Basic Study on Seismic Performance Analysis of RC Tunnel Junction Using Large-scale Finite Element Method

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SUMMARY:
This paper studies the applicability of large scale finite element method to evaluate the seismic performance of an RC tunnel junction, which is a critical component as it connects a main tunnel and ramp tunnels. Making use of a most advanced code of finite element method, which is able to solve a model of more than 10,000,000 degree-of-freedom, we implement Maekawa's non-linear elasto-plastic concrete constitutive relations and particle discretization scheme for failure analysis. Results of the numerical computation for the junction model are presented, and discussions are made on the usefulness of the large-scale finite element method for the seismic performance evaluation.

Keywords: nonlinear, large-scale, finite element method, concrete constitutive relations, particle discretization scheme

1. INTRODUCTION

Accurate estimate of dynamic response of a structure subjected to ground motion is of primary importance for aseismic design. Experimental analysis is a standard choice even though it is costly, labor-intensive and time-consuming, especially for a structure with complicated configuration or consisting of composite structures.

Numerical analysis is an alternative of estimating the seismic response. In earthquake engineering, a sophisticated structure component element has been used for the numerical analysis. While it is able to reproduce non-linear behavior, it must require tuning for its parameters by using the experimental results of structure components.

The use of solid elements becomes more standard for the numerical analysis of finite element method (FEM) since it is able to analyze a model more than 1,000,000 degree-of-freedom, taking advantage of the progress of computer hardware and software. The numerical analysis requires fewer efforts than the experimental analysis, if such a large scale computation is possible, since tuning a solid element needs only material sample experiment.

In this paper, we investigate a possibility of applying large-scale FEM with solid element to seismic performance evaluation. The target structure is an RC tunnel junction. This structure has complicated configuration as it has to bear large force that is required to carry in connecting a ramp tunnel and two main tunnels. A most advanced code of FEM, called ADVENTURE, is used. The key issue is to enhance the code with features which are essential for the analysis of an RC structure, namely, non-linear concrete constitutive relations and brittle failure due to multiple cracking.

The contents of this paper are organized as follows. In section 2, treatment of Maekawa's concrete constitutive relations including reformulation and algorithm is presented. In section 3, treatment of brittle failure is explained. Results of numerical computation of the junction model are presented and discussed in section 4. Some concluding remarks are given in section 5.

2. TREATMENT OF MAEKAWA'S CONCRETE CONSTITUTIVE RELATION

Maekawa and his colleagues have been developing original concrete elasto-plastic constitutive relations which are attaining the highest reputation in concrete engineering ([Maekawa(2003)]). While it is well validated, the formulation of the constitutive relation is less transparent to be implemented in a general purpose FEM. Moreover, it allows negative slope for an apparent relation between stress and strain, which will be a hinge for the modern FEM that uses a fast solver based on the conjugate gradient method. We thus reformulate Maekawa's concrete relation and develop an algorithm which accounts for the negative slope of the apparent stress strain relation.

2.1. Reformulation

Maekawa's constitutive relation is rationally formulated in terms of elastic strain and its invariants. When the elastic strain and stress denoted by $\boldsymbol{\epsilon}^E$ and $\boldsymbol{\sigma}$, the set of equations are expressed as follows:

$$\boldsymbol{\sigma} = \mathbf{c} : \boldsymbol{\epsilon}^E, \tag{2.1}$$

$$\mathrm{d}\boldsymbol{\epsilon}^P = \boldsymbol{\ell} : \mathrm{d}\boldsymbol{\epsilon}^E. \tag{2.2}$$

Here, $\mathbf{c}$ is isotropic elasticity tensor, which is a forth-order tensor-valued nonlinear function of elastic strain invariants, and $\boldsymbol{\ell}$ is another forth-order function of $\boldsymbol{\epsilon}^E$.

