have investigated the elongation of the vibration period either on the basis of the measured response of instrumented buildings during earthquakes (e.g., Trifunac *et al*. 2001, Clinton *et al*. 2006, Masi and Vona 2010), or by experimentally studying the successive inelastic episodes of full scale structural models (e.g., Pinho and Elnashai 2000, Jeong and Elnashai 2004, Zembaty *et al*. 2006). Notwithstanding the advancements already made, the vast majority of research results are case-specific, due to the limited number of structural systems and seismic motions studied. On the other hand, the degree of period elongation that is predicted by numerical analysis is also controversial as it varies from a factor of 1.50-1.70 (e.g., Dunand *et al*. 2006, Michel and Gueguen 2010) up to 2.0-2.5 (e.g., Mucciarelli *et al*. 2004, Calvi *et al*. 2006). This modification of the dynamic characteristics during strong ground motion is an issue that also affects the earthquake record selection procedure, as most seismic codes prescribe the desirable spectral matching bandwidth as a function of the fundamental period. This is of particular interest in the case of Eurocode 8, which in contrast to the U.S. codes, enforces a wider spectral matching range that extends up to twice the fundamental period of the building. The latter requirement commonly leads to over-conservative or even unrealistic estimates of seismic demand at the periods closer to the fundamental one, for reasons that are described elsewhere (Sextos *et al*. 2011).

This study presents the results of a parametric analysis that was performed for a group of reinforced concrete buildings (Papanikolaou and Elnashai 2005) with different structural configuration and dynamic characteristics that were designed for different levels of ductility according to Eurocode 8. The buildings were transformed into equivalent inelastic single-degree-of-freedom (SDOF) nonlinear systems (Fajfar 2000) and were subjected to a large number of earthquake motions using different hysteretic rules in terms of stiffness degradation, strength deterioration and pinching. The results that follow highlight the correlation depicted between the earthquake intensity imposed and the corresponding period elongation predicted.

2. EARTHQUAKE STRONG-MOTION RECORDS

In contrast to the seismic hazard-specific earthquake record selection procedure that is followed for the assessment of a specific structure, this study required the formation of an ensemble of earthquake records that presented a wealth of different characteristics. Therefore, a set of 20 records was formed from the PEER Strong Ground Motion Database (available at http://peer.berkeley.edu/smcat) varying in terms of frequency content, amplitude (PGA), soil profile and near/far-field conditions. Given that the aim was to investigate the upper bound of period shift under strong ground motion, short duration motions were filtered-out (i.e., a $M_s > 5.5$ criterion was adopted), as they would clearly lead to lower ductility demand. From the resulting catalogue of eligible records, only those associated with severe structural damage near the recording sites were eventually selected. Then, each record was scaled 10 times within the range $0.15\ g < a_g < 1.50\ g$ to ensure that all structures will undergo the same stages of seismic behavior, that is, elastic response, first cracking, post-yield and severe damage. Eventually, a total number of 2000 analyses (20 records x 10 levels of intensity x 5 buildings x 2 hysteretic rules) were performed for the SDOF systems studied.

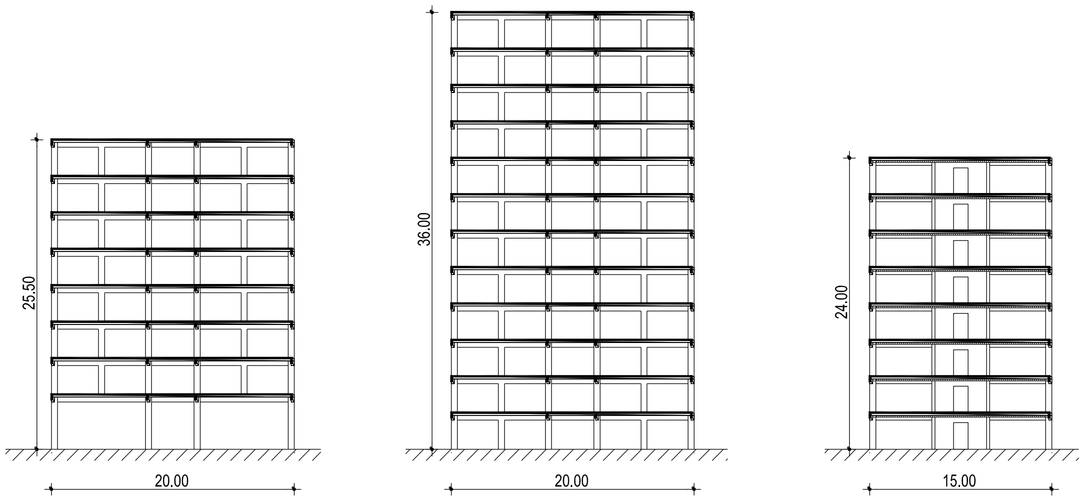
Table 2.1. Earthquake strong-motion records selected

