Analytical Study of Residual Seismic Performance by Estimation of Response Reduction Ratio and Equivalent Damping for Aftershocks

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SUMMARY:
A practical method for the evaluation of residual seismic capacity of existing buildings is proposed based on the equivalent linear analysis using elastic response spectra. In order to develop an instrument to determine the residual seismic performance of existing buildings after an earthquake, several numerical simulations are carried out and their responses are analyzed by means of the comparison between the capacity curve and the response spectra of earthquake records and artificial waves. These numerical simulations are performed in a single-degree of freedom (SDOF) system, which was widely set in order to obtain diverse conditions, such as different elastic periods, nonlinear properties and hysteretic models. The improved relationships to estimate safely the response reduction ratio and the equivalent damping for aftershocks are proposed based on the analytical results.

Keywords: Aftershock, response reduction ratio, equivalent damping, residual seismic performance

1. INTRODUCTION

Large earthquakes have harmful effects and their consequences are devastating for human life, it is noted in that the most earthquakes-related deaths are caused by the collapse of structures, this structure's state may be not necessarily caused by the main shock rather for the following shakes, in some cases more destructive than the main shock, because of the seismic capacity degradation of existing structures due to main shock. On the other hand, the partial damages due to main shock and the time between main shock and aftershock may be short, and they make difficult the quick inspection (or other direct way to be in the know of the actual state of the existing structure). That's why; it is desirable to count with an instrument to estimate the residual seismic capacity of damaged structures.

This study attempts to provide relationships to estimate the residual seismic performance of existing structures after an earthquake by means of comparison between the capacity curve and the demand curve; it provides a quick method to estimate the earthquake response. The substitute damping model is one of the most straight forward approximate analysis technique to estimate whether a building survive an earthquake and; if it does survive, how damaged the building suffered.

The performance point is estimated as the maximum earthquake response (horizontal deformation) of a structure by the intersection of the capacity curve, which represents the whole structural performance of the building, and a reduced response spectrum (demand curve) for the considered earthquake. This method provides the inelastic response which can be verified directly by the visual representation of the performance point for an assumed earthquake.

Fig. 1.1 shows the concept of the residual seismic evaluation method. The demand spectrum with 5% of viscous damping ratio is applied for the elastic range, represented by the curve-1. When the structure exceeds the yield point (A) the damping also increases because the hysteresis damping depends directly on the maximum deformation (ductility), as explained later on. Then, the demand spectrum is reduced as the curve-2 in order to find an apparent damping which can be used by the equivalent linear analysis to obtain the response same as the maximum response during the main shock at point (B). This additional damping is supposed to be highlighted in this paper, since the equivalent damping increases along with the damage level. Later, the maximum response during the aftershock can be anticipated with the same method, as shown in Fig. 1.1.

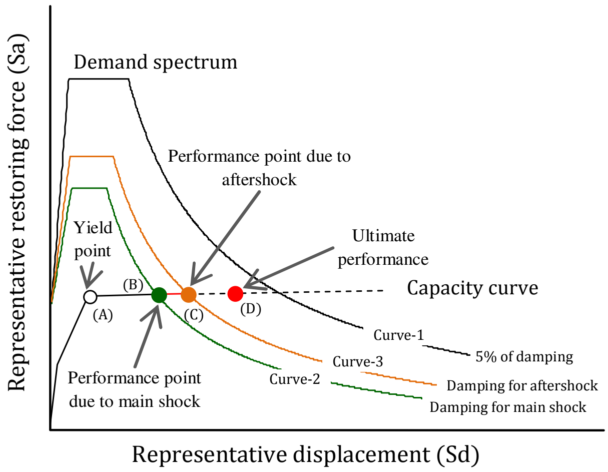


Figure 1.1. Capacity curve and Demand curve

On the other hand, the aftershock is a smaller quake than the main shock; therefore a long motion can be considered from the beginning of the main shock to the end of the maximum expected aftershock. In this sense, this maximum expected aftershock can be assumed same as the main shock. Then, the demand spectrum for the maximum expected aftershock is same as that for the main shock, since the maximum response calculated by the elastic analysis is independent upon the number of times that the motion is inputted.