The following $\mathrm{d}\boldsymbol{\sigma}$-$\mathrm{d}\boldsymbol{\epsilon}$ relation is easily derived using the above two equations, Eqn. 2.1 and Eqn. 2.2:

$$\mathrm{d}\boldsymbol{\sigma} = \mathbf{c}^{EP} : \mathrm{d}\boldsymbol{\epsilon}. \tag{2.3}$$

Here, $\mathbf{c}^{EP}$ is forth-order elasto-plasticity tensor given as

$$\mathbf{c}^{EP} = (\mathbf{c} + \boldsymbol{\nabla}\mathbf{c} : \boldsymbol{\epsilon}^E) : (\mathbf{I} + \boldsymbol{\ell})^{-1}, \tag{2.4}$$

where $\boldsymbol{\nabla}\mathbf{c} : \boldsymbol{\epsilon}^E$ is a fourth-order tensor with its component being given as $(\partial c_{ijpq} / \partial \epsilon^E_{kl})\epsilon^E_{pq}$ and $(.)^{-1}$ denotes inverse of a fourth-order tensor $(.)$.

A key feature of the constitutive relation is that the elastic strain invariants give the direction of the plastic strain increment. The amplitude of the plastic strain increment will be determined by satisfying a condition which corresponds to the progress of damage. The plastic strain increment denoted by $\mathrm{d}\boldsymbol{\epsilon}^P$ is expressed in terms of the direction and the incremental quantity as follows,

$$\mathrm{d}\boldsymbol{\epsilon}^P = dg\mathbf{d}. \tag{2.5}$$

Here, dg is the incremental quantity and $\mathbf{d}$ is the incremental direction given as,

$$\mathbf{d} = DP\boldsymbol{\delta} + \frac{\boldsymbol{\epsilon}^E}{J_2^E} \ . \tag{2.6}$$

Here, $\boldsymbol{\delta}$ is Kronecker's delta, D and P are functions of $\boldsymbol{\epsilon}^E$.

Regarding the condition of damaged progress as a yield condition, we can reformulate Maekawa's constitutive relation in a form similar to non-associated flow rule, which describes the plastic strain increment as a gradient of a plastic potential and determines its amplitude to satisfy a yield condition. The shear plastic relation between the second invariant of plastic strain J_2^P and the second invariant of elastic strain J_2^E in Maekawa's constitutive relation is employed as the yield function, which is written as

$$f(J_2^P, J_2^E) = J_2^P - H(J_2^E) = 0. \tag{2.7}$$

According to the consistency condition, $\mathrm{d}f = 0$ is calculated. With some mathematical manipulation, dg, the amplitude of $\mathrm{d}\boldsymbol{\epsilon}^P$, is obtained as $dg = \frac{H'}{2(1+H')}\frac{\boldsymbol{e}^E:\mathrm{d}\boldsymbol{\epsilon}}{J_2^E}$, which can be explicitly calculated from $\mathrm{d}\boldsymbol{\epsilon}$, and elasto-plastic tensor $\mathbf{c}^{EP}$ is reformulated as follows:

$$\mathbf{c}^{EP} = (\mathbf{c} + \nabla\mathbf{c} : \boldsymbol{\epsilon}^E) : (\mathbf{I} - \mathbf{L}). \tag{2.8}$$

For $\mathbf{c}^{EP}$ given by Eqn. 2.8 does not involve tensor inversion, we further seek to obtain an explicit expression of $\nabla\mathbf{c}$ in order to reduce computational load. This reformulation results in a more transparent form of the constitutive relation. Indeed, it is straightforward to implement the constitutive relation into a general purpose FEM code, which usually includes a functionality of non-linear elasto-plastic analysis that is based on an associated flow rule or a similar one.

2.2. Algorithm

A fast solver is based on the conjugate gradient method, and solves a linear matrix equation by minimizing the error of the equation. An FEM with such a solver requests positive-definiteness of a global stiffness matrix, which is guaranteed by making elasticity or elasto-plasticity tensor positive-definite. Softening, stress drop due to strain increase, is a key feature of concrete. Hence, it is not possible to apply a fast solver to analyse a concrete structure or an RC structure.

To tackle this essential difficulty of the concrete constitutive relation, we develop an algorithm of solving a matrix equation, the matrix of which is not positive definite. The matrix is replaced by a suitable positive-definite matrix, and an additional term is added to the original matrix equation so that the replacement is compensated. The details are explained as follows.

