

# Fundamental Study on Energy Transformation in Dynamic Systems Controlled by Uplifting Slide Shoe



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## SUMMARY

The Uplifting Slide Shoe (UPSS) is proposed as one of the methods for upgrading the seismic performance of multi-span continuous girder bridges. UPSS converts a part of the superstructure's inertia force (kinetic energy) by the horizontal motion during an earthquake into the vertical force (potential energy) by the superstructure's sliding up a slope, and controls the horizontal force transmitted of substructure. In this study, so as to understand the transition energy during an earthquake in the vibration system using the UPSS, using a model of a point mass system, the transition of basic energy during an earthquake is studied. Kinetic energy due to the horizontal movement of the superstructure is stored temporarily as the gravitational potential energy of the UPSS and strain energy of the superstructure, and consumed by viscous damping and stable friction on the sliding plane.

*Keywords: Uplifting slide shoe, UPSS, energy transformation, gravitational potential energy, slide bearing*

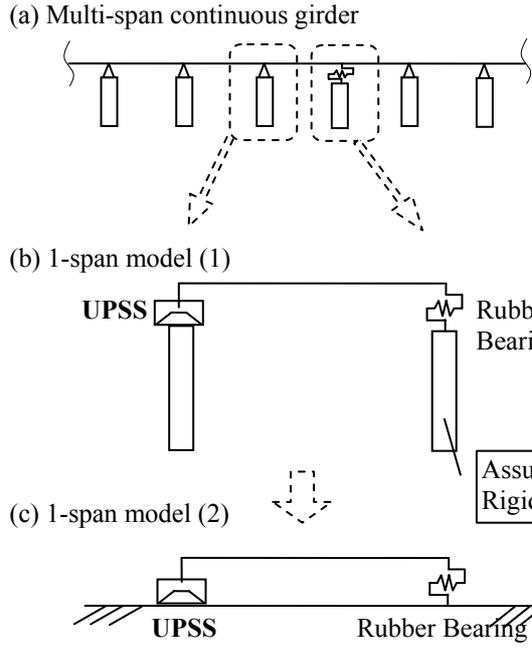
## 1. INTRODUCTION

After the 1995 Hyogo-ken Nanbu earthquake, the seismically-isolated bridge has been widely used as an earthquake resistant structure of continuous elevated bridges. However, since its displacement in case of an earthquake increases, the size of expansion joints tends to be larger than the case of the conventional bridges. Some problems arise due to the size of increase of expansion joints such as traffic-induced noise and vibration, and construction and maintenance cost increase. In order to alleviate this problem, a new type of slide bearing called "Uplifting Slide Shoe (UPSS)" was developed (Igarashi et al. 2010). The UPSS consists of a horizontal section and slope sections to control the seismic response. This mechanism mainly transforms the kinetic energy generated by seismic excitation into the gravitational potential energy due to the displacement in the vertical direction. It is verified on the past research that horizontal displacement can be reduced as the slope angle increases. Various kinds of energy values are involved in the seismic response of UPSS, and the proportion of these types of energy can be different depending on the bearing design.

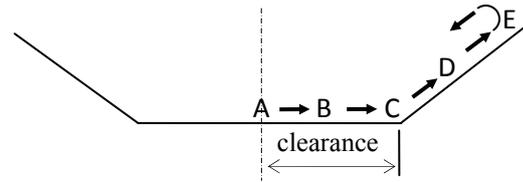
Generally, the energy associated with UPSS in case of an earthquake is roughly divided into potential energy and dissipated energy. The strain energy and gravitational potential energy are of the former type, and friction damping, viscous damping, etc. are the latter. The kinetic energy is temporarily stored by sliding up on the slope, and would be consumed later. Using a simple model of a single mass supported by UPSS, non-linear dynamic analysis was carried out to evaluate the energy transformation for selected values of parameters such as the angle of the inclined plane and the friction coefficient. Based on the analysis results, the amount of the potential energy by the UPSS depending on the slope angle is discussed.

