Seismic Analysis of Structures Resting on Two Parameter Elastic Foundation

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15 WCEE
LISBOA 2012

SUMMARY:
In this paper, a structural system composed of a 3-D frame superstructure and a mat foundation resting on a two-parameter elastic subsoil is modelled by finite elements and the seismic behaviour of the system is evaluated. The 3-D superstructure is idealized by frame elements while the mat foundation is idealized by thick shell finite elements. In the modelling of subsoil behaviour, the part of the governing equation that belongs to Winkler behaviour is represented by elastic springs and the part that belongs to elastic shear layer is taken into account by a four-noded quadrilateral subsoil finite element with four degrees of freedom (d.o.f.). Elastic time-history analyses are performed to examine the seismic behaviour of the structural system. Horizontal and vertical components of 1999 Kocaeli-Turkey earthquake are utilized in the time history analyses. The internal force and displacement histories of 3-D structure for different subsoil parameters are also given.

Keywords: Time-history analysis, two-parameter elastic foundation, Pasternak model, Vlasov model

1. INTRODUCTION

In modern design and analysis of structures, the superstructure-foundation-soil interaction has to be taken into account in a sophisticated way, which is sufficiently accurate but simple enough for practical purposes. The concept of a plate resting on an elastic foundation has been an important tool for the modelling and analysis of structural, highway, geotechnical and railroad engineering problems. Extensive research in this area has been reported in the literature.

In order to model soil behaviour, several approaches have been developed in the past. The oldest, most famous and most frequently used soil model is the one devised by Winkler, E. (1867), in which the beam-supporting soil is modelled as a series of closely spaced, mutually independent, linear elastic vertical springs. The Winkler model has been extensively used to solve many soil-foundation interaction problems and has given satisfactory results for many practical problems. In that method, it is assumed that the deflection at each point is proportional to the pressure applied to that point and completely independent of the pressures or deflections occurring at the neighbouring points along the foundation. In the Winkler model, the properties of soil are described only by the parameter C, which represents the stiffness of the vertical spring. One of the major disadvantages of this model is that a plate undergoes rigid body displacements without any bending moments and shear forces in it when subjected to uniform loads. Moreover, the use of the Winkler model involves difficulties in determining the value of C.

Discontinuous nature of Winkler's model gives rise to the development of various forms of two-parameter elastic foundation models. Some of the major two-parameter elastic foundation models are Filonenko-Borodich, M.M. (1940), Hetenyi, M. (1946, 1950), Pasternak, P.L. (1954) and Vlasov, V.Z., and Leont'ev, U.N. (1966). Filonenko-Borodich, Hetenyi, Pasternak and Vlasov-Leont'ev have attempted to make the classical Winkler model more realistic by postulating a two-parameter model. Their models take into account the effect of shear interaction among adjacent points in the foundation. In these models, the first parameter represents the stiffness of the vertical spring, as in the Winkler model, whereas the second parameter is introduced to account for the coupling effect of the linear elastic springs. It is worth mentioning that the interaction enabled by this second parameter also allows the consideration of influence of the soil on either side of the plate. In all these models, the first and second parameters have to be determined experimentally.

Vlasov and Leont'ev (1966) have introduced another arbitrary parameter,γ, dependent on the soil material and the thickness of the soil layer. However, they did not report the method of determining this parameter. In the works of Vallabhan, C.V.G., Straughan, W.T. and Das.Y.C. (1991), Vallabhan, C.V.G., Daloglu, A. (1999), it has been shown how the soil parameter ,γ , can be estimated by using an iterative computational procedure for plates. These three-parameter models constitute a generalization of two-parameter models, the third parameter being used to make them more realistic and effective. When the γ parameter is determined, the first and second parameters of soil can easily be calculated. One of the basic features of the three-parameter models is the flexibility and convenience that they offer in the determination of the level of continuity of the vertical displacements at the boundaries between the loaded and unloaded surfaces of the soil.