Hence, the maximum response due to main shock at point (B) may be exceeded by the maximum response due to aftershock at point (C). Thus, the apparent damping for the aftershock is less than that for the main shock; in consequence, the damping decreases for the aftershock as the curve-3, shown in Fig. 1.1.

2. SUBSTITUTE DAMPING MODEL

The equation of motion of SDOF system is given by Eqn. 2.1. Where, m is the mass, c is the damping coefficient and k represents the stiffness.

$$m \cdot \ddot{x} + c \cdot \dot{x} + k \cdot x = -m \cdot \ddot{x}_0 \tag{2.1}$$

The energy response of the system during a motion in Eqn. 2.2, assuming this system vibrates from 0 to t seconds, can be calculated by multiplying the velocity and performing integration in Eqn. 2.1 by t.

$$\underbrace{\frac{1}{2} m \cdot \dot{x}(t)^2}_{T} + \underbrace{\int_0^t c \cdot \dot{x}^2 \cdot dt}_{D} + \underbrace{\frac{1}{2} k \cdot x(t)^2}_{V} = \underbrace{-\int_0^t m \cdot \ddot{x}_0 \cdot \dot{x} \cdot dt}_{L} \tag{2.2}$$

Where, the kinetic energy is represented by T, the work done by the damping force by D, the potential energy by V, and the work done by the external force (earthquake) by L. Since the work done by the external force becomes equal to the work done by the damping force, it is possible to incorporate a substitute damping parameter, h_s. Then, the substitute damping is expressed in terms of ground acceleration, velocity, and equivalent frequency, ω_e, (or equivalent period) in Eqn. 2.3 (Gulkan and Sozen, 1974).

$$h_s = \frac{-\int_0^t \ddot{x}_0 \cdot \dot{x} \cdot dt}{2 \cdot \omega_e \cdot \int_0^t \dot{x}^2 \cdot dt} \tag{2.3}$$

2.1. Hysteresis Damping

If a steady-state response is assumed, the work done by the external force is the same as the work done by the damping force in one-cycle; then an equivalent damping at the resonance can be expressed in terms of the hysteretic energy dissipation, ΔW, and maximum strain energy, W, as shown in Eqn. 2.4.

$$h_{eq} = \frac{1}{4\pi} \cdot \frac{\Delta W}{W} \tag{2.4}$$

Taking into account the nonlinearity of the system, two hysteretic models are assumed, such as a) perfect elastoplastic and, b) stiffness degradation models; then the equivalent damping produced by the hysteresis energy dissipation can be derived on the hysteresis loop according to the geometrical stiffness method (Jennings, 1968) which is expressed in terms of ductility factor, μ. Eqn. 2.5-a, b represent the equivalent damping for perfect elastoplastic and stiffness degradation bilinear models, respectively.

$$h_{eq} = 2/\pi \cdot (1 - 1/\mu) \tag{2.5-a}$$

$$h_{eq} = 1/\pi \cdot (1 - 1/\sqrt{\mu}) \tag{2.5-b}$$

Generally speaking, the substitute damping factor can be derived into the form of the expression in Eqn. 2.6 (Jacobsen, 1930). This expression is represented in terms of ductility factor, μ, initial viscous damping ratio, h_0, and the coefficient, γ.

$$h_s = h_{eq} + h_0 = \gamma \cdot (1 - 1/\sqrt{\mu}) + h_0 \tag{2.6}$$

Notification from Ministry of Land, Infrastructure and Transportation (MLIT) of Japan #1457-6 gives estimated equations for the damping factor due to the damage of the structure. Eqn. 2.7-a in case of material which constitutes the member, and joint connected to the adjacent member are rigid and, Eqn. 2.7-b in case of other members or brace members where the buckling strength is degraded by the compressive forces when seismic force acts.

