Travel Time Tomography using Neural Networks

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SUMMARY:
Travel time tomography, a highly effective method that utilizes artificial or natural earthquakes to estimate the properties of subsurface structures, is widely used in various fields ranging from civil engineering to academia. However, the resolution of the results of traditional travel time tomography analysis is low because grid spacing is constrained by the geometry between source and receiver locations. In the case of natural earthquake tomography, in particular, the ray paths between sources and receivers are nonhomogeneously distributed, and the grid spacing must be set on the basis of sparse ray path areas. To solve this problem, Oda and Ishiyama (2011) developed a new tomography method that uses a multilayer neural network. However, as this method assumes that a ray path is a straight line from the source to the receiver, errors are produced in the case of complex subsurface structures. Therefore, in this study, we employed the pseudo-bending ray tracing method to improve the neural network tomography method. We tested this improved tomography method in three-dimensional numerical experiments and obtained satisfactory results.

Keywords: Travel time tomography, Natural earthquake, Pseudo-bending method, Neural network, Back-propagation training algorithm

1. INTRODUCTION

It is important to estimate subsurface structures accurately to prevent earthquake disasters, and travel time tomography is one of the most effective methods for this estimation. Travel time tomography using artificial or natural earthquakes is widely used in fields ranging from civil engineering to academia as a powerful method for estimating subsurface structures. The travel time tomography method for estimating these structures was developed in the late 1970s (Dines and Lytle, 1979). The method first applies the algebraic reconstruction technique or the simultaneous reconstruction technique to the travel time data, and the iterative least-square technique is then used as the main method of analysis.

Traditional travel time tomography is useful for imaging subsurface structures. However, this technique has a significant limitation: the subsurface structures must be discretized into many grids or small elements, and the grid spacing or element size must be manually optimized on the basis of ray path density. Moreover, even when a high-density ray path area is in a target zone, the grid spacing must be set according to the lowest ray path density area. Such a constraint indicates that the process is inefficient. In addition, choosing an appropriate initial model is very important for achieving results with high accuracy.

To address these problems, Oda and Ishiyama (2011) developed a tomography method that uses a multilayer neural network. This method exploits the excellent capability of multilayer neural networks for approximating arbitrary functions (Funahashi, 1989; White, 1990); the network training is performed by minimizing the squared residuals of an integral equation. Oda and Ishiyama validated the reliability of their new method with numerical experiments. However, their method assumes that the ray path is a straight line from the source to the receiver, and this assumption produces inaccurate results in the case of complex subsurface structures. Therefore, in this study, we improved the method so that it calculates ray paths by using the pseudo-bending ray tracing method (Um and Thurber, 1987).

2. METHOD

We employed a multilayer neural network that was trained by minimizing an object function composed of appropriately prepared equation residuals. By choosing an appropriate combination of the squared residuals of various equations and minimizing it, the trained neural network easily produced a solution for a complicated system. Because the interpolation and smoothing functions are inherently included in the neural network, the difficulty of subsurface estimation that arises from an imperfect set of ray path data is significantly reduced.

The travel time T_i^j along the ray path between the *i*th source and the *j*th receiver is the integration of the slowness of the ray along the path, given as

$$T_i^j = \int_{\vec{r}_i}^{\vec{r}_j} S(\vec{r})\,ds \qquad (i = 1,\cdots,I \quad ; \quad j = 1,\cdots,J) \tag{2.1}$$

where $S(\vec{r})$ is the slowness at a position $\vec{r}$, and I and J are the total number of sources and receivers, respectively. To determine the value of the function $S(\vec{r})$, we used a neural network with input units for $\vec{r}$ and an output unit for $S(\vec{r})$. To evaluate the residuals of the integral Eqn. 2.1, the equation is discretized as

$$T_i^j = \sum_{q=1}^{Q} \alpha S(\vec{r}) \qquad (i = 1,\cdots,I \quad ; \quad j = 1,\cdots,J) \tag{2.2}$$

where q and α denote a sampling point and the corresponding weight for the numerical integration, respectively, and Q is the total number of sampling points along the ray path (Fig. 1).