Although static and dynamic behaviours of beams and plates on two-parameter elastic foundations are examined by many researchers, the structures with mat foundations on two-parameter subsoil are not examined enough. One of the works on earthquake analysis of the plates on two-parameter elastic foundation is Ayvaz, Y., Daloglu, A. and Dogangun A. (1998). In this study, a 3-D sample frame with shear-walls on Winkler and two-parameter elastic subsoil is analysed by elastic time-history analysis using a well-known computer program SAP2000 (2010), under lateral and vertical real earthquake records from 1999 Kocaeli earthquake. Elastic subsoil parameters are calculated depending on the variable compressible subsoil depths and deformations of the subsoil surface according to Vallabhan, C.V.G., Straughan, W.T. and Das.Y.C. (1991).

Subsoil parameters are calculated according to the subsoil surface deformations due to the dead loads of the structure and they are taken as constant during the analyses assuming that the parameters do not change under dynamic effects. It is also assumed that the mass of the subsoil does not change with depth and the mass of the soil at one third of the total soil depth is considered as constant in the analyses. The 3-D structure and mat foundation are idealized by frame and shell finite elements and the subsoil zone under and outside the mat foundation is idealized as shear layer by a four d.o.f. quadrilateral subsoil shear element. Since the soil element that exists in the literature Celik, M. and Saygun A. (1999), Darilmaz, K. (2009), is not included in the computer program library, it is derived from the existing layered shell element with orthotropic material by defining appropriate boundary conditions and elastic properties.

2. MATHEMATICAL FORMULATION OF FOUR D.O.F. SUBSOIL SHEAR ELEMENT

In this section, mathematical formulation of the four d.o.f. subsoil shear element representing the shear layer will be given. The other finite elements representing the remaining parts of the structural system will not be mentioned since they are well-known finite elements such as quadrilateral Mindlin plate elements and plane stress elements and frame elements. The first parameter of the soil that belongs to the Winkler behaviour is represented by elastic springs at the nodes of the subsoil shear and plate finite elements.

Subsoil reactions of a plate resting on a two-parameter elastic foundation may be given depending on the displacement function *w* of the subsoil surface as

$$q_z = Cw - 2C_T\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \tag{2.1}$$

In this equation, C and $2C_T$ are the Winkler and shear parameters representing the elastic spring and shear layer behaviours of the subsoil, respectively. These parameters are constant according to Pasternak subsoil assumption whereas they may be stated in terms of compressible subsoil depth H and subsoil surface displacement parameter γ according to Vallabhan, C.V.G., Straughan, W.T. and Das.Y.C. (1991).

$$C = E_s \frac{(1-\nu_s)}{(1+\nu_s)(1-2\nu_s)} \frac{\gamma}{H} \frac{(\sinh 2\gamma + 2\gamma)}{4\sinh^2 \gamma} \tag{2.2}$$

$$2C_T = G_s \frac{H}{\gamma} \frac{(\sinh 2\gamma - 2\gamma)}{4\sinh^2 \gamma} \tag{2.3}$$

Here, E_s and G_s are the modulus of elasticity and shear modulus of the subsoil, respectively. Subsoil surface displacement parameter γ may be given depending on the subsoil surface deformation as in the following.

$$\gamma^2 = H^2 \frac{(1-2\nu_s)}{2(1-\nu_s)} \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left[\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 \right] dxdy}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} w^2 dxdy} \tag{2.4}$$

By using the finite element formulation of Celik, M. and Saygun A. (1999), Darilmaz, K. (2009), the subsoil finite element stiffness matrix representing the shear layer behaviour may be defined as

$$\left[C_T\right] = 2C_T \iint_A \left(\frac{\partial N_j}{\partial x}\frac{\partial N_i}{\partial x} + \frac{\partial N_j}{\partial y}\frac{\partial N_i}{\partial y} \right) dxdy \tag{2.5}$$

where N_i is the shape function of node i. The element stiffness matrix of the subsoil shear element may be obtained by using the linear isoparametric shape functions and numerical integration. The displacements of the quadrilateral subsoil shear element are given in Fig. 2.1.