$$h_{eq} = 0.25 \cdot (1 - 1/\sqrt{\mu}) \tag{2.7-a}$$

$$h_{eq} = 0.20 \cdot (1 - 1/\sqrt{\mu}) \tag{2.7-b}$$

2.2. Response Reduction Ratio

The response reduction ratio due to the damped vibration is given as the relation between the nonlinear response and the equivalent elastic response, $F_h = S_a(h)/S_a(h_0)$. Notification from MLIT of Japan #1457-6 also gives an estimated equation for the response reduction ratio, F_h, due to the damping factor, h, Eqn. 2.8.

$$F_h = \frac{1.5}{1 + 10 \cdot h} < 1 \tag{2.8}$$

As section 2.1, if the steady-state response is assumed and the spectral acceleration is considered as $S_a \approx |\ddot{x} + \ddot{x}_0|_{max}$; the expression in Eqn. 2.9 to estimate the response reduction ratio is derived in this paper. It is expressed in terms of frequency ratio, p/ω (p is the excitation frequency and ω the natural frequency); damping factor, h; and initial viscous damping ratio, h_0.

$$F_h = \sqrt{\frac{[1-(p/\omega)^2]^2+4\cdot h_0{}^2\cdot(p/\omega)^2}{1+4\cdot h_0{}^2\cdot(p/\omega)^2}\cdot\frac{1+4\cdot(p/\omega)^2\cdot h^2}{[1-(p/\omega)^2]^2+4\cdot(p/\omega)^2\cdot h^2}} \quad (2.9)$$

In a practical way, Eqn. 2.9 is redefined in terms of damping factor, h, and coefficients, γ_1, γ_2 and γ_3, as shown in Eqn. 2.10. It is an approach in order to perform a relationship which can safely estimate the response reduction ratio and not underestimates the inelastic response.

$$F_h{}^* = \sqrt{\frac{\gamma_1+\gamma_2\cdot h^2}{1+\gamma_3\cdot h^2}} \quad (2.10)$$

3. PROCEDURE OF STUDY

3.1. Aftershock Assumption

The concept of equivalent viscous damping is based on the assumption of a steady forced vibration and perfect elastoplastic bilinear model for the nonlinearity; in case of the random motion, the response differs significantly from the steady-state response, although the concept of equivalent viscous damping may hold good if the coefficient in Eqn. 2.6 is reduced as shown in Eqn. 2.7. Thereby, it can be also assumed if the coefficient in Eqn. 2.7 is conveniently reduced as shown in Eqn. 2.11. The motion was inputted twice in order to simulate the aftershock scenario; in the second time, the motion was reentered after the system reached rest by means of input of zero ground acceleration.