Ordinary elasto-plastic governing equation for displacement incremental $\mathrm{d}\mathbf{u}$ from the equilibrium equation for stress increment $\mathrm{d}\boldsymbol{\sigma}$ is:

$$\nabla \cdot \left(\mathbf{c}^{EP} : (\nabla \mathrm{d}\mathbf{u})\right) = \mathrm{d}\mathbf{f}, \tag{2.9}$$

where $\nabla\mathrm{d}\mathbf{u}$ is gradient of $\mathrm{d}\mathbf{u}$, $\mathrm{d}\mathbf{f}$ is external force. In the discretised form for the FEM analysis, the governing equation can be written as:

$$\mathbf{K}^{EP}\mathrm{d}\mathbf{u} = \mathrm{d}\mathbf{f}, \tag{2.10}$$

where $\mathbf{K}^{EP} = \sum_e \mathbf{B}_e^T \mathbf{c}^{EP} \mathbf{B}_e$ is stiffness matrix which can lose either symmetric or positive definiteness for the case of Maekwa's concrete constitutive relation.

In order to solve the difficulty described above, an alternative governing equation is derived using the following constitutive relation:

$$\mathrm{d}\boldsymbol{\sigma} = \mathbf{c} : \mathrm{d}\boldsymbol{\epsilon} + \mathrm{d}\boldsymbol{\sigma}^*. \tag{2.11}$$

Here, $\mathrm{d}\boldsymbol{\sigma}^*$ is a second-order tensor which is given as:

$$\mathrm{d}\boldsymbol{\sigma}^* = -\mathbf{c} : \mathrm{d}\boldsymbol{\epsilon}^P + (\nabla\mathbf{c} : \mathrm{d}\boldsymbol{\epsilon}^E) : \boldsymbol{\epsilon}^E. \tag{2.12}$$

Accordingly, the alternative governing equation is replaced as follows:

$$\mathbf{K}\mathrm{d}\mathbf{u} = \mathrm{d}\mathbf{f} + \mathrm{d}\mathbf{f}^*, \tag{2.13}$$

where $\mathbf{K} = \sum_e \mathbf{B}_e^T \mathbf{c} \mathbf{B}_e$ and $\mathrm{d}\mathbf{f}^* = (\mathbf{K} - \mathbf{K}^{EP})\mathrm{d}\mathbf{u}$.

Since $\mathbf{c}$ is symmetric and positive definiteness, it is guaranteed that the global stiffness matrix $\mathbf{K}$ is symmetric and positive definite, and hence a fast solver can be applied. For the second term in the right side, we do not need to calculate the complicated $\mathbf{K}^{EP}$ matrix; instead, the vector term is directly derived from $\mathrm{d}\boldsymbol{\sigma}^*$ in which $\mathrm{d}\boldsymbol{\epsilon}^P$ and $\nabla\mathbf{c}$ are explicitly calculated from variables of previous step during iteration.

3. TREATMENT OF BRITTLE FAILURE

It is essential to consider cracking that takes place in concrete for the evaluation of ultimate seismic capacity of an RC structure. However, numerical analysis of cracking in concrete is much more difficult than cracking in metallic materials. While the analysis of a major crack subjected to tensile force is a key issue for metallic materials, more complicated processes of cracking need to be analysed for concrete; numerous micro-cracks are initiated subjected to compression or shearing forces, and they grow with branching and kinking until they finally form macro-cracks which lead to collapse of the structure.

Cracking functionality is implemented in modern FEM packages. However, it is not suitable to analyse the complicated cracking process in concrete. This is mainly because this functionality is developed to analyse a single crack or a few cracks. Furthermore, the functionality is not able to determine configuration of multiple cracking, since it is based on fracture mechanics of a single crack.