## 2. TYPES OF ENERGY ASSOCIATED WITH DYNAMIC ACTION OF UPSS

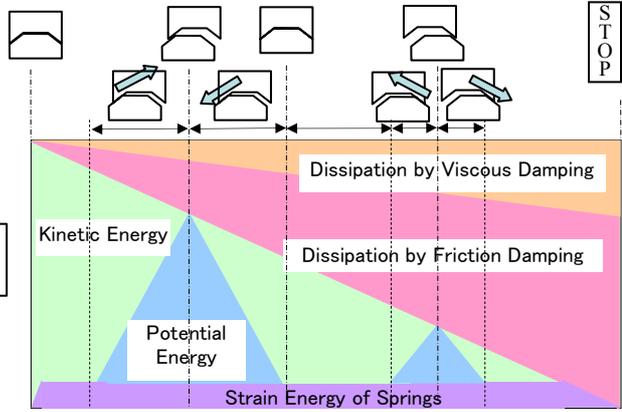
Various energies are involved in the dynamic action of UPSS. In order to simplify the situation, a single span is taken from a continuous girder with UPSS, as shown in Fig. 1. The various types of



**Figure 1.** Extraction of a model



**Figure 2.** Diagram for action state of UPSS



**Figure 3.** Relation between displacement and energy of UPSS

energy values are calculated at the states A though E as the sequence during the sliding on the three planes (a level part and two slope parts) as shown in Fig. 2.

First, the case without energy dissipation such as viscous damping and friction damping is discussed as the most fundamental case.

In state A, kinetic energy  $E_{K,0}$  is generated by the initial velocity as expressed by Eqn. 1.

$$E_{K,0} = \frac{1}{2} m V_0^2 \quad (1)$$

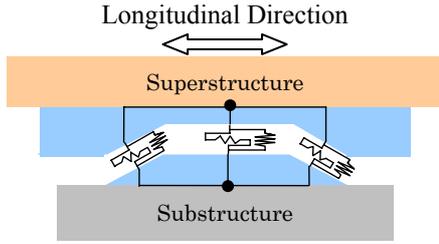
where,  $m$  is the mass of the point mass which represents the superstructure, and  $V_0$  is initial horizontal velocity. A the point mass slides on the plane, and when the displacement reaches the clearance limit of UPSS contacting to the right slope, an impact force takes place at the same time, as indicating in previous research. Then the mass slides up the slope in the state D and E, the kinetic energy is converted to the potential energy  $E_g$  expressed by Eqn. 2.

$$E_g = mgh \quad (2)$$

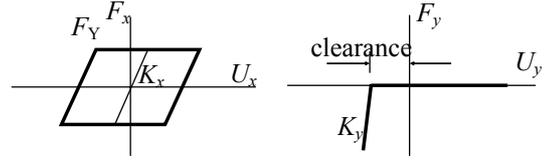
where,  $g$  is gravity acceleration ( $=9.80665 \text{ m/sec}^2$ ) and  $h$  is the height from the horizontal sliding level. Moreover, in the process of state D, the kinetic energy of the point mass decreases as it slides up on the slope, eventually reaching zero, and stops on the slope (state E). The energy at this time is only the gravitational potential energy, and a point mass begins to slide down on the slope. In this state, the gravitational potential energy decreases and is again converted into a kinetic energy. In addition, strain energy due to the spring of the slider exists, although the amount is not significant.

Next, the friction damping and viscous damping are taken into consideration. If the point mass is moving to the  $x$  direction on the plane with friction forces, the force expressed by Eqn. 3. acts in the direction opposite to the velocity, and its energy dissipation  $E_\mu$  is expressed by Eqn. 4. (state B and D.)

$$F_x = \mu N \quad (3)$$



**Figure 4.** Analytical model of UPSS



(a) Tangential spring (b) Normal spring  
**Figure 5.** Nonlinear spring model

$$E_\mu = \int F_x v_x dt \quad (4)$$

where,  $\mu$  is the coefficient of friction,  $N$  is the normal force. And  $v_x$  is the velocity of the sliding,  $F_x$  is the active force in the opposite direction as the sliding. In addition, energy dissipation due to viscous damping exists. The energy dissipation  $E_R$  by the viscous damping is expressed by Eqn. 5.