Seismic event	Date	Recording Station	M	R (km)	Soil type	PGA (g)
Cape Mendocino, CA	25.04.1992	Petrolia	7.01	4.510	C	0.662
Chi-Chi, Taiwan	20.09.1999	CHY080	6.20	29.48	C	0.473
Coalinga, CA	02.05.1983	Parkfiled-Fault Zone #14	6.36	38.54	D	0.274
Coyote Lake, CA	06.08.1979	Gilroy Array #6	5.74	4.37	C	0.440
Hector Mine, CA	16.10.1999	Hector	7.13	26.53	C	0.266
Imperial Valley, CA	15.10.1979	Delta	6.53	33.73	D	0.285
Imperial Valley, CA	15.10.1979	El Centro Array #10	6.53	26.31	D	0.224
Kern County, CA	21.07.1952	Taft Lincoln School	7.36	43.49	C	0.173
Kobe, Japan	16.01.1995	Amagasaki	6.90	38.79	D	0.364
Kobe, Japan	16.01.1995	Kobe University	6.90	25.40	B	0.290
Kobe, Japan	16.01.1995	Takatori	6.90	13.12	D	0.616
Loma Prieta, CA	18.10.1989	Emeryville	6.93	45.50	D	0.256
Managua, Nicaragua	23.12.1972	Managua, ESSO	6.24	5.68	D	0.337
Northridge, CA	17.01.1994	Jensen Filter Plant	6.70	10.20	C	0.592
Northridge, CA	17.01.1994	Castaic-Old Bridge Route	6.69	40.68	C	0.520
Northridge, CA	17.01.1994	Santa Monica City Hall	6.69	22.50	D	0.369
San Salvador, El Salv.	10.10.1986	National Geographical Inst.	5.80	9.54	D	0.612
Westmorland, CA	26.04.1981	Westmorland Fire Station	5.90	7.02	D	0.496
Whittier Narrows, CA	01.10.1987	Castaic-Old Ridge Route	5.99	19.81	D	0.332
Victoria, Mexico	09.06.1980	Cerro Prietto	6.33	33.73	C	0.621

3. STRUCTURAL MODELS

3.1. Multiple Degree of Freedom Systems

The structures selected for this investigation have been extensively studied in the past (Fardis 1994, Mwafy and Elnashai 2001) and represent a wide range of realistic structural configurations in terms of the earthquake resistant system (frame or dual), their overall height (and subsequently, fundamental period) and ductility level (Classes High and Low according to the Eurocode 8). These buildings are illustrated in Fig. 3.1 and can be classified into three main categories: regular 12-storey frames of low and high ductility (codified as 12RFDCL and 12RFDCH respectively), regular 8-storey dual systems again of low and high ductility (8SWDCL and 8SWDCH) and irregular in elevation 8-storey frame of high ductility (8IFDCH). The irregular building was studied in order to investigate the range of applicability of the envisioned transformation of the MDOF to the SDOF system (Fajfar 2000). All structures were first modeled as two-dimensional MDOF systems using the computer code Zeus-NL (Elnashai *et al*. 2002). Cubic 3D elasto-plastic elements were used considering concrete behavior under cyclic loading, residual strength and stiffness degradation through a fiber approach.



ID: 8IFDCH
Number of storeys: 8
Structural system: Frame
Regularity: Irregular in height
Typical storey height: 3.0 m (first: 4.5 m)
Ductility Class: High
Fundamental period: 0.581 s

ID: 12RFDCH / 12RFDCL
Number of storeys: 12
Structural system: Frame
Regularity: In height
Typical storey height: 3.0 m
Ductility Class: High / Low
Fundamental period: 0.715 s / 0.752 s

ID: 8SWDCH / 8SWDCL
Number of storeys: 8
Structural system: Dual (shear wall-frame)
Regularity: In height
Typical storey height: 3.0 m
Ductility Class: High / Low
Fundamental period: 0.532 s / 0.565 s

Figure 3.1. Configuration of the structures considered

A bilinear elasto-plastic model with kinematic strain-hardening was adopted for the reinforcing steel (yield strength, f_y= 585 MPa), while the uniaxial nonlinear constant confinement model (compressive strength, f_c=30 MPa) was chosen for concrete (Papanikolaou and Elnashai 2005). The fundamental periods of the five structural models were determined as follows: 12RFDCH: T=0.715 s, 12RFDCL: T=0.752 s, 8SWDCH: T=0.532 s, 8SWDCL: T=0.565 s and 8IFDCH: T=0.581s.

