Probabilistic Cost-Benefit Ratio of Risk Mitigation Measures

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SUMMARY:
For seismic risk reduction over an essential buildings portfolio should be necessary to carry out a structural upgrade that implies significant monetary amounts. A methodology to evaluate the probability distribution (PDF) of the benefit-cost ratio (B/C) is developed here; this is a ground-breaking tool to analyze net benefits of risk mitigation measures (RMM) like seismic code enforcement and earthquake retrofitting. An analytical solution for the PDF of the net present value of losses (NPV(β_i)) is presented; the solution is verified using a Monte Carlo method. The model and derived tools are described based on three portfolios of public buildings (Education, Health and Administration) in Bogota, Colombia. The analysis results are the PDF for the NPV(β_i), for both, the unretrofitted and the retrofitted structures states, allowing the estimation of the probability that B/C be positive.

Keywords: Benefit-Cost, BCA, seismic retrofitting, risk mitigation, vulnerability reduction, risk mitigation measures

1. INTRODUCTION

In practical terms, the only effective way to reduce vulnerability in several infrastructure components exposed to natural hazard events is through retrofitting of structural and non-structural elements. Structural retrofitting process requires usually a high upfront investment to reduce vulnerability and consequently the risk. The reduction is materialized as a reduction of expected losses in the long term due the future events occurrence. Those losses diminish corresponds not only to a direct physical loss, but also to loss of contents, and for lost profits because of activities interruption. Indirect impacts such as effects on people, injured or dead, and indirect social effects are generally difficult to quantify and rarely taken into account.

These ideas leads the need of benefit-cost analysis - BCA - for different risk mitigation measures - RMM -, with a clear criterion to define an optimal level of intervention, or a tool to propose a priority scheme among technically feasible alternatives, all above in a context of limited available resources. Thus, the benefits are the savings in expected future losses (including any direct, indirect, lost profits and in general all the losses associated with an element), while the costs are the value of implement a specific RMM.

The assessment of the expected losses is based on the events recurrence and its intensities magnitudes. Given the uncertainty associated with the occurrence of future events a simulation process is used, based on a probabilistic model, characterized with historical events recurrence information. For each event in the analysis it is necessary to obtain the net present value of future loss to compare with the upfront investment of the retrofitting.

Risk analyzes based on B/C has two major advantages:

a) Provides information to justify different RMM, because each one can be assessed in terms of economic impact or may also in terms of social impacts.
b) Represent a technical and clear prioritization of RMM for different components, or, to define the retrofitting activities to be performed according with the maximum B/C criterion.

In the analysis, it is necessary to have a reliable estimate of the upfront required for each RMM. Also it should be established a confident relationship between proposed interventions and the expected vulnerability reduction. The economic benefits that would arise from RMM should be brought to its NPV for an appropriate economic comparison, at an acceptable discount rate.

Figure 1.1 shows the schema of a typical BCA, in which for appropriate comparison, it is necessary to evaluate NPV of future costs and benefits according to an implementation of a certain RMM and compare it with the required upfront investment.

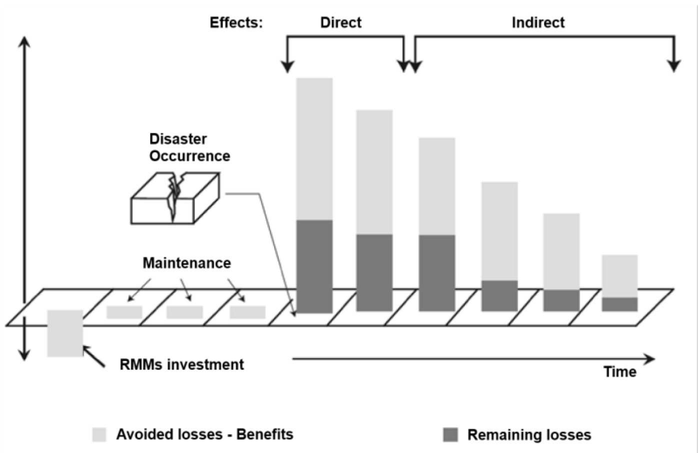


Figure 1.1. Analysis of NPV of costs, benefits and upfront investment in RMMs

The benefit-cost ratio, $\boldsymbol{Q}$, is defined as the ratio of savings (reduction in losses) because of the implementation of RMM, and the upfront cost of the intervention. Evaluation of $\boldsymbol{Q}$ is proposed as follows:

$$Q = \frac{L_R - L_U}{R} \tag{1.1}$$

Where $\boldsymbol{L_U}$ is the NPV of future losses in the state of none implementation of a RMM, $\boldsymbol{L_R}$ corresponds to the NPV of future losses in the state of implemented RMM, these both, which are random variables with known probability distribution can therefore be calculated. $\boldsymbol{R}$ is the cost or value of upfront investment for the RMM.