$$E_R = (\alpha \cdot m + \beta \cdot K_e) \int v^2 dt \quad (5)$$

where  $\alpha$  and  $\beta$  are the coefficients applied to the point mass and element stiffness in the Rayleigh damping, and  $K_e$  is the element stiffness. Transition of energy values and the states of UPSS can be depicted as in Fig. 3. During the sliding up on the slope, the kinetic energy is temporarily converted to gravitational potential energy. Furthermore, the initial kinetic energy would decrease according to the increase of the energy dissipation by the friction damping accompanying the sliding on the horizontal plane and side slopes in addition to viscous damping, resulting in the zero velocity when the kinetic energy becomes zero.

Furthermore, at the time of transition from the horizontal planes to the right or left slope, it is assumed that the work  $E_n$  for the perpendicular direction given by equation 6. is done to the point mass.

$$E_n = \int F_y v_y dt \quad (6)$$

where  $v_y$  is the velocity in the direction perpendicular to the sliding, and  $F_y$  is the force in same direction.

### 3. VERIFICATION OF SEISMIC ANALYTICAL MODEL

#### 3.1. UPSS Model

The analysis model for expressing the action of UPSS is shown in Fig. 4. The perpendicular and horizontal spring in each local coordinate system are treated as a spring set, and a total of 3 sets are arranged on the level plane and on the both side slopes. Each spring possesses nonlinear perfect elasto-plastic characteristics for the sliding direction and resists only compression in the perpendicular direction to the plane, as shown in Fig. 5.

#### 3.2. Model for the Study

The single mass system model consisting of a UPSS with a rubber bearing shown in Fig. 6. is used for the analysis, in order to evaluate the fundamental action of UPSS. This model is taken from a 12-span continuous girder bridge (40m x 12) in accordance with Fig. 1. The angles considered are 5 degrees and 15 degrees. The 5 degrees case is intended to be similar to pendulum type sliding bearings slope angles, while the 15 degrees case is selected so as to the horizontal resistance force balance to the

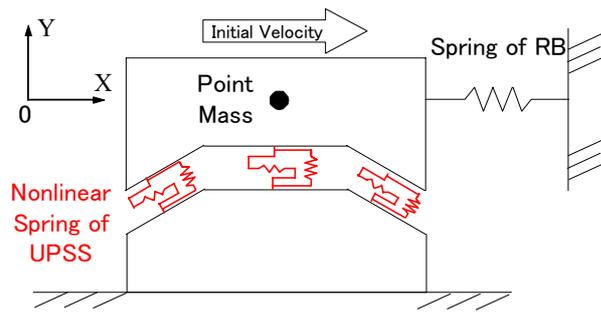


Figure 6. Model for dynamic analysis

Table 1. Cases of analysis

Case	Rubber Bearing	Slope Angle	Friction Damp. ( $\mu$ )	Viscous Damp. ( $h$ )
05-Simple	Non	5 degrees	0	0 %
05-00	Included		0.05	3 %*
05-10			0.10	
15-Simple	Non	15 degrees	0	0 %
15-00	Included		0.05	3 %*
15-10			0.10	

\* Excluding UPSS springs

inertia force due to the ground acceleration on the slope.

As shown in Table 1, the behaviour of the simple UPSS model was examined first, and the model with a spring to represent a rubber bearing was examined. The first two-digit number (05 and 15) of a case denotes the slope angle. If the word “Simple” is the second part of the case name, it means that the rubber bearing is not included in the model, and friction and viscous damping are neglected. Otherwise, the second part denotes the friction coefficient, and the rubber bearing is considered. Furthermore, viscous damping of 3 % is assumed in the model (05 and 10) according to Japanese Specifications for Highway Bridges except for “00” cases. Viscous damping was not included in the nonlinear springs of UPSS. The mass of the superstructure is  $2.066 \times 10^3$  ton, and the spring stiffnesses were 1.887 and  $0.726 \times 10^7$  kN/m for perpendicular and sliding directions, respectively. The stiffness of the RB spring is  $6.189 \times 10^4$  kN/m.