3.2. Equivalent Single Degree of Freedom Systems

A transparent formulation, based on Fajfar N2 method (Fajfar and Gašperšič 1996, Fajfar 2000), was utilized in order to transform the five MDOF systems described above, into equivalent inelastic SDOF systems. Having defined analytically the parameters of the equivalent inelastic SDOF system (i.e., height, mass, deflection shape, force-displacement relationship at yield and ultimate deformation), the five SDOF systems were modeled in IDARC-2D (Reinhorn *et al.* 2009) using the bilinear backbone curve that resulted from the Standard Pushover (SPO) analysis of the MDOF system. The embedded yield-oriented model was also used to account for stiffness degradation, strength deterioration, non-symmetric response and pinching.

A modeling issue of particular interest was to investigate the implications of the very initial (essentially purely elastic) stiffness of the MDOF systems on period elongation using a tri-linear backbone curve. Such an assumption would have the advantage that the SDOF systems studied would have identical elastic period with the MDOF ones. As it was found that first cracking stage is not only rapidly exceeded but has also negligible impact on the period elongation, the effective stiffness was determined on the basis of the yield point of the bi-linear SPO:

$$T^* = 2\pi\sqrt{\frac{m^* D_y^*}{F_y^*}} \tag{3.1}$$

where F^*_y and D^*_y are the yield strength and displacement of the SDOF system. The resulting elastic periods of the equivalent SDOF systems were found equal to 0.966 s, 1.038 s, 0.707 s, 0.723 s and 0.804 s for the 12RFDCH, 12RFDCL, 8SWDCH, 8SWDCL and 8IFDCH buildings respectively.

It is recalled that the vibration periods of the SDOF systems were higher than the fundamental periods of the corresponding MDOF systems (by a factor of 1.28-1.38), since the latter have been derived through eigenvalue analysis using elastic and not effective properties. With respect to the degrading hysteresis model adopted, there are four parameters that control the inelastic loading reversals: α accounting for stiffness degradation, β_1 and β_2 for strength deterioration and a slip parameter γ that controls the pinching due to the closing cracks during the reloading phase. Two degradation levels (mild and severe) were examined in order to envelope the period shift under strong ground motion.

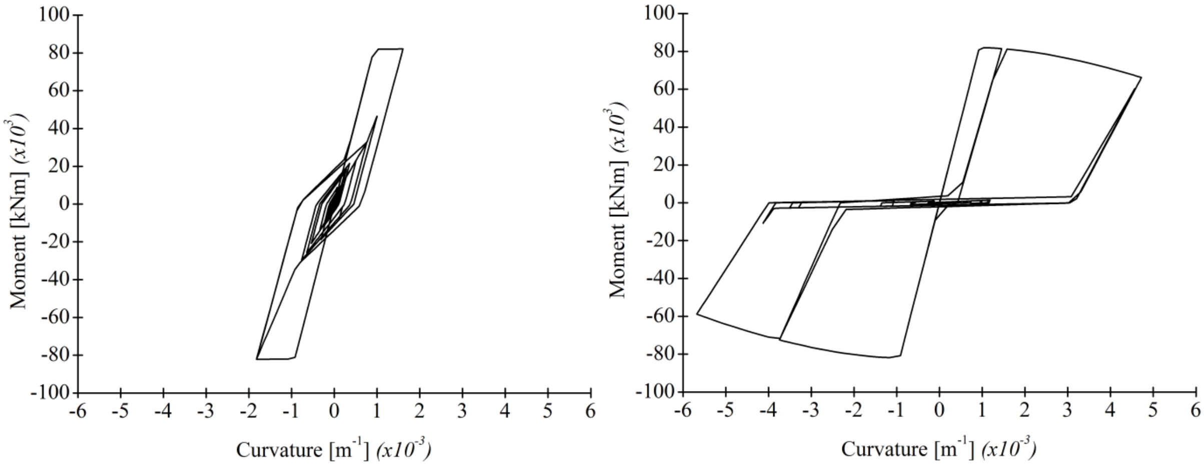


Figure 3.2. Moment-curvature relationship of the 12RFDCH-SDOF system when subjected to the Emeryville record (scaled to a PGA of 0.90g) for the case of a mild (left) and severe (right) degradation rule.

4. VALIDATION OF THE SDOF SYSTEMS

To validate the above procedure, the predominant inelastic period predicted by the MDOF and the equivalent SDOF systems, was comparatively assessed for all five buildings studied. The predominant inelastic period was identified through the Fast Fourier Transformation (FFT) of the relative acceleration at the top with respect to the base acceleration. The validation procedure involved the performance of 1000 additional nonlinear response history analyses for the MDOF systems examined (5 models x 20 ground motions x 10 scaling factors). It is important to note that the MDOF structural systems, modeled using ZeusNL (at least in the version used to produce these results), account explicitly only for stiffness degradation while there is also a minor effect of the pinching load on the closure of the open cracks; hence, the elongation of the fundamental period of vibration of the MDOF systems is primarily attributed to the spread of cracks and the section yielding.