2. SOLUTION FOR NET PRESENT VALUES OF FUTURE LOSSES

2.1. Analytical probability distribution of NPV of future losses

The BCA applied to risk analysis is a systematic procedure for evaluating the decisions resulting from strategic management of risk. Following the sequence given by Smyth (2004) for a simplified BCA, Ordaz (2008) presents analytical solution to evaluate the *pdf* of NPV of future losses ***L***, from multiple seismic events $\boldsymbol{\beta_i}$, equation (2.1).

$$L = \sum_{1}^{\alpha} \beta_i e^{-\gamma t_i} \tag{2.1}$$

Since the unknowledge of exact sequence of earthquakes by means of their times of occurrence and their magnitudes, the B/C should be considered as a random variable. Its *pdf* is calculated in terms of NPV of the sum of all probable future losses and their variability, and also considering the random occurrence of seismic events. Equations (2.2) and (2.3), are the mean ***E(L)*** and variance ***VAR(L)*** respectively of the NPV of future losses ***L***.

$$E(L) = E(\beta_A) / \gamma \tag{2.2}$$

$$VAR(L) = VAR(\beta_A) / 2\gamma \tag{2.3}$$

Where $\boldsymbol{\beta_A}$ is the annual loss, $\boldsymbol{\gamma}$ is the value of money over time or discount rate.

$\boldsymbol{\beta_A}$ is the sum of all accumulated losses for a year, and $\boldsymbol{E(\beta_A)}$ is the expected annual loss, also known as pure premium, this value is a frequent result obtained from a probabilistic risk analysis -PRA.

Interestingly, according to equation (2.2), the NPV of losses is the value of $\boldsymbol{E(\beta_A)}$ divided by the discount rate $\boldsymbol{\gamma}$. In principle, one could assume that ***L*** is normally distributed since it is calculated as the sum of a large number of random variables, but for usual values of $\boldsymbol{\gamma}$ (2 to 5% per year) the number of terms that actually contribute to ***L*** is small. Using Monte Carlo simulations (Cardona et al, 2006) a normal distribution was verified as not a good approach, and in fact, each term in ***L*** is approximately the product of two independent random variables each one that follows a Beta distribution, and the product of these two variables may also be a Beta distribution. It would in this case that ***L*** could consist of sum of random variables approximately follow a Beta distribution (Cardona et al, 2008a, b). However, in the limit, for example when the expected value of ***L*** is smaller than the maximum possible value of it. Ordaz (2008) determined that ***E(L)*** is approximately a gamma distribution with parameters ***r*** and $\boldsymbol{\lambda}$ calculated as follows:

$$r = \frac{E^2(L)}{VAR(L)} \tag{2.4}$$

$$\lambda = \frac{r}{E(L)} \tag{2.5}$$

2.2. Actual probability distribution of NPV of future losses

Using the loss exceedance curve - LEC - as a result of risk analysis, it is possible to generate necessary scenarios of future losses to compute values of ***L*** to determine the *pdf* of the variable. Now if the probability of exceed a certain loss, $\boldsymbol{\beta o}$, corresponds to the integral under the *pdf* from 0 to $\boldsymbol{\beta o}$ and given that the losses for various events are independent and equally distributed following a Poisson distribution (Ordaz, 2000; Ordaz & Santa-Cruz, 2003), then $P(\beta<\beta o)$ is equal to the probability

P(β>0) minus the ratio of the number of events with minor losses to ***βo*** and the number of total events, or equivalently, *ν(βo)/ν(0)*.

$$P(\beta < \beta_o) = \int_0^{\beta_o} p(\beta_i) d\beta = 1 - \frac{\nu(\beta o)}{\nu(0)} \tag{2.6}$$

From here *p(βi)* is:

$$p(\beta_i) = -\frac{1}{\nu_0} \frac{d\nu(\beta_i)}{d\beta_i} \tag{2.7}$$

Knowing the *pdf* of losses, ***p(βi)***, you can generate random losses with that corresponding distribution by a set of random numbers ***u***, where 0 <***u*** <1. Thus one can calculate the loss associated with a probability as shown in Figure 2.1.

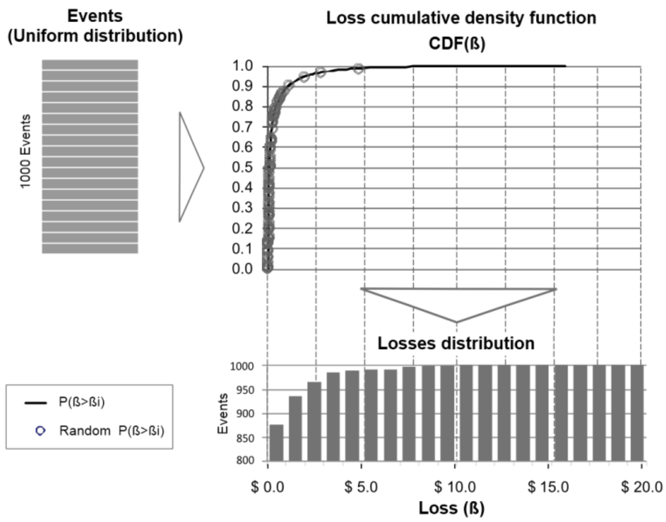


Figure 2.1. Loss events generation process.