## 4. ANALYTICAL RESULT

### 4.1. Simple UPSS Case

The computed transition of energy for Case 05-Simple is shown in Fig. 7. Since friction and viscous damping are not considered only the kinetic energy and the potential energy are found and they are alternatively transformed each other.

### 4.2. Case of UPSS with Rubber Bearing

The following four types of results are shown for the cases with the rubber bearing.

- Time History of Displacement Response ( for horizontal (X) and vertical (Y) )
- 2-dimensional displacement response trajectory
- Time History of Energy Transformation
- Relationship between Horizontal Force and Displacement

The time history of the energy transformation (c) deals with various kinds of energies shown in Table 2. , which also shows the notation for the name of energy. It should be noted that each “Energy Name” symbol denotes the sum of the corresponding energy values listed in the “Type of Energy” column. The table also shows whether the energy value is of conservative type or dissipative type, and the equation to be used for the calculation of the value. The energy transition plots presented in the ensuing sections are based on a similar diagram for the energy of UPSS already shown in Fig. 3. The values of gravitational potential energy (Potential-E) and strain energy of the spring representing rubber bearings (RB-E) are shown in the manner such that the latter is stacked on the former in the plots. On the other hand, the dissipated energy by viscous damping of the point mass representing the superstructure (Nodal-D-E) and the accumulated energy dissipation on the three sliding surfaces (UPSS-H-E, UPSS-R-E, UPSS-L-E) excluding the negligibly small strain energy of the springs are

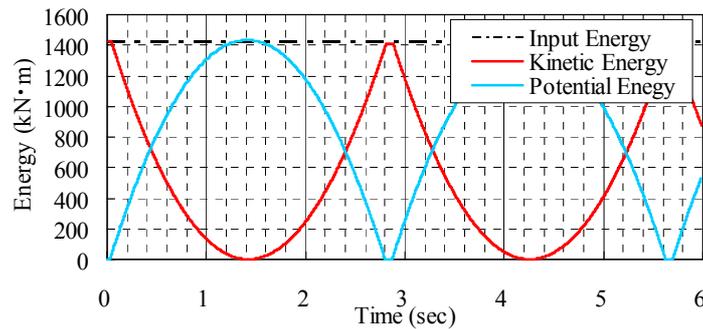
**Table 2.** Classification of Energies for Analysis

Energy Name	Type of Energy	Dissipative	Conservative	Eqn.
Nodal-D-E	Viscous Damping of Mass	○		(5)
UPSS-H-E (Spring on the Horizontal Plane of UPSS)	Damping by Friction	○		(4)
	Viscous Damping	○		(5)
	Work for Normal Direction		○	(6)
	Strain Energy of Spring for Sliding Direction		○	**
	Strain Energy of Spring for Normal Direction		○	**
UPSS-R-E (Spring on the Right Slope of UPSS)	Damping by Friction	○		(4)
	Viscous Damping	○		(5)
	Work for Normal Direction	○		(6)
	Strain Energy of Spring for Sliding Direction		○	**
	Strain Energy of Spring for Normal Direction		○	**
UPSS-L-E (Spring on the Left Slope of UPSS)	Damping by Friction	○		(4)
	Viscous Damping	○		(5)
	Work for Normal Direction	○		(6)
	Strain Energy of Spring for Sliding Direction		○	**
	Strain Energy of Spring for Normal Direction		○	**
Kinetic-E	Kinetic Energy		△*	(1)
RB-E	Strain Energy of RB		○	***
Potential-E	Potential Energy of UPSS		○	(2)

\* Although it is conservative.

\*\* Not included in the plot since the value is negligibly small.

\*\*\* RB-E was calculated as  $(\text{Spring Constant} \times \text{Square of Horizontal Displacement}) / 2$



**Figure 7.** Energy transformation

shown by the values consecutively subtracted from the total energy. Therefore, the kinetic energy of the point mass (Kinetic-E) is expressed as the gap between the dissipated energy associated with the sliding surfaces and the two major conservative energy values in the energy transition plots.