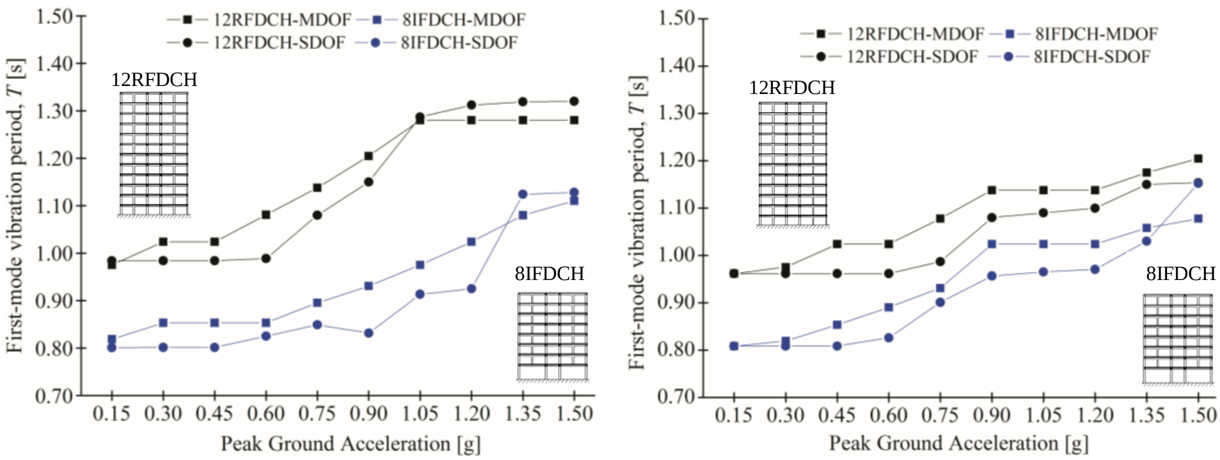


Figure 4.1. First-mode vibration periods predicted by the MDOF and SDOF (mild degradation) systems for the 12RFDCH and 8IFDCH buildings when subjected to the Whittier Narrows (left) and Hector Mine (right) earthquake strong motions.

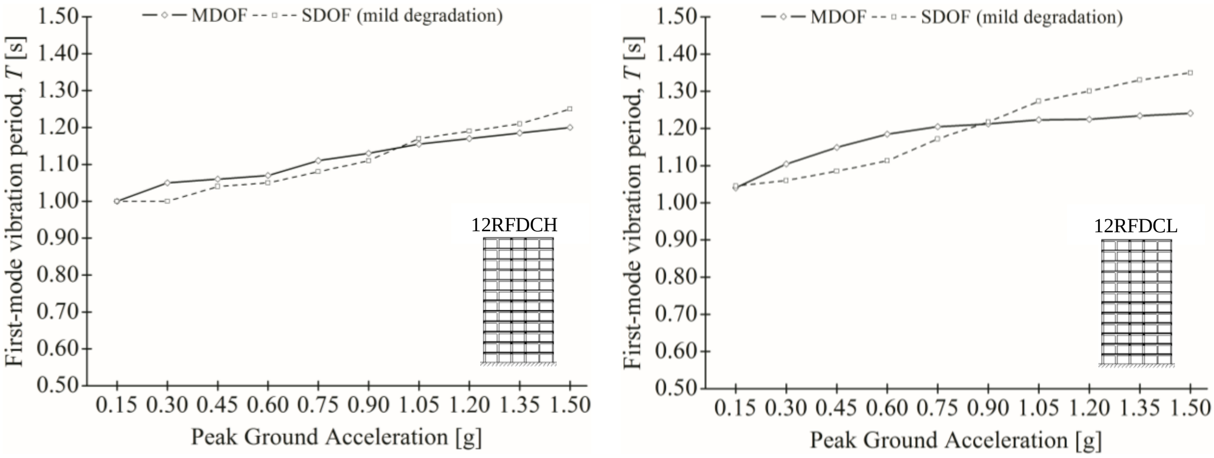


Figure 4.2. Mean first-mode vibration periods predicted by the MDOF and the SDOF (mild degradation) systems for the 12RFDCH (left) and 12RFDCL (right) buildings.