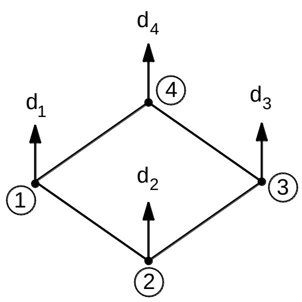

Figure 2.1. Four d.o.f. subsoil shear element

Since C and $2C_T$ parameters depend on the subsoil surface deformation, calculation of these parameters needs a successive approximation. In this study, subsoil surface deformations due to the dead loads are calculated by a successive approximation and they are assumed unchanged due to the seismic effects. Moreover, since the Winkler parameter is represented by nodal springs, stiffness matrix [C] belonging to the Winkler subsoil is not given here.

On the other hand, the integrals in equation (2.6) may be calculated in terms of the nodal displacements of the subsoil shear element

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right]dxdy=\sum_{i=1}^{n}\frac{1}{2C_T}[d]^T[C_T][d] \tag{2.6}$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} w^2 dxdy=\sum_{i=1}^{m}Ad_i^2 \tag{2.7}$$

where, n, m and A represent the element number, the node number and the tributary area of node i, respectively. Therefore, the mode shape parameter γ may be calculated at the end of any analysis step in terms of vertical displacements of the foundation-subsoil system.

3. NUMERICAL APPLICATION

In this section, dynamic analysis results and comparisons of a ten story L shaped shear wall frame structural system resting on a two-parameter elastic foundation are given. Plan and perspective views of the 3D building are shown in Fig. 3.1 and Fig. 3.2, respectively.