We are developing an alternative which is capable of determining multiple-crack configuration in a straightforward manner, based on the strength of material. This alternative is based on a new discretization scheme which discretizes a smooth function to a function with numerous discontinuities since a crack is treated as a discontinuity in displacement function. This scheme is called Particle discretization scheme (PDS) ([Hori(2005)] and [Wijerathne(2009)]). The key point of this scheme is the use of two sets of characteristic functions, in terms of which a function and its derivatives are discretized; while a function is discretized as a function with discontinuity, its derivative can be rigorously computed, and hence the scheme is applicable to solving differential equations.

4. NUMERICAL EXAMPLE

4.1. Problem Setting

In order to examine the basic usefulness of the large-scale FEM that is implemented with Maekawa's concrete constitutive relation and PDS, we solve an RC structure member, which is used for a junction of underground tunnels, connecting a ramp tunnel with two main tunnels. The ultimate capacity of the structure member is a key factor which determines the seismic performance of the junction since it transmits forces of the ramp tunnel to the main tunnel. We need to examine whether the numerical analysis that uses the large-scale FEM will be an alternative of experimental analysis for the capacity evaluation.

The target structure member is shown in Fig. 4.1 and the layout of reinforcements within the structure is shown in Fig. 4.2. This member is a composite structure, consisting of a steel girder which is embedded in an RC body. It is the girder that is used to transmit forces and distribute stress in connecting the main tunnel to the ramp tunnels. Thus, the ultimate capacity of the girder, regarding bending, shearing and compressive forces, needs to be evaluated, by analysing the stress transfer from the girder to the embedded body.

An analysis model of the structure member is meshed using tetrahedral solid elements; the number of nodes, elements and degree-of-freedom is 2,551,983, 14,944,479 and 7,655,949, respectively. Fig. 4.3 and Fig. 4.4 show the close-up view of the mesh of reinforcements and concrete, respectively. The material properties of concrete and steel are summarized in Table 4.1; as a remarkable feature of Maekawa's concrete constitutive relations, the concrete material parameters which have to be experimentally measured are Young's modulus and the compressive strength, with the other parameters being predetermined. A workstation with Quad-Core AMD Opteron (tm) Processor 8379 HE (2.4GHz 128 GB RAM) is used.

The boundary conditions are applied in such a way that the displacements of surface A and B and bottom surface (see Fig. 4.1) are fixed in the x-, y-, z-direction, and static load, which is shown as a red arrow in Fig. 4.1, is applied at the loading panel located on the girder; the load is 200kN, and bending moment is generated on the girder and transferred to the RC ramp tunnel as desired.

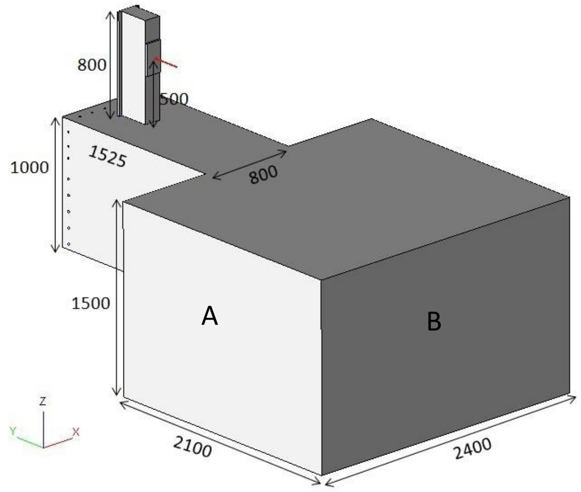

Figure 4.1. Tunnel junction member model (unit: mm).