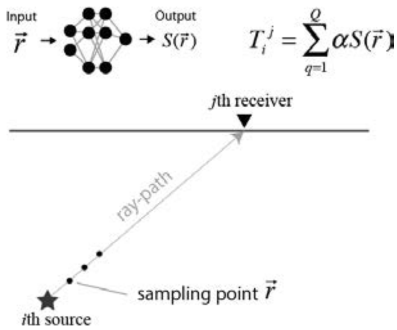


Figure 1. Schematic diagram of the travel time computation. First, the slowness at all sampling points along a ray path is obtained by the neural network, and the travel time is then calculated using Eqn. 2.2.

To calculate the ray paths, we employed the pseudo-bending method, which is commonly used in natural earthquake tomography. To estimate the slowness $S(\vec{r})$, we took the squares of the residuals of the above integral equations (E_{ij}) as the neural network's object function:

$$E_{ij} = (T_i^{j,NN} - T_i^{j,Obs})^2 \tag{2.3}$$

where $T_i^{j,NN}$ and $T_i^{j,Obs}$ denote the travel time calculated by the neural network and the observed travel time, respectively. We employed a back-propagation algorithm as the training method. In the conventional back-propagation training process, the target values for the output data are given to the network as teacher data. In our study, however, the teacher data are not available; therefore, we modified the back-propagation training algorithm such that the numerically calculated line is integrated along the ray paths. The weights were updated with each calculation of a single line (ray path) integral.

3. NUMERICAL EXPRIMENTS

To examine the applicability of our new method, we applied it to three-dimensional natural earthquake tomography problems.

3.1 Horizontally layered medium

For the numerical experiments, we employed two cases of natural earthquake tomography: horizontally layered medium and irregular ground medium. In these cases, the area to be analyzed was $20km \times 20km \times 16km$, 36 seismic stations were randomly placed on the surface, and 200 earthquakes were randomly generated at depths of 5–15 km. Therefore, the total number of ray paths was 7,200. Synthetic observed travel time data were calculated using the pseudo-bending ray tracing method.

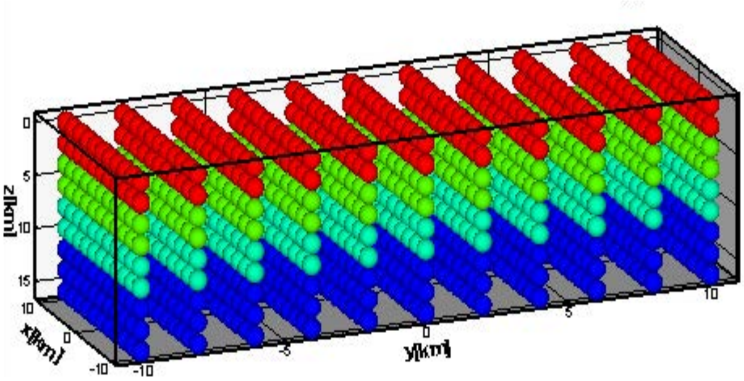
We employed a four-layered neural network in which the first three layers contained 3, 30, and 30 units, and the output layer contained a single unit. The total number of weights was therefore 1,020. We input the coordinate values (*x, y, z*) of each sampling point to obtain the velocity *Vp* for the sampling point from the neural network. The number of sampling points after the learning process was $N = 21 \times 21 \times 17$, which implies that the horizontal and vertical resolutions were both 1.0 km.