3.2. Method of Analysis

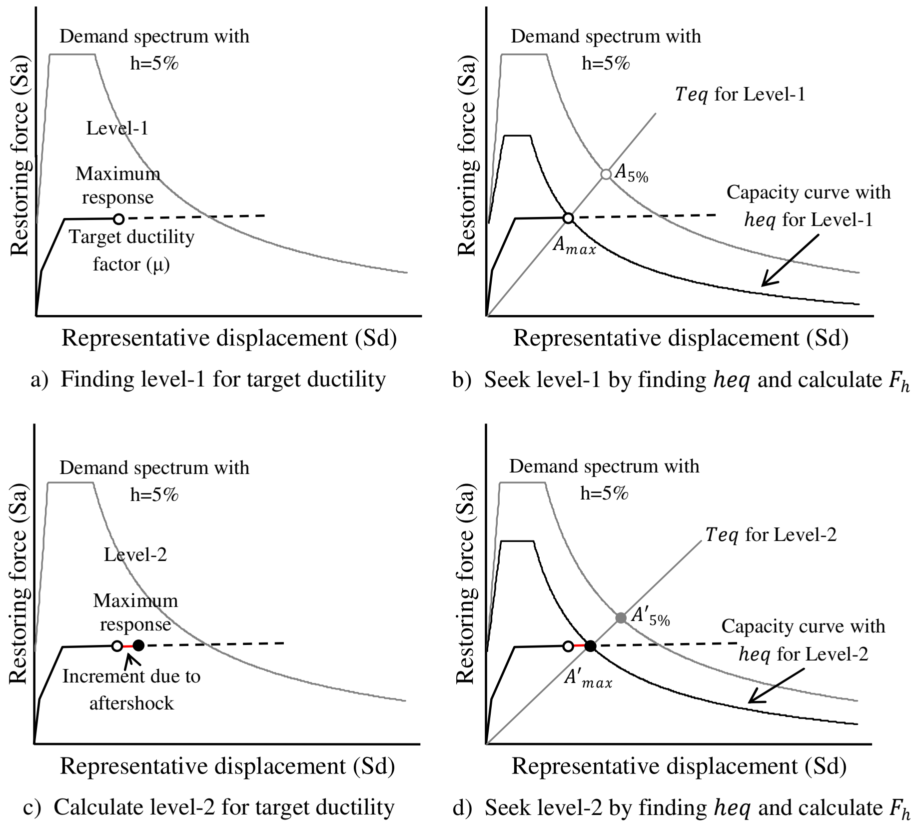


Figure 3.1. Procedure of Analysis

Fig. 3.1 shows the method to perform the residual seismic performance analysis which is described as follows:

a) Find the adequate system with the selected input parameters to reach the target ductility factor (μ) under the selected ground motion; and determine the maximum response, displacement and representative restoring force coordinates on the capacity curve (Level-1). The maximum representative restoring force is $\mathrm{A_{max}}$.

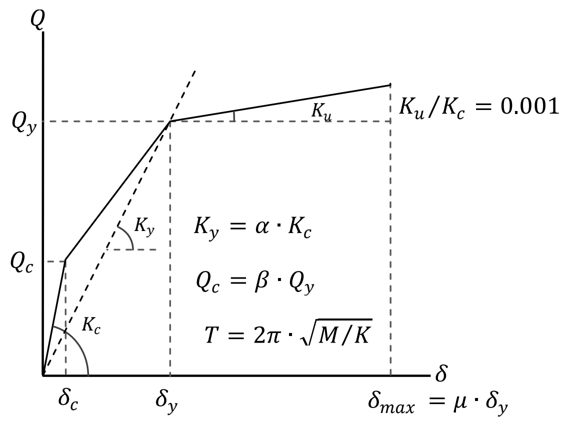
b) Calculate the equivalent period at Level-1 and perform the elastic analysis of the system with a viscous damping ratio of 5% and determine the maximum response; the maximum representative restoring force is $\mathrm{A_{5\%}}$. Seek the point established on the step a) by means of conducting the elastic analysis of the system for finding an equivalent damping ratio $\mathrm{h_{eq}}$. Calculate the reduction ratio of acceleration for the main shock $\mathrm{F_h} = \mathrm{A_{max}}/\mathrm{A_{5\%}}$.

c) Resume the nonlinear analysis performed on the step a). After Level-1, input zero ground acceleration in the measure that the response converges to zero, and then input the same ground motion again. Determine the maximum response for the aftershock as a point on the capacity curve (Level-2). The maximum representative restoring force for the aftershock is $\mathrm{A'_{max}}$.

d) Calculate the equivalent period at Level-2 and perform the elastic analysis of the system with a viscous damping ratio of 5% and determine the maximum response; the maximum representative restoring force is $\mathrm{A'_{5\%}}$. Seek the point established on the step c) by means of conducting the elastic analysis of the system for finding an equivalent damping ratio $\mathrm{h'_{eq}}$. Calculate the reduction ratio of acceleration for the aftershock $\mathrm{F'_h} = \mathrm{A'_{max}}/\mathrm{A'_{5\%}}$.

3.3. Parameters of Analysis

Five types of hysteretic models are chosen; such as: degrading bilinear, Takeda, Takeda-slip, degrading trilinear and origin-oriented models. The parameters to define the nonlinearity are shown in Fig. 3.2.



- Elastic period (T): from 0.2 seconds to 2.0 seconds, with an interval of 0.2 seconds.
- Degrading factor of yielding stiffness (α): from 0.5 to 0.9, with an interval of 0.1.
- Reduction ratio of yielding force (β): 0.25, 0.50 and 0.75.