From the above procedure a loss event arrangement according with the *pdf*, ***p(βi)***, is obtained as shown. Consequently, if the losses by earthquakes follow a *Poisson* process, the time between the occurrences of events follows an *Exponential* distribution which parameter is the rate of occurrence of events, $\boldsymbol{\nu_o}$.

$$p(\Delta T) = \nu_o e^{-\nu_o \beta_i} \tag{2.8}$$

Since the *Exponential* distribution function has an event generating function, it is not necessary to use simulations as in previous case. This generating function is presented in Equation 2.9 where ***u*** is a uniformly distributed random number:

$$E(\Delta Ti) = -\frac{Ln(u)}{\nu_o} \tag{2.9}$$

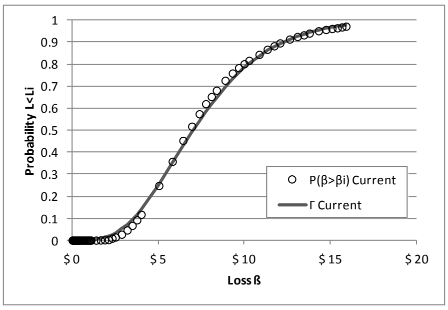


Figure 2.2. Comparison between analytical model and simulations.

Calculated the expected value of loss for an event ***i***, ***E(βi)***, and the time when such loss occurs, ***E(ΔTi)***, it is possible to calculate the NPV of losses, ***E(L)*** (Equation 2.1). This operation is performed as many times as necessary to obtain a number of comparison data with respect to the Gamma *pdf* with parameters obtained analytically. Figure 2.2 shows the results obtained with the analytical model proposed by Ordaz (2008) and simulated losses obtained with the results from seismic PRA.

This methodology allows the comparisons of probabilistic expected benefits of a RMM (preventive oriented strengthening seismic buildings), so this approach is useful for planning an optimal strategy of risk mitigation in accordance with BCA developed in Arámbula (2001).

3. COMPREHENSIVE ANALYSIS OF BENEFITS

For a comprehensive analysis of potential benefits accruing from an intervention or structural retrofitting expected losses should be considered the following:

(a) Direct: structure, contents, human.
(b) Indirect: loss of profit or activities interruption, maintenance, indirect social effects, environmental effects.

A comprehensive analytical losses model should include all the above components projected in time.

It should be noted, however, that not all losses or impacts can be measured in economic terms. For example the loss of life or indirect social impacts, such as those associated with possible disruption in hospital services are not easily quantifiable in economic terms, so generally would not be additive, but should be complementary.

Another important aspect in the BCA is to establish relations between the costs of implement a RMM and the vulnerability reduction and/or losses reduction. Seen as the cost of take the structure from a vulnerable state to an acceptable level of safety, and estimate the vulnerability associated with that state. In most cases the relationship between the retrofitting cost and the decrease in vulnerability is unique to each building, thus not easy to establish generalized models. Therefore it is important the specialists criteria to achieve a tight relationship.

3.1. Prioritization of RMMs by sector

On large cities is common to have an annual fixed resources as part of investment on risk mitigation budget. In that case, the prioritization problem arises, and decides which sector and/or which buildings should be subject of a RMM becomes complex considering the interaction of the following variables:

a) The hazard is different for each buildings type and building locations.

b) The variability in building behavior. There are buildings in very bad conditions (high vulnerability) and recently constructed buildings in accordance with building codes (low vulnerability).
c) The effects upon the occurrence of an event are different for each building. Additionally, indirect effects on occupants, consequences and costs of losses due to functionality interruption (business interruption - BI) are different in each case.
d) There are different possible RMM. From a simple strengthening of a relevant structural elements (probably a low-cost intervention) to a comprehensive intervention of structural and non-structural elements (probably a very expensive intervention but reliable).

This problem can be solved doing a BCA not only for individual buildings, but for a group of building from different sectors with the aim to define differential budget allocations, or for different future investment plans.

Table 3.1 shows a summary of information available from three sectors; the information includes the number of buildings, replacement values and share percentage of the total portfolio.