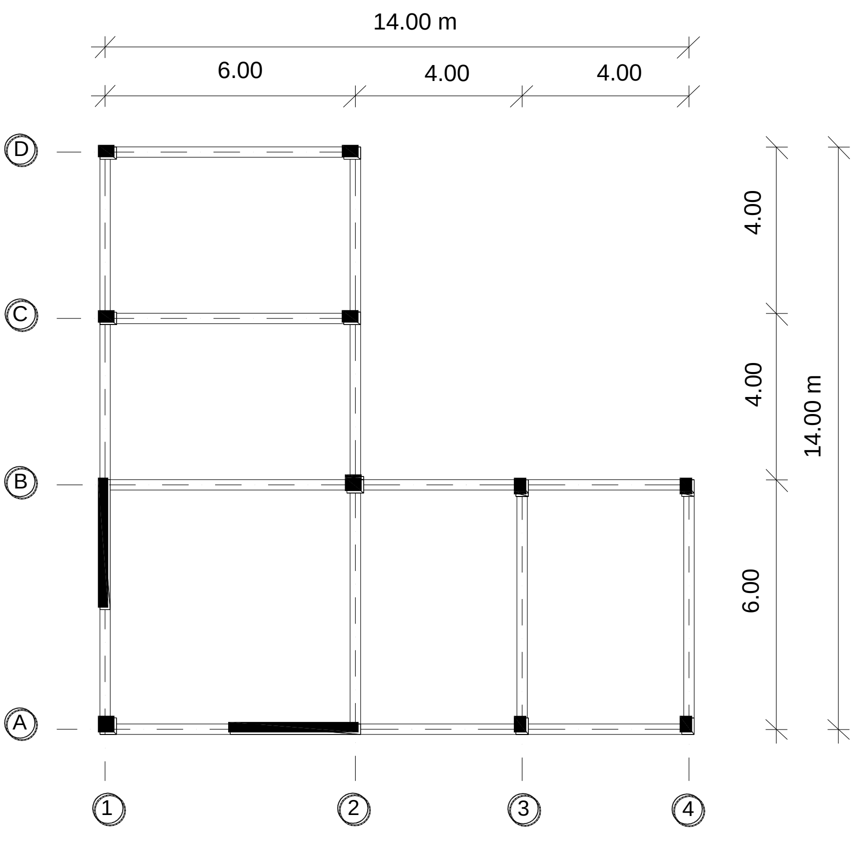

Figure 3.1. Plan view of the 3D building

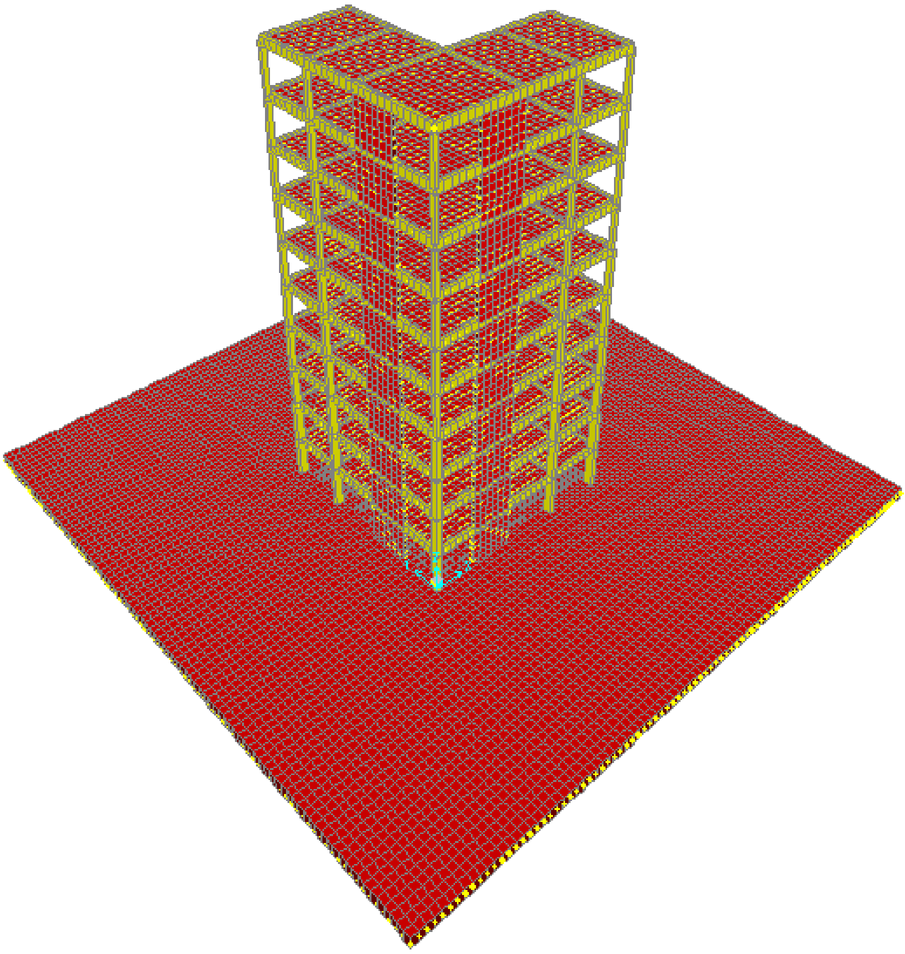

Figure 3.2. Perspective view of the 3D building

The dimensions of the beams are chosen as 250x600 mm for the entire building and the cross-sectional dimensions of the shear walls are chosen as 250x3000 mm which are constant along the height of the building. The column dimensions are given in Table 3.1.

Table 3.1. Dimensions of the columns in mm

Story	Column		
	A1	B2	Others
9 - 10	300 x 300	300 x 300	300 x 300
7 - 8	300 x 300	400 x 400	300 x 300
5 - 6	300 x 300	400 x 400	300 x 400
3 - 4	400 x 400	500 x 500	300 x 500
1 - 2	400 x 400	500 x 500	300 x 600

The building is modelled in SAP2000 structural analysis program and time-history analyses are performed. Fifteen-second interval of the lateral and vertical records of 1999 Kocaeli Earthquake are used in time-history analyses. First, mode shape parameters γ are calculated due to the dead loads for 3m, 6m, 9m, 12m and 15m compressible subsoil depths and C and $2C_T$ parameters which are obtained from dead load analyses by successive approximation, are used in time history analyses as constant values. Subsoil parameters for various compressible subsoil depths are given in Table 3.2.