Images of the subsurface structures estimated by our new method and two others are shown in Fig. 2. Fig. 2a shows the true model that was used for calculating the synthetic observed travel-time data; Fig. 2b represents the estimation produced by the widely used natural earthquake tomography code SIMULPS12 (Evance et al., 1994); Fig. 2c represents the estimation produced by the neural network method with straight ray paths (Oda and Ishiyama, 2011); and Fig. 2d represents the estimation produced by our new method. To evaluate the results, we defined the velocity error E_{Vp} and travel time error $E_{traveltime}$ as

$$E_{Vp} = \frac{\sum_{n=1}^{N} \left| Vp_{true}^{n} - Vp_{estimation}^{n} \right|}{N}$$
$$E_{traveltime} = \frac{\sum_{p=1}^{P} \left| T_{observation}^{p} - T_{estimation}^{p} \right|}{P} \tag{3.1}$$

where *N* is the number of estimated points after the learning process, *P* is the number of ray paths, Vp_{true}^{n} is the velocity at point *n* of the true model, and $Vp_{estimate}^{n}$ is the estimated velocity obtained through one of the methods. $T_{observation}^{p}$ and $T_{estimate}^{p}$ are the observed and estimated travel time, respectively, of the *p*th ray path. These error functions denote the average velocity and travel time errors. In this experiment, the E_{Vp} of SIMULPS was 0.521 km/s, the E_{Vp} of

(a)



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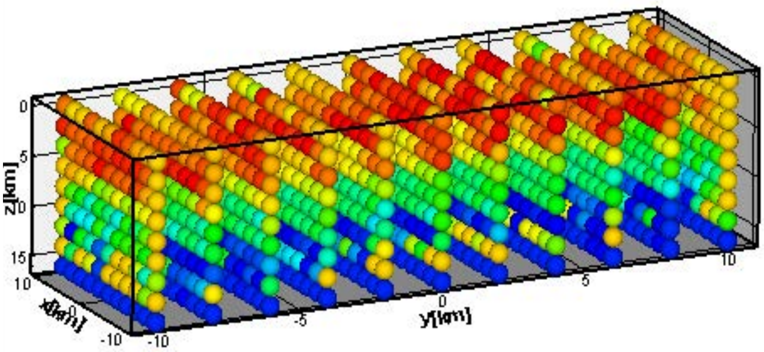

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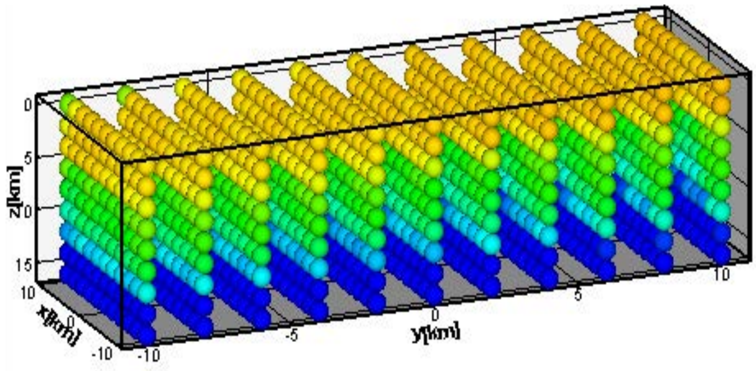

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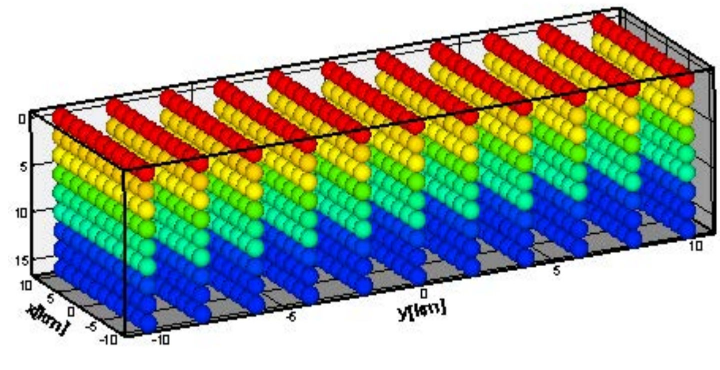