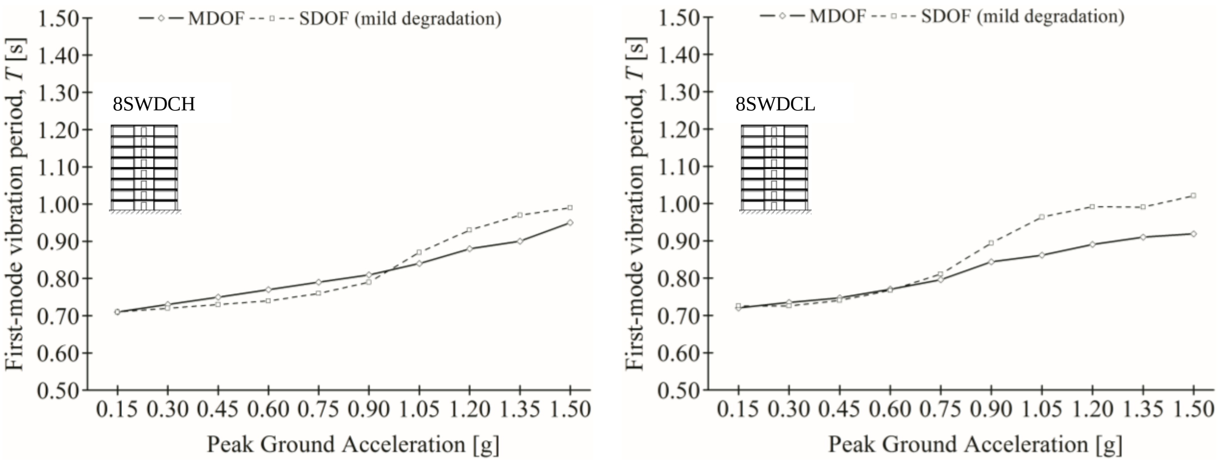


Figure 4.3. Mean first-mode vibration periods predicted by the MDOF and the SDOF (mild degradation) systems for the 8SWDCH (left) and the 8SWDCL (right) buildings.

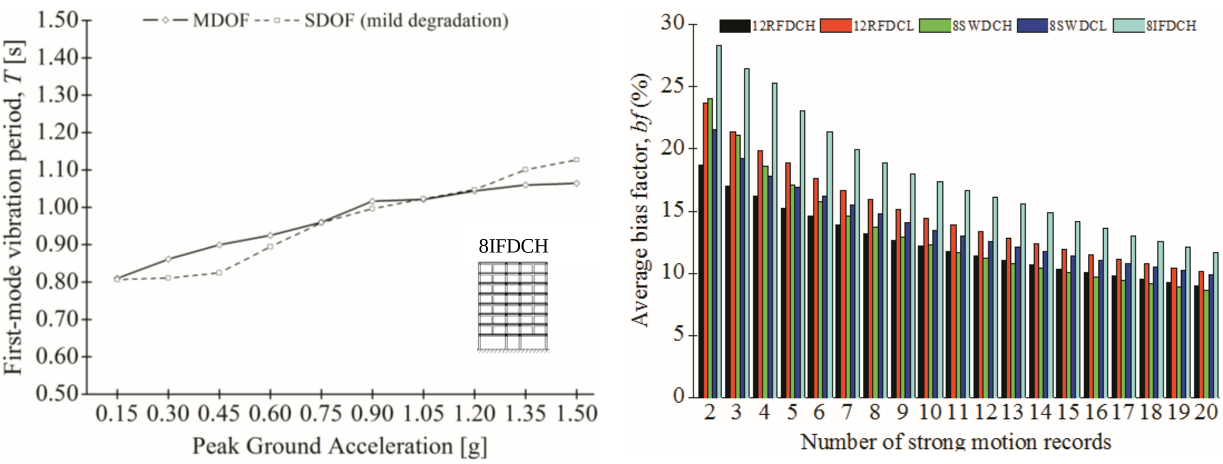


Figure 4.4. Mean first-mode vibration periods predicted by the MDOF and the SDOF (mild degradation) systems for the 8IFDCH buildings (left). Average bias factor of the SDOF first-mode vibration period estimates as a function of the number of earthquake strong motions used (right).

On the other hand, the SDOF systems modeled in IDARC2D account explicitly for stiffness degradation, strength degradation and pinching but at the same time they neglect the inelastic response at a member level. For this reason, the predictions regarding the period elongation of the SDOF systems were based on the assumption of an (average) mild degradation rule. Figure 4.1 plots some indicative results of period elongation results for the 12RFDCH and 8IFDCH buildings, subjected to the multiply scaled records of the Whittier Narrows and Hector Mine (corresponding to low and high frequency content respectively). A reasonably good agreement is observed, at least from a qualitative point of view, even for significantly high levels of ground motion intensity and severe associated inelastic response. It is also noted that this reasonable matching is evident for all buildings studied and for all earthquake records used.