Table 3.2. Subsoil parameters for various compressible subsoil depths

Compressible subsoil depth (H)	Winkler parameter (C, kN/m^3)	Shear parameter ($2C_T$, kN/m)
H=3 m	27750.000	25600.000
H=6 m	14000.000	49100.000
H=9 m	9450.000	71000.000
H=12 m	7200.000	91200.000
H=15 m	5850.000	109900.000

No damping is considered for the structure and foundation system. 14 m of a subsoil zone is considered from the external borders of the building. The subsoil zone and mat foundation are divided into 7744 elements. Subsoil shear elements are layered under and outside the mat foundation. In SAP2000 model, the subsoil shear elements are represented by layered shell elements with orthotropic material. In the definition of orthotropic material, the shear modulus is taken as equal to the subsoil shear parameter $2C_T$ and elasticity modulus is taken as a large number. The rotational degrees of freedom of the subsoil shear elements are fixed and vertical displacements of common nodes of the plate elements and subsoil shear elements are constrained by using the weld joint property of SAP2000. The Winkler behaviour of the subsoil is represented by elastic nodal springs. Three different subsoil conditions such as two-parameter elastic foundation, one parameter elastic foundation and fixed base (no foundation effect) are considered and time history analyses are performed for various subsoil depths i.e. various subsoil parameters. Maximums and minimums of the top displacement, base shear of the corner column (A-1), base shear of the shear-wall and the total base shear are obtained and compared for different subsoil conditions and subsoil parameters. Since the building is symmetrical, time history analyses are performed only in one direction. The displacement and base shear histories are given in Fig. 3.3 to Fig. 3.6 and their maximum, minimum and absolute maximum values are given in Table 3.3 to Table 3.6 together with corresponding times and compressible subsoil depths.

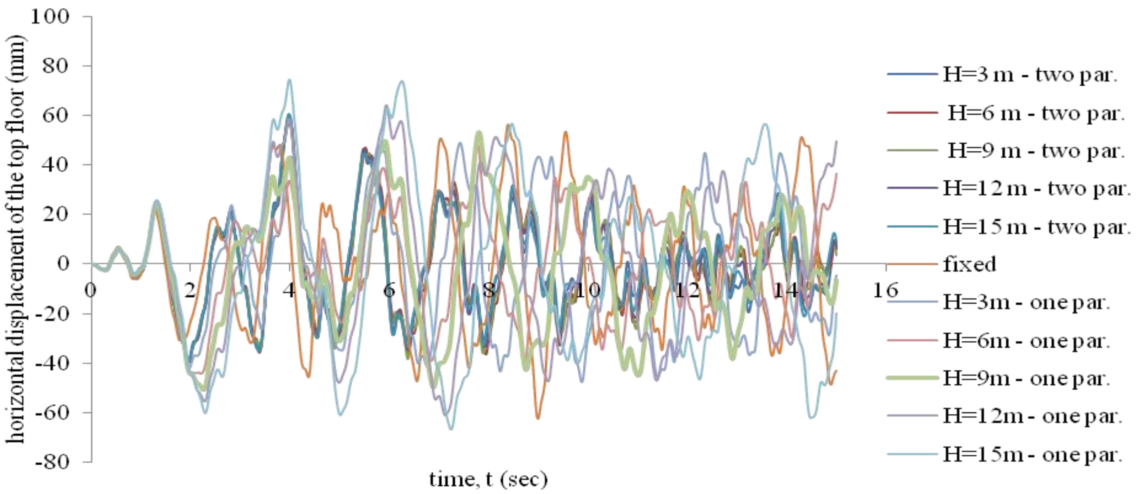

Figure 3.3. Variation of top floor horizontal displacement in earthquake direction with time and compressible subsoil depth (H)

Table 3.3. Maximum and minimum horizontal displacements of the top floor

		maximums				minimums				maximum absolute value	
two-parameter foundation	H=3 m	60.290	mm	3.990	sec	-39.820	mm	1.950	sec	60.290	mm
	H=6 m	57.190	mm	3.980	sec	-40.032	mm	1.955	sec	57.190	mm
	H=9 m	58.243	mm	3.965	sec	-39.658	mm	1.950	sec	58.243	mm
	H=12 m	59.885	mm	3.975	sec	-39.804	mm	1.950	sec	59.885	mm
	H=15 m	60.816	mm	3.980	sec	-39.178	mm	1.950	sec	60.816	mm
fixed	fixed	56.092	mm	8.390	sec	-62.494	mm	8.990	sec	62.494	mm
one-parameter foundation	H=3 m	48.926	mm	7.425	sec	-47.749	mm	9.870	sec	48.926	mm
	H=6 m	48.123	mm	7.775	sec	-44.121	mm	2.210	sec	48.123	mm
	H=9 m	53.430	mm	7.800	sec	-50.722	mm	2.260	sec	53.430	mm
	H=12 m	63.925	mm	5.940	sec	-60.951	mm	7.125	sec	63.925	mm
	H=15 m	74.372	mm	3.990	sec	-66.662	mm	7.240	sec	74.372	mm