All the above parameters are set to find the ductility factors (μ): 1, 2, 3, 4 and 5.

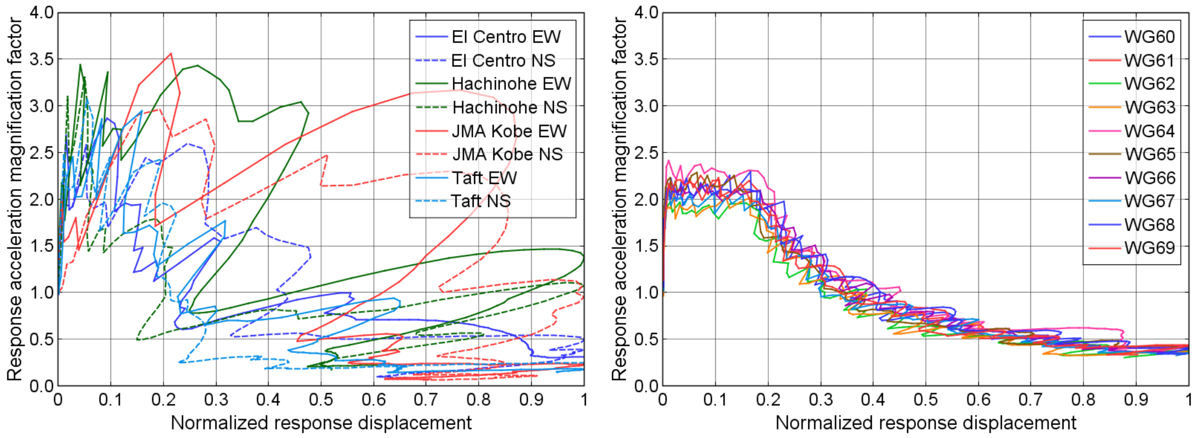
Figure 3.2. Parameters of study

For bilinear model the degrading factor of yielding stiffness, α, and the reduction ratio of yielding force, β, are neglected.

The ground motions selected for this study are: 8 earthquake records, such as El Centro, Hachinohe, JMA Kobe and Taft in NS and EW components and; 10 artificial waves, such as WG60 to WG69. Table 3.1 shows the peak ground acceleration (PGA) of all input motions and their normalized demand spectra are shown in Fig. 3.3.

Table 3.1. Peak ground acceleration of input motions

Artificial Waves		Earthquake records	
Motion	PGA (cm/s^2)	Motion	PGA (cm/s^2)
El Centro EW	210.10	WG60	581.16
El Centro NS	341.70	WG61	560.79
Hachinohe EW	182.90	WG62	627.50
Hachinohe NS	225.00	WG63	612.05
JMA Kobe EW	617.14	WG64	524.94
JMA Kobe NS	817.82	WG65	548.17
Taft EW	175.90	WG66	575.02
Taft NS	152.70	WG67	600.80
-	-	WG68	570.37
-	-	WG69	568.02



a) Earthquake Records b) Artificial waves

Figure 3.3. Normalized Demand Spectra

3.3. Integral Method

In this study, the mass was arbitrarily chosen to be the unit, the analysis was performed using Newmark-β numerical method (β=1/4), and the time interval was reduced to 0.005 seconds in order to improve the numerical integration. The damping matrix is taken as proportional to the instantaneous stiffness ([C] =2ho/ωo [K']), and the initial viscous damping ratio (ho) is 5%.