Most importantly, the agreement between the MDOF and SDOF predictions is significantly improved if the comparison is made on the basis of the mean period elongation using the entire set of earthquake records (Figs. 4.2, 4.2 and 4.4 left). In this case, the bias factor (*bf*) between the two prediction lines (mean inelastic period T_i^{MDOF} and T_i^{SDOF} versus PGA of the MDOF and SDOF systems respectively) is given by the following expression:

$$bf = \sqrt{\frac{1}{N}\sum\left(\frac{T_i^{MDOF}-T_i^{SDOF}}{T_i^{MDOF}}\right)^2}\text{ , } i=1 \text{ to } 10 \text{ levels of strong motion intensity} \tag{4.1}$$

As it is shown in Fig. 4.4 (right) the bias factor of the mean inelastic period for the entire set of strong ground motions drops down to less than 10% independently of the building examined and despite the compromising modeling assumptions that were inevitably made.

5. VARIATION OF PERIOD ELONGATION WITH GROUND MOTION INTENSITY

Having obtained a level of confidence on the equivalent SDOF systems of the five buildings studied, the severely degrading hysteretic rule was also adopted as a means to provide an upper bound of the potential period shift. Again, the inelastic period ($T_{in.}$) was assessed through the FFT transformation and the period shift ratio ($T_{in.}/T_{el.}$) was derived on the basis of the elastic period ($T_{el.}$) of the original MDOF system (see §3.1). Fig. 5.1 depicts an example of such an evolution of the fundamental frequency of vibration with increasing PGA for the 12RFDCH building and two different strong motions (Northridge and Imperial Valley). Fig. 5.2 illustrates another example of the period elongation as a function of ground motion intensity, for the 12RFDCH equivalent system, when modeled with mild and severe degradation rules and is subjected to the (high frequency content) Northridge and the (low frequency content) Coyote Lake earthquake ground motions.

It is clearly observed that period elongation estimates from the mildly degrading system are, naturally, always lower than the ones derived when the severe degradation rule is adopted. As it is also anticipated, the inelastic response is triggered at the same level of PGA (i.e., 0.3 g for both the two seismic motions used) for the two SDOF systems. Once this threshold value is exceeded, the fundamental period is monotonically lengthened as the ground motion becomes more intense. Note that the period shift ratio, that corresponds to the lowest plotted PGA value to 0.15 g, is not unity simply because the inelastic period is normalized to the (effective) fundamental period of the reference MDOF systems and not to the initial (elastic) vibration period (for the reasons explained in §3.2).

Fig. 5.3 plots the mean period shift ratio ($T_{in.}/T_{el.}$) predicted for the entire set of strong motions, as a function of the adopted intensity measure (PGA). A monotonic increase is again observed. As anticipated, the assumption of a severe degrading rule (right figure) leads to higher period elongation as opposed to mild degradation (left figure). This deviation is, reasonably, equal to zero for low levels of PGA (i.e., 0.15 g) and generally increases with strong motion intensity, reaching 16.5% for the extreme value of 1.50 g. This behavior is also anticipated given the fact that the constitutive differences of the two models become more significant only well beyond yield.

The absolute amplitude of the period elongation observed for the five buildings studied is of particular interest. The highest mean period shift ratios were emerged from the irregular 8IFDCH system which is the most vulnerable structure studied. It can be also seen that for all buildings, the assumption of a severe degradation rule leads to an upper bound of the period shift ratio that ranges from 2.04 to 2.29 (Fig, 5.3, right) for the extreme case of a peak ground acceleration of 1.50 g. This range of period elongation is higher compared to range 1.78 to 1.91 predicted for the same buildings when adopting the mild degradation rule.