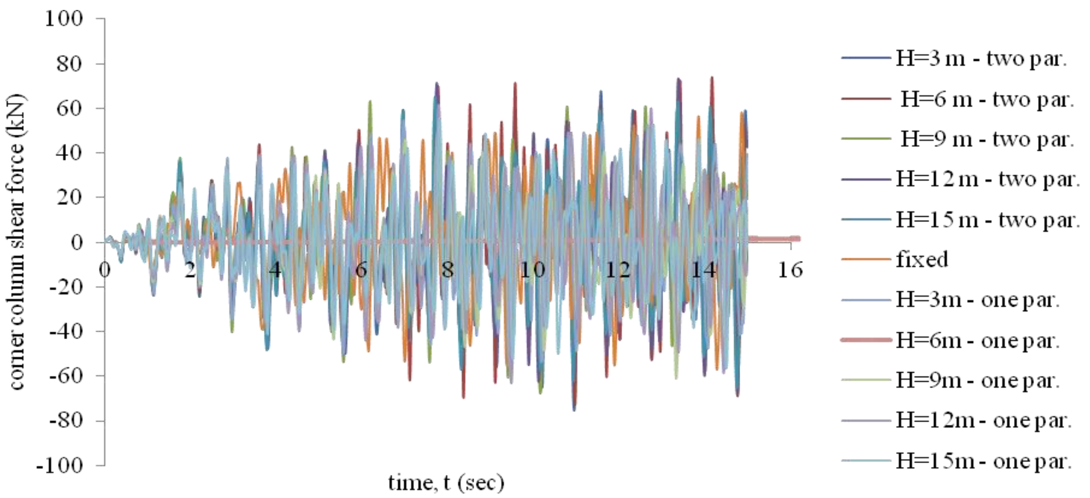

Figure 3.4. Variation of corner column base shear with time and compressible subsoil depth (H)

Table 3.4. Maximum and minimum base shears of the corner column

		maximums				minimums				maximum absolute value	
two-parameter foundation	H=3 m	71.447	kN	7.755	sec	-75.216	kN	10.960	sec	75.216	kN
	H=6 m	73.947	kN	14.175	sec	-73.011	kN	10.980	sec	73.947	kN
	H=9 m	63.088	kN	6.200	sec	-67.930	kN	10.170	sec	67.930	kN
	H=12 m	73.565	kN	13.390	sec	-68.390	kN	14.765	sec	73.565	kN
	H=15 m	65.472	kN	7.715	sec	-65.347	kN	14.755	sec	65.472	kN
fixed	fixed	58.076	kN	14.870	sec	-57.074	kN	14.290	sec	58.076	kN
one-parameter foundation	H=3 m	54.340	kN	6.980	sec	-55.223	kN	9.445	sec	55.223	kN
	H=6 m	61.121	kN	6.995	sec	-59.344	kN	10.230	sec	61.121	kN
	H=9 m	52.235	kN	12.710	sec	-61.123	kN	13.340	sec	61.123	kN
	H=12 m	61.725	kN	7.795	sec	-62.891	kN	9.495	sec	62.891	kN
	H=15 m	55.361	kN	7.820	sec	-56.666	kN	14.510	sec	56.666	kN