#### 4.2.1. Results of 5-degree slope cases

The results of the analysis of 5-degree cases (05-00, 05-05, 05-10) using the initial velocities of 50 kine and 120 kine are shown in Figs. 8. and 9., respectively. In those cases, the friction coefficient of 0.05 and viscous damping ratio of 3% are assumed. It can be seen in the time histories that the peak displacement on the negative side is smaller than that on the positive one due to energy dissipation during the half cycle of the response. The energy transition plots shown in Fig. 8(c). and Fig. 9(c) indicate that stable and continuous energy dissipation (Nodal-D-E, UPSS-H-E, UPSS-R-E) takes place since the onset of the dynamic response of the system. Moreover, energy transformation process is smooth at the time of crossing the boundary between the horizontal and slope sliding surfaces which also causes energy dissipation. The horizontal forces during up-sliding and down-sliding are found to be approximately 2600kN and 740kN, respectively, as shown in Fig. 9(c). On the other hand, the predicted values calculated by static equilibrium are 2764kN and 754kN, respectively. These horizontal forces can reasonably be estimated using the static equilibrium approximation even with the effect of friction.

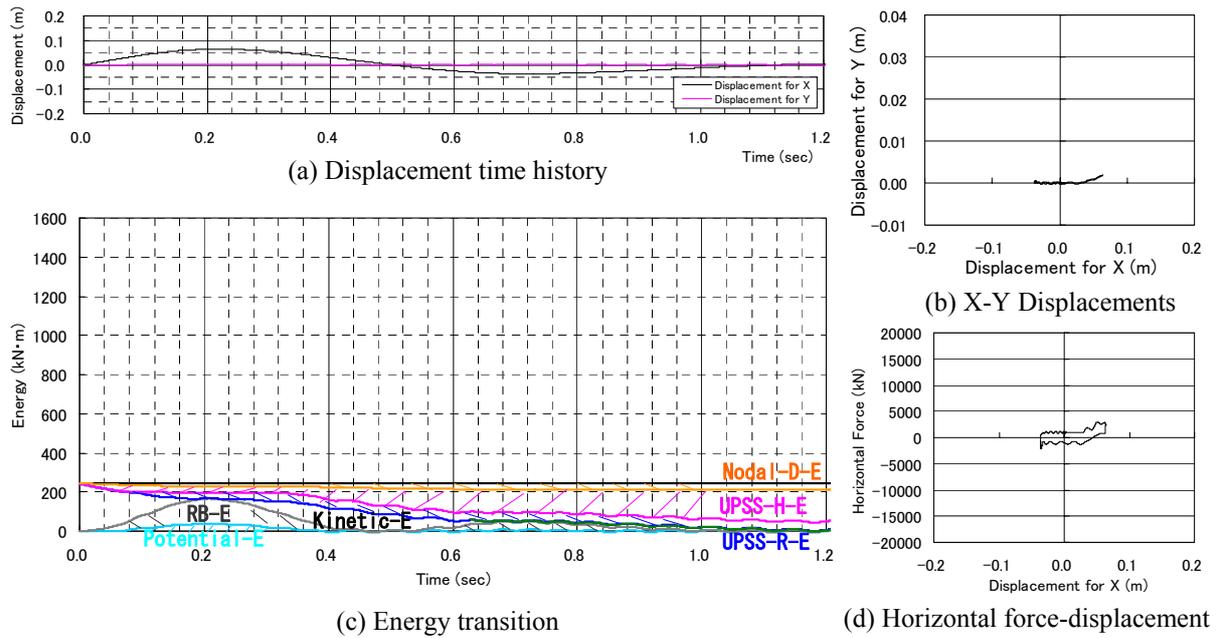


Figure 8. Results of 5-degree slope case ( $V_0 = 50$  kine,  $\mu = 0.05$ ,  $h = 3\%$ )