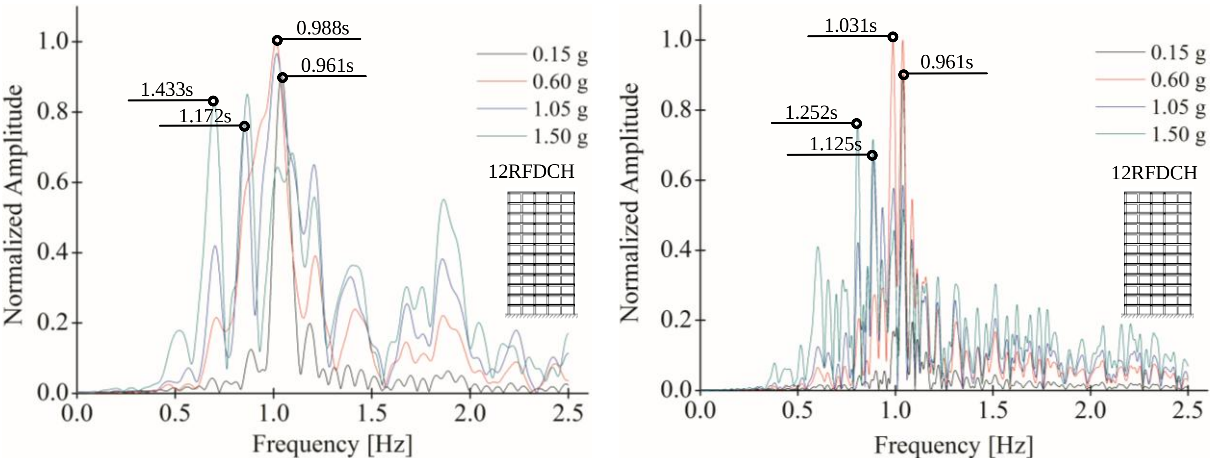


Figure 5.1. Evolution of the frequency spectrum for the 12RFDCH equivalent SDOF system when subjected to increasing levels of the Northridge (left) and Imperial Valley (right) seismic motions.

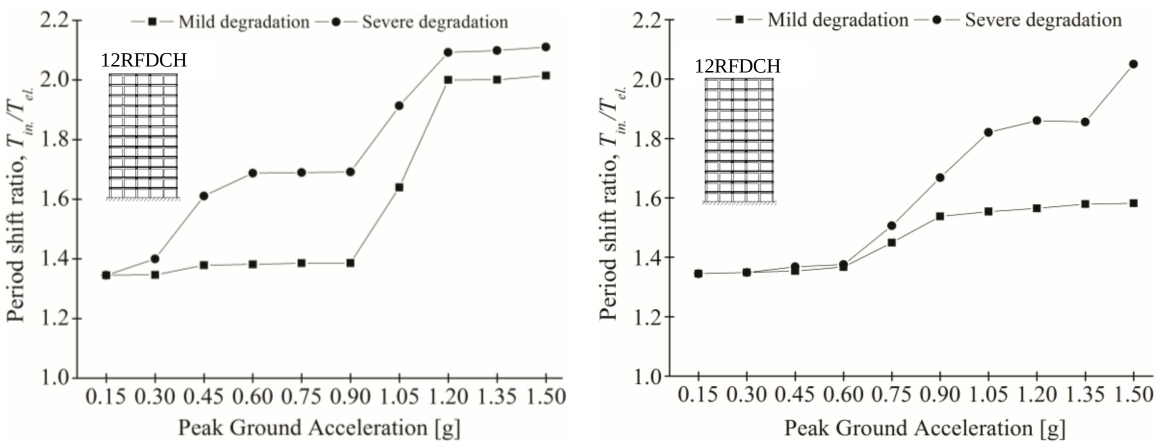


Figure 5.2. Period shift ratio as a function of PGA for the 12RFDCH equivalent system modeled with two degradation rules adopted and subjected to the Northridge (left) and Coyote Lake (right) seismic motions.

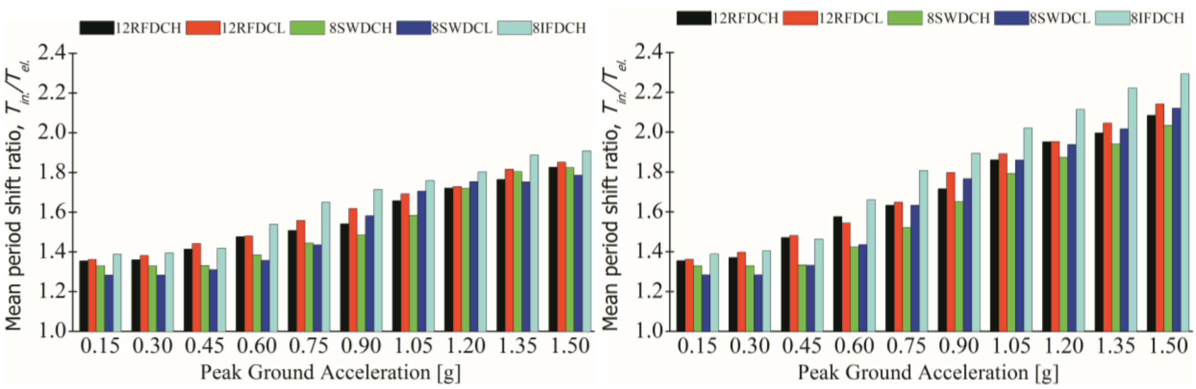


Figure 5.3. Mean period shift ratio as a function of PGA for the case of mild (left) and the severe (right) degradation rules.