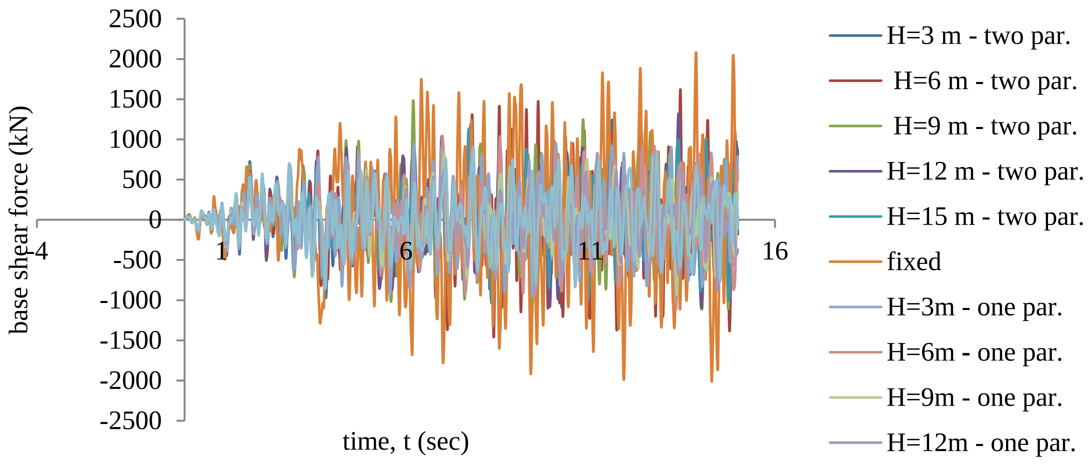

Figure 3.5. Variation of base shear of shear-wall with time and compressible subsoil depth (H)

Table 3.5. Maximum and minimum base shears of shear-wall

		maximums		minimums		maximum absolute value
two-parameter foundation	H=3 m	1257.771 kN	13.395 sec	-1209.871 kN	10.970 sec	1257.771 kN
	H=6 m	1621.436 kN	13.435 sec	-1461.347 kN	8.380 sec	1621.436 kN
	H=9 m	1484.391 kN	6.200 sec	-1179.094 kN	9.415 sec	1484.391 kN
	H=12 m	1320.473 kN	13.390 sec	-1102.802 kN	14.005 sec	1320.473 kN
	H=15 m	1136.573 kN	7.710 sec	-1037.633 kN	8.320 sec	1136.573 kN
Fixed	fixed	2081.897 kN	13.865 sec	-2013.959 kN	14.290 sec	2081.897 kN
one-parameter foundation	H=3 m	954.079 kN	10.065 sec	-960.293 kN	9.445 sec	960.293 kN
	H=6 m	1043.093 kN	6.990 sec	-1110.685 kN	13.325 sec	1110.685 kN
	H=9 m	835.909 kN	7.035 sec	-936.363 kN	13.335 sec	936.363 kN
	H=12 m	863.380 kN	7.795 sec	-923.877 kN	9.495 sec	923.877 kN
	H=15 m	907.611 kN	7.830 sec	-862.628 kN	3.810 sec	907.611 kN

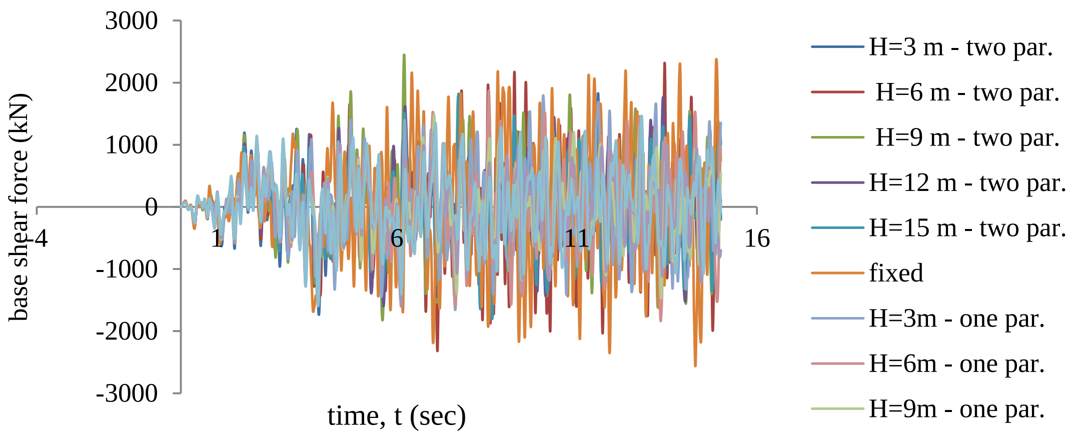

Figure 3.6. Variation of total base shear with time and compressible subsoil depth (H)