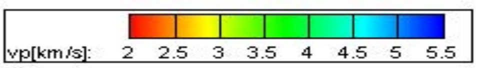
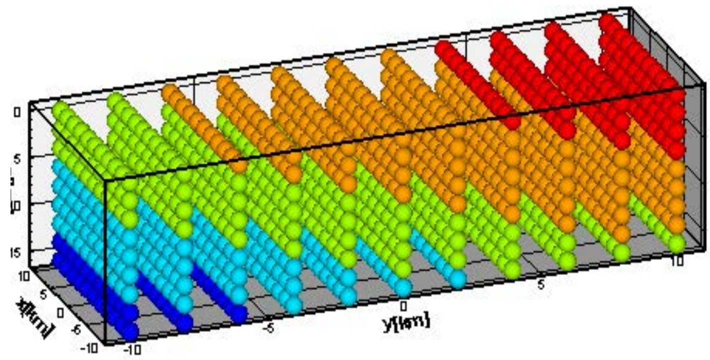
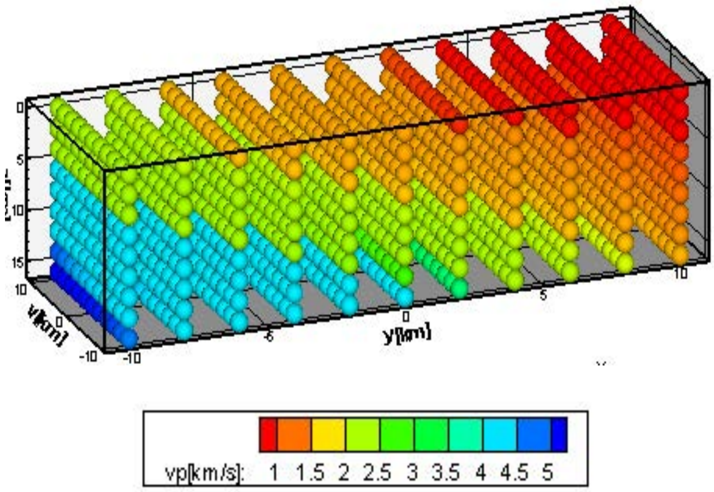


Figure 2. Results for the horizontally layered medium experiment: (a) true model, (b) reconstructed image using a conventional method (SIMULPS12), (c) reconstructed image using a neural network (Oda and Ishiyama, 2011), and (d) reconstructed image using our new method.

(a)



(b)



(c)

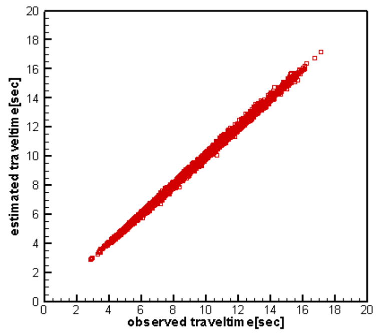


Figure 3. Results for the irregular ground medium case: (a) true model, (b) reconstructed image using our new method, and (c) correlation between observed and estimated travel times.

the neural network using straight ray paths was 0.682 km/s, and the E_{Vp} of our new method was 0.263 km/s. The $E_{traveltime}$ of the neural network method using straight ray paths was 0.123 s and that of our new method was 0.019 s.

3.2 Irregular ground medium

In the second experiment, we employed the irregular ground medium shown in Fig. 3a. The analysis area and the number and locations of seismic stations and earthquakes were the same as in the first experiment. The estimated subsurface structure corresponded to the correct model in both the shape of the layer boundaries and the P-wave velocity values. The correlation diagram shown in Fig. 3c indicates that the travel time of each ray path was estimated with high accuracy using the neural network. In this case, the E_{Vp} was 0.095 km/s and the $E_{traveltime}$ was 0.081 s.

4. CONCLUSIONS

In this study, we added a ray tracing function to the travel time tomography method that uses a neural network. The method we employed for computing this function uses the residual minimization training neural network, which is trained by minimizing an object function composed of the sum of the squared residuals of an integral equation.

We applied our new method to natural earthquake tomography through three-dimensional numerical experiments. We obtained a subsurface structure with a higher accuracy and a relatively high resolution than those obtained by the conventional travel time tomography method.

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