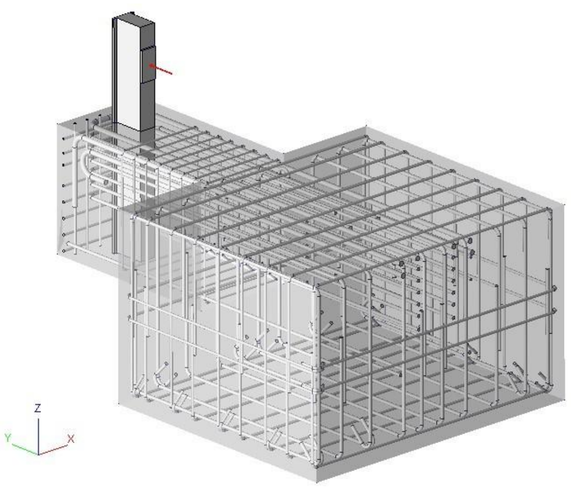

Figure 4.2. Model of reinforcements and girder

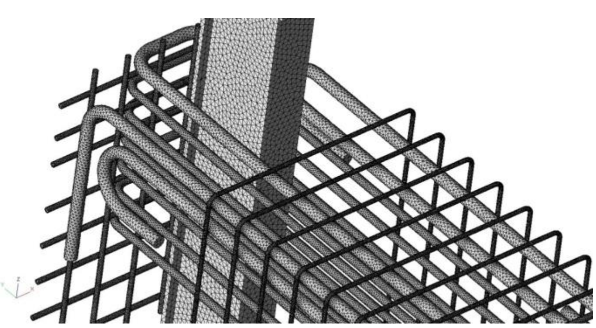

Figure 4.3. Analytical model of reinforcements and girder.

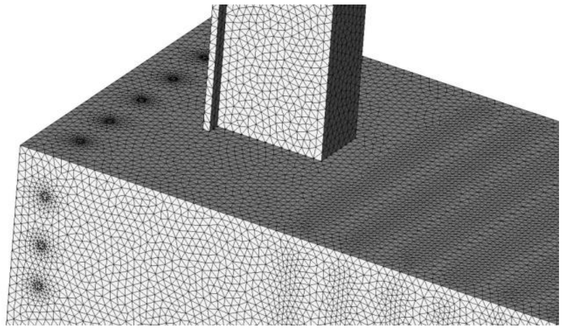

Figure 4.4. Analytical model of concrete and girder

Table 4.1. Material properties.

Material	Young's modulus	Poisson's ratio	Yield stress	Compressive strength
Concrete	25 GPa	0.2	-	25 MPa
Reinforcements	210 GPa	0.3	345 MPa	-
Steel	210 GPa	0.3	325 MPa	-

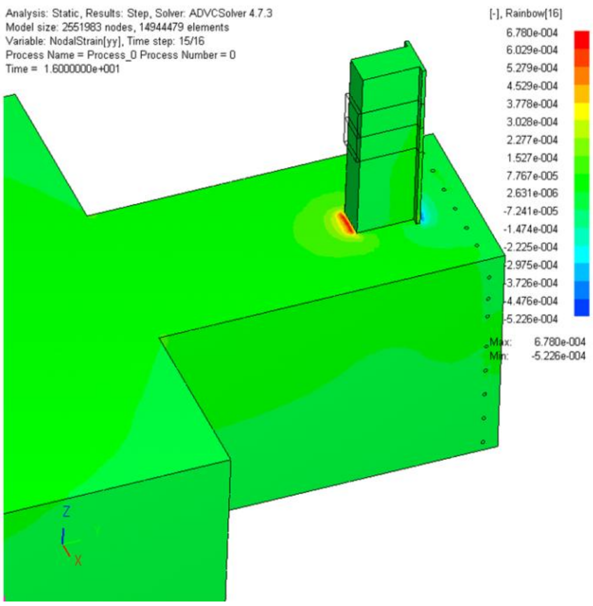

Figure 4.5. Strain distribution.

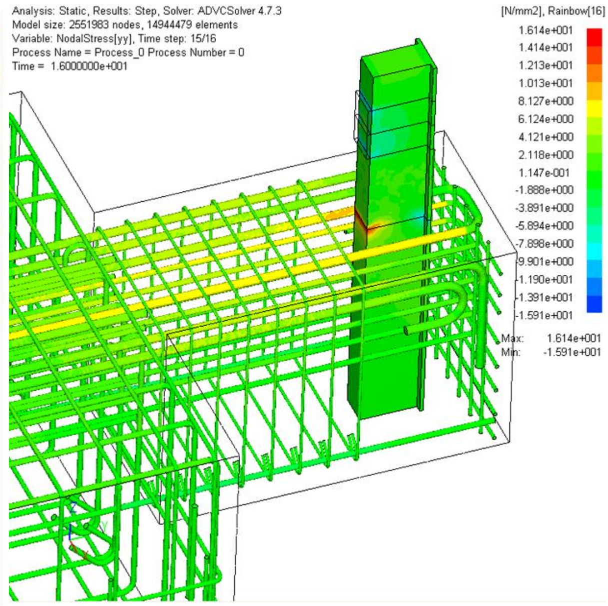

Figure 4.6. Stress distribution for rebars and girder with failure analysis.