For more frequent levels of seismic intensity (that is, for intensity measures with higher probability of exceedance) the mean period shift ratios are significantly lower. For instance, for twice the design earthquake at high seismicity zones in Europe, which roughly corresponds to a design peak ground acceleration equal to 2x0.36 g=0.72 g, the mean period shift lies in the range of 1.51-1.80 for the worst case of severe degradation. The corresponding period shift for mild degradation is lower (1.42-1.63).

This observation has a significant implication in the procedure put forward by Eurocode 8 - Part 1 (EC8 2004) in selecting and scaling earthquake records for the purpose of response history analysis. It is recalled that the mean of the 5%-damped elastic spectrum calculated from all time histories should not be less than 90% of the corresponding value of the 5%-damped EC8 elastic response spectrum, in the range of periods from $0.2T_1$ and $2T_1$, where T_1 is the fundamental period of the structure in the direction where the accelerogram is applied (§3.2.3.1.2.4 of EC8). It has been shown (Sextos *et al.* 2011) that the long period bound of $2T_1$ is one of the two main factors that introduce large variability in the structural response within a given set of earthquake records (the other factor being the lack of an upper bound for the spectral accelerations of the mean spectrum, similar to the lower bound of 90%).

The numerical predictions of the period shift ratios presented herein, do not confirm the fundamental period elongation hinted by the upper bound of $2.0T_1$ since, as already stated, they have been found to lie within the range of 1.42-1.80 for a peak ground acceleration of 0.72 g. This difference is even more pronounced bearing in mind that the above period shift ratios have been derived with respect to the *elastic* period of the buildings studied. On the contrary, the factor 2.0 of Eurocode 8 refers to the fundamental period of the design spectrum which is typically derived through modal analysis with *cracked section* properties (i.e., after a uniform stiffness reduction of 50% for all beams and columns). As the ratio between the elastic and the effective period may vary by 30% (see §3.2), it is evident that the maximum mean inelastic period that was predicted in this study for the most vulnerable building (8IFDHC), the most unfavorable case of (severe) strength and stiffness degradation and for twice the design earthquake in a high seismicity area, did not exceed 1.80/1.30=1.38 times the corresponding effective period. This value contradicts the duplication of the effective fundamental period that is prescribed in the code. It is also noted that any other potential source of period elongation (for instance, soil compliance) is *already* accounted for in the finite element model used to predict the fundamental period T_1, hence there is no other mechanism than structural yielding that may further lengthen the period of vibration.

Based on the above, the range of spectral matching ($0.2T_1<T_1<2T_1$) prescribed in EC8-Part 1 could be revised to ($0.2T_1<T_1<1.50T_1$) at least for the case of new buildings designed for low or moderate levels of ductility and low-to-medium stiffness degradation. It is recalled that the upper bound of period elongation which is proposed herein (i.e., $1.5T_1$) is also in agreement with the provisions of EC8-Part 2 (EC8 2004) for bridges, ASCE 07-10 (ASCE 2010) and the current guidelines provided by NEHRP (FEMA P-750 2009).

6. CONCLUSIONS

A parametric analysis was employed to predict the period elongation for a group of RC buildings with different structural configurations and dynamic characteristics. The buildings were all designed according to Eurocode 8. The original MDOF structures were transformed to equivalent inelastic SDOF systems with two different hysteretic degrading rules. After extensive validation to confirm that the ability of the equivalent SDOF systems to accurately predict period elongation, the mean period shift ratios were derived for 20 selected seismic motions, scaled to 10 strong motion intensity levels each. The conclusions drawn are summarized as follows:

(a) there is a monotonic and almost proportional trend between the ground motion intensity, as expressed in terms of PGA, and the period elongation. This trend was found independent of the characteristics of the buildings and the hysteretic rules.

(b) the mean period elongation for the five buildings studied varied in the range of 1.78-2.29 at the extreme level of a PGA equal to 1.50 g, for the case of mild and severe hysteretic degradation respectively.
(c) this range of variation was found significantly lower (i.e., 1.42 to1.80) for ordinary levels of ground motion intensity (for instance, for twice the design earthquake at the regions of high seismicity in Europe, indicatively corresponding to a PGA of 0.72 g).
(d) in the most unfavorable case of the irregular building characterized by severe strength and stiffness degradation, the mean inelastic period did not exceed 1.38 times the corresponding effective period (again for a PGA of 0.72 g).

Based on the above observations, it is proposed to revise the highly conservative period range prescribed by Eurocode 8 for spectral matching ($0.2T_1<T_1<2.00T_1$) to ($0.2T_1<T_1<1.50T_1$) at least for the case of new buildings designed for low or moderate levels of ductility and low-to-medium stiffness degradation. It is deemed that this revision, which is also in line with the U.S. codes and standards, will also reduce the structural response scatter attributed to the width of the spectral matching range and further contribute towards a more reliable and stable estimate of the structural response under strong ground motion.

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