4. ANALYSIS

In order to obtain the target relationships based on several systems; 24,400 and 30,500 simulations are performed under earthquake records and artificial waves, respectively. The equivalent damping relationship for aftershock is proposed in Eqn. 2.11-a and 2.11-b which are conveniently reduced from Eqn. 2.7-a and 2.7-b, respectively.

$$h_{eq} = 0.12 \cdot (1 - 1/\sqrt{\mu}) \tag{2.11-a}$$

$$h_{eq} = 0.08 \cdot (1 - 1/\sqrt{\mu}) \tag{2.11-b}$$

The response reduction ratio relationship is proposed in Eqn. 2.12 based on Eqn. 2.10, which is analyzed for systems under main shocks and aftershocks.

$$F_h{}^* = \sqrt{\frac{1.1+12\cdot h^2}{1+52\cdot h^2}} \tag{2.12}$$

The values shown in Table 4.1 and 4.2 represent systems in which the response from analysis exceeds the estimated response. In other words, this value indicates how much the response can be underestimated by means of Eqn. 2.7, 2.8, 2.11 and 2.12. Column (1) to (5) in Table 4.1 and 4.2 show the error rate of the estimation by the relationships for main shock and aftershock presented in this paper.

Table 4.1. Error rate of the response under Main shock

Model	Motion	Number of data	For Main Shock				
			(1) h_{eq} estimated w/0.25	(2) F_h estimated w/h_{eq}-0.25	(3) $F_h{}^*$ estimated w/h_{eq}-0.25	(4) F_h from Analysis	(5) $F_h{}^*$ from Analysis
Bilinear	EQ	334	0.42	0.48	0.16	0.68	0.13
	Artificial	495	0.57	0.58	0.10	0.48	0.01
Takeda	EQ	5338	0.36	0.46	0.14	0.73	0.12
	Artificial	7472	0.22	0.28	0.00	0.59	0.00
Takeda-slip	EQ	5504	0.42	0.50	0.18	0.71	0.10
	Artificial	7472	0.22	0.28	0.00	0.59	0.00
Trilinear Degrading	EQ	5018	0.38	0.48	0.17	0.74	0.12
	Artificial	7393	0.41	0.45	0.09	0.56	0.00
Origin-Oriented	EQ	3963	0.77	0.68	0.42	0.62	0.09
	Artificial	6843	0.91	0.79	0.65	0.32	0.01
Total	EQ	20157	0.46	0.51	0.20	0.71	0.11
	Artificial	29675	0.44	0.39	0.10	0.55	0.00
	All	49832	0.45	0.44	0.14	0.61	0.05

Table 4.2. Error rate of the response under Aftershock

Model	Motion	Number of data	For Aftershock				
			(1) h_{eq} estimated w/0.12	(2) F_h estimated w/h_{eq}-0.12	(3) $F_h{}^*$ estimated w/h_{eq}-0.12	(4) F_h from Analysis	(5) $F_h{}^*$ from Analysis
Bilinear	EQ	334	0.49	0.37	0.10	0.59	0.09
	Artificial	495	0.52	0.43	0.13	0.41	0.02
Takeda	EQ	5338	0.43	0.40	0.12	0.61	0.09
	Artificial	7472	0.14	0.10	0.01	0.51	0.00
Takeda-slip	EQ	5504	0.45	0.42	0.13	0.56	0.06
	Artificial	7472	0.14	0.10	0.01	0.51	0.00
Trilinear Degrading	EQ	5018	0.51	0.46	0.20	0.60	0.10
	Artificial	7393	0.42	0.39	0.11	0.45	0.00
Origin-Oriented	EQ	3963	0.90	0.66	0.45	0.52	0.07
	Artificial	6843	0.98	0.74	0.47	0.28	0.01
Total	EQ	20157	0.55	0.44	0.17	0.58	0.08
	Artificial	29675	0.41	0.21	0.05	0.48	0.00
	All	49832	0.46	0.30	0.10	0.52	0.03

The damping factor is overestimated by Eqn. 2.7 in 45% of cases under main shock. In similar percentage of cases, Eqn. 2.11 overestimates the damping factor under aftershock. The relationship between damping factor (h) and ductility factor (μ) for systems with Takeda model, shown in Fig. 4.1.