Table 3.1 Summary of available information for building of three sectors: education, health and central government

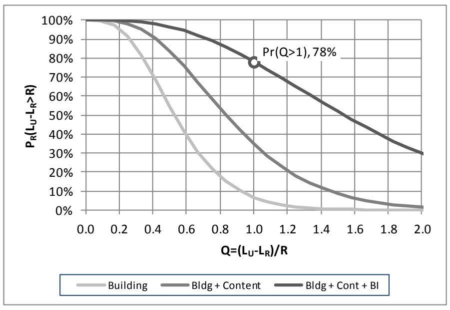
Sector	Building Count	Exposed value			
		Building	Contents	BI	Total
	[units]	[x10³ USD]	[x10³ USD]	[x10³ USD]	[x10³ USD]
Education	**691**	**146,902**	**103,212**	**293,804**	**543,918**
Health	**39**	**17,676**	**7,867**	**35,351**	**60,894**
Administrative	**94**	**31,352**	**18,137**	**62,704**	**112,193**
Total	**824**	**195,930**	**129,216**	**391,859**	**717,005**

Table 3.2 presents the expected values of ***Q*** for the three sectors and for different analysis. Additionally shows the probability of B/C ratio be positive.

Table 3.2 Analysis results by sector

Sector	Variable	Building	Building & Contents	Building, Contents & BI
Education	**E(Q)**	**0.92**	**1.59**	**3.15**
	Pr(Q>1)	**43%**	**70%**	**87%**
Health	**E(Q)**	**0.38**	**0.68**	**2.79**
	Pr(Q>1)	**45%**	**74%**	**98%**
Administrative	**E(Q)**	**0.42**	**0.70**	**1.45**
	Pr(Q>1)	**7%**	**35%**	**78%**

Meanwhile, Figure 3.1 presents the *cdf* of the BCA from buildings, buildings & contents, and, buildings-contents & BI, for the Administration sector; Figure 3.2 shows the comparison of the three sectors for buildings-contents & BI losses.



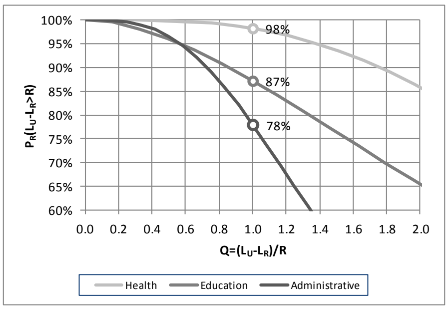
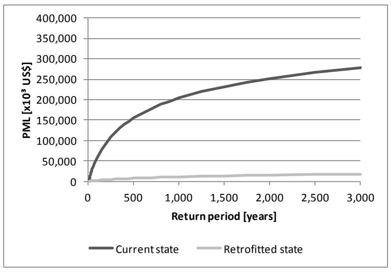
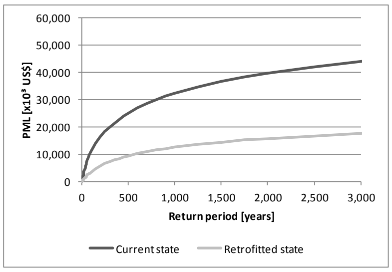


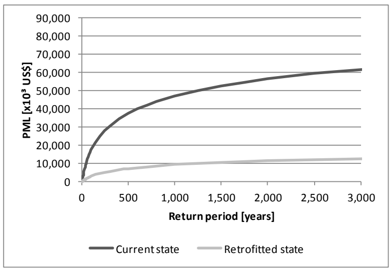
Figura 3.1 Cumulative density function of *Q*

Figura 3.2. probability to have a positive B/C ratio

Next figures show the probable maximum loss curves –PML-, of the three sectors for retrofitted and unretrofitted conditions.







Education sector

Health sector

Administrative sector

Figures 3.3, 3.4, 3.5. Probable maximum loss curves for Education, Health and Administrative sectors respectively

From social point of view the benefits take into account direct and indirect effects over the population. Although the costs are directly related to investment on RMMs, the benefits should be translated as reduction in population effects such as decrease in the number of affected people, the number of people requiring medical assistance and the expected number of casualties.

4. CONCLUSIONS

The evaluation of the probability distribution function of the B/C ratio is a good tool for decision making by analyzing the net benefits from RMMs for both structural rehabilitation to building codes enforcement.

In this way it is possible to develop mitigation and risk management policies that conducts to prioritize investment and resources, establish reserve funds for construction and renovation plans of public infrastructure. Meanwhile, on private sector these mechanisms are important for decisions making mainly for physical plant investment and to determine the risk level to assume in future construction projects.

Due to the stochastic nature of natural phenomena, the NPV of losses, such as earthquakes, is highly uncertain quantity. Therefore, decisions should not be established solely based on expected values. Therefore, it is necessary to use methodologies to determine the likelihood of B/C ratios greater than unity and select the alternative with the highest probability.

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