Table 3.6. Maximum and minimum total base shears

		maximums		minimums		maximum absolute value
two-parameter foundation	H=3 m	1827.590 kN	11.585 sec	-1737.070 kN	3.835 sec	1827.590 kN
	H=6 m	2317.040 kN	13.435 sec	-2320.314 kN	7.125 sec	2320.314 kN
	H=9 m	2449.338 kN	6.200 sec	-1825.874 kN	5.600 sec	2449.338 kN
	H=12 m	1764.547 kN	13.390 sec	-1646.140 kN	8.645 sec	1764.547 kN
	H=15 m	1820.911 kN	7.715 sec	-1801.100 kN	8.650 sec	1820.911 kN
Fixed	fixed	2378.069 kN	14.870 sec	-2564.061 kN	14.290 sec	2564.061 kN
one-parameter foundation	H=3 m	1795.250 kN	10.065 sec	-1658.065 kN	7.620 sec	1795.250 kN
	H=6 m	1869.914 kN	8.555 sec	-1838.634 kN	13.325 sec	1869.914 kN
	H=9 m	1478.773 kN	7.035 sec	-1469.141 kN	13.335 sec	1478.773 kN
	H=12 m	1335.586 kN	12.760 sec	-1603.578 kN	6.115 sec	1603.578 kN
	H=15 m	1464.404 kN	12.800 sec	-1597.676 kN	3.810 sec	1597.676 kN

According to Table 3.2, for the low depths of compressible soil layer, C and $2C_T$ values are at the same order, however, C values decrease while $2C_T$ values increase, with the increasing depths of compressible soil layer. As seen from Fig. 3.3 and Table 3.3, the top displacement of the building on the two-parameter subsoil remains at the same order with the fixed base situation while the top displacement of the building on the Winkler subsoil increases depending on the increasing compressible subsoil depth. From the comparison of the corner column base shear with the shear-wall base shear, column base shear of the building on the Winkler subsoil is at the same order with that of the fixed base, but the column base shear of the building on the two-parameter subsoil is bigger than that of the fixed based building. However, the column base shear of the building on the two-parameter subsoil decreases with the increasing compressible subsoil depth, Fig. 3.4, Table 3.4. Either the shear-wall base shear of the building on the two-parameter subsoil or that of the building on the Winkler subsoil is less than that of the fixed base situation, according to Fig. 3.5 and Table 3.5. From the evaluation of the results in Fig. 3.6 and Table 3.6, the total base shear of the building with fixed base is bigger than the value of either the building on the Winkler subsoil or the building on the two-parameter subsoil. For the 6m and 9m compressible subsoil depths, the total base shear of the buildings on the two-parameter subsoil is close enough to the total base shear of the building with fixed base.

4. CONCLUSIONS

In this study, seismic behaviour of an L shaped 3-D building with mat foundation on a two-parameter elastic subsoil is evaluated. Time-history analyses are performed using the well-known computer program SAP2000. The lateral and vertical records of 1999 Kocaeli earthquake are used in time-history analyses. The first and second parameters C and $2C_T$ of the subsoil are calculated depending on the depth of the compressible subsoil layer from the dead load analysis by successive approximation and it is assumed that the parameters C and $2C_T$ remain unchanged with the seismic excitation. According to the analyses, especially the base shears change considerably when the two-parameter subsoil is considered. In this case, the column base shear increases while the base shear of the shear-wall decreases comparing to the fixed base solution. The displacements and base shears of the building on the two-parameter subsoil are found to be more realistic comparing to those of the buildings either on the Winkler subsoil or the fixed base. For the more general evaluation, much more numerical solutions need to be done for various subsoil and building types and earthquake records. Moreover, the change of the mode shape parameter γ with seismic effects should also be considered in the analyses. The comparison with the solutions of elastic half-space and other soil-structure interaction methods is also needed.

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