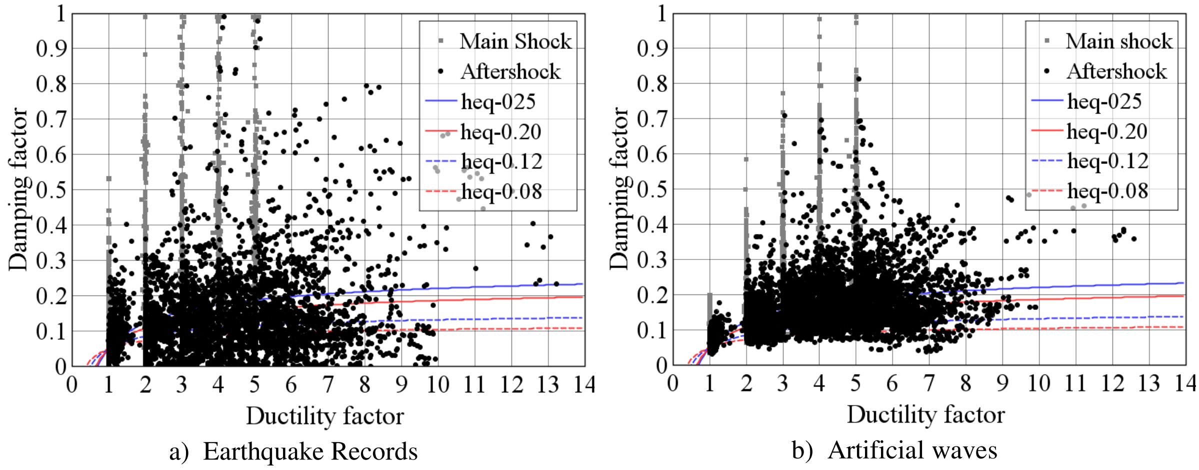


Figure 4.1. Relationship between damping factor and ductility factor for Takeda Model

Fig. 4.2 shows the relationship between response reduction ratio (F_h) and damping factor (h) obtained from analysis for systems with Takeda model. The average of error rates of the response reduction ratio relationship given by Eqn. 2.8 are 61% and 52% for main shock and aftershock, respectively. While, using the response reduction ratio relationship given by Eqn. 2.12 are 5% and 3%, for main shock and aftershock, respectively. Eqn. 2.8 and 2.12 are represented by curves “#1457-6” and “Upper”, respectively, in Fig. 4.2 and 4.3.

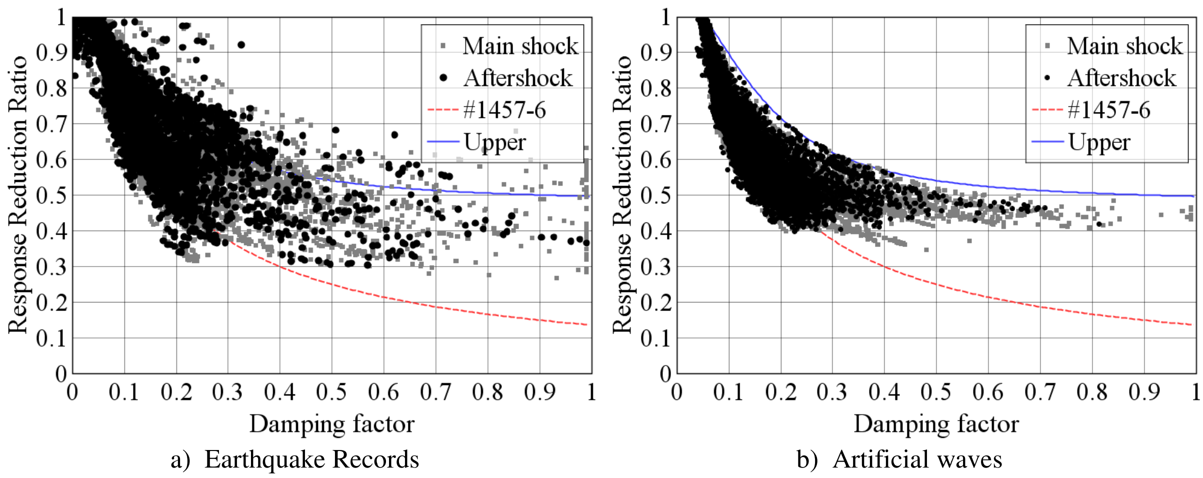
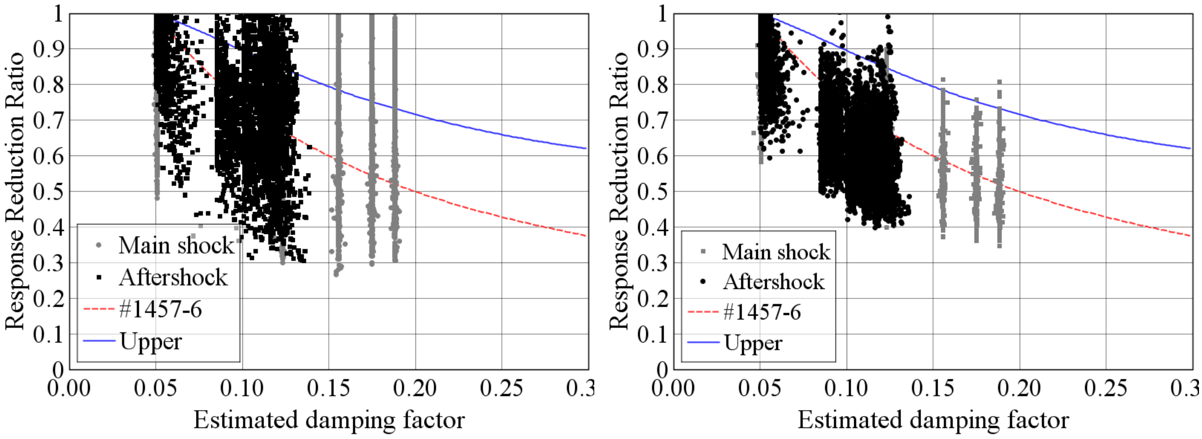