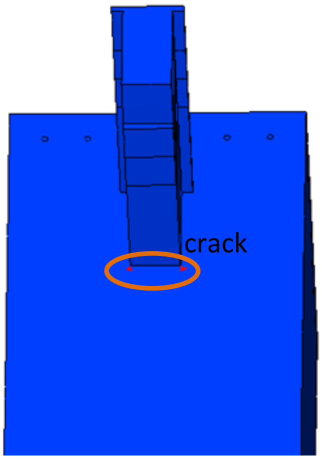

Figure 4.7. Cracking state.

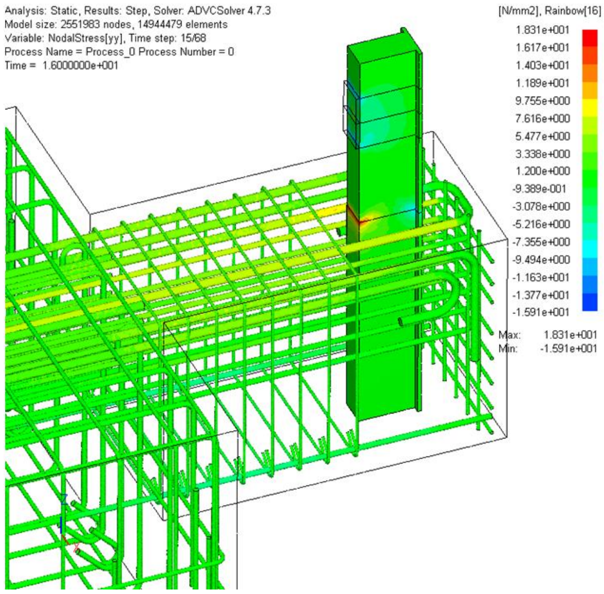

Figure 4.8. Stress distribution of rebars and girder without failure analysis.

4.2. Results and Discussion

As preliminary results, a strain distribution of the entire model and a stress distribution of reinforcing bars at the end of loading are shown in Fig. 4.5 and Fig. 4.6, respectively. As demonstrated by Fig. 4.5, largest strain occurs at the connection part between the girder and the RC body, which coincides with the PDS-FEM result predicting the cracks at the same place; see Fig. 4.7.

As an attempt to show the influence of concrete cracks, the same model is analysed without failure analysis, and the results are compared with those of the foregoing model. As illustrated in Figs. 4.8 and 4.6, the girder suffers from larger tensile stress without the concrete cracks, while, in comparison, the presence of cracks have reduced the stress of the girder where cracks occur by redistributing the stress and activating the tensile behavior of the reinforcing bars. This result shows that crack propagation affect the distribution of stress in the steel components, making the reinforcing bars active and reducing the excessive tensile stress on the girder.

The results of the current example simulation appear reasonable. At least, it is shown that the FEM code implemented with Maekawa's concrete constitutive relations and PDF is executable. While verification of the code and the validation of the model must be done, we can confirm the potential usefulness of this FEM, in order to analyse a 3D analysis model of an RC structure subjected to quasi-static/dynamic loading.

5. CONCLUDING REMARKS

We implement most advanced concrete constitutive relations and failure analysis functionalities to a large-scale FEM code. The FEM code is executable, indicating that the implementation is successfully made. As an example problem, we analyse the performance of a steel girder which is embedded in an RC body. While further efforts are needed, it is shown that the implemented concrete constitutive relation and the failure analysis work so that non-linear plastic deformation and cracking are computed.

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