Figure 4.2. Relationship between response reduction ratio and damping factor for Takeda Model

Fig. 4.3 shows the relationship between response reduction ratio obtained from analysis and the estimated damping factor obtained by using Eqn. 2.7 and Eqn. 2.11 for main shock and aftershock, respectively. It is the most representative relationship to analyze the substitute damping model since the response reduction ratio (F_h) is calculated from damping factor (h) which depends directly on ductility factor (μ).

This figure also shows the comparison between the analytical response and estimated response since the relationship given by Notification from MLIT of Japan #1457-6 given by Eqn. 2.8 and the proposed response reduction ratio relationship given by Eqn. 2.12 are also plotted. It means that systems where the response is underestimated by the relationships are represented by the points above the curve defined by Eqn. 2.8 and 2.12.



a) Earthquake Records b) Artificial waves

Figure 4.3. Relationship between response reduction ratio and estimated damping factor for Takeda Model

The error rates of the response reduction ratio relationship in Eqn. 2.8, using the damping factor relationships for main shock (Eqn. 2.7) and aftershock (Eqn. 2.11) are in average 44% and 30%, respectively; while using the proposed response reduction ratio relationship in Eqn. 2.12, using the same damping factor relationships for main shock (Eqn. 2.7) and aftershock (Eqn. 2.11) are 14% and 10%, respectively.

5. CONCLUDING REMARKS

Nonlinear analysis and linear analysis were conducted to analyze the residual seismic performance. Different parameters and input motions performed a total of 54,900 simulations, and 91% of them reached convergence errors smaller than 5%.

Analysis carried out by Takeda, Takeda-slip and trilinear degrading models are quite similar, which have no large difference in comparison to bilinear model; these hysteretic models are more adequate to be used with the damping substitute model. Origin-oriented model does not hold good with the damping substitute model due to the limited energy dissipation.

The conducted analysis demonstrates that the response reduction ratio relationship holds good for main shock and aftershock due to the similarity of results as shown in Fig. 4.2. The proposed response reduction ratio relationship given by Eqn. 2.12 can safely estimate the inelastic response in 86% and 90% of cases for main shock and aftershock using their respective damping factor relationships, such as Eqn. 2.7 and Eqn. 2.11 for main shock and aftershock, respectively.

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