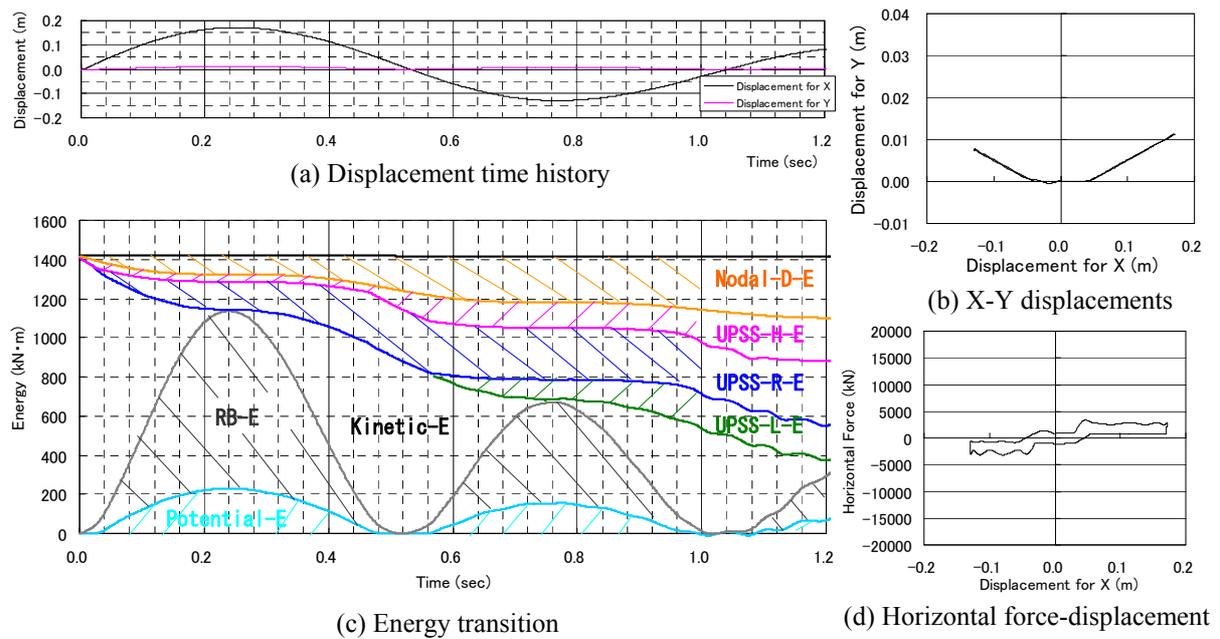


Figure 9. Results of 5-degree slope case ( $V_0 = 120$  kine,  $\mu = 0.05$ ,  $h = 3\%$ )

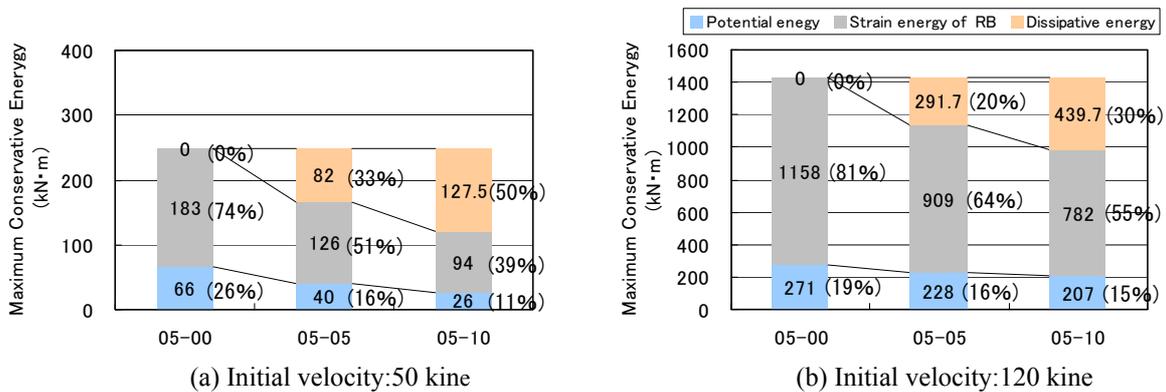


Figure 10. Maximum conservative energy (5-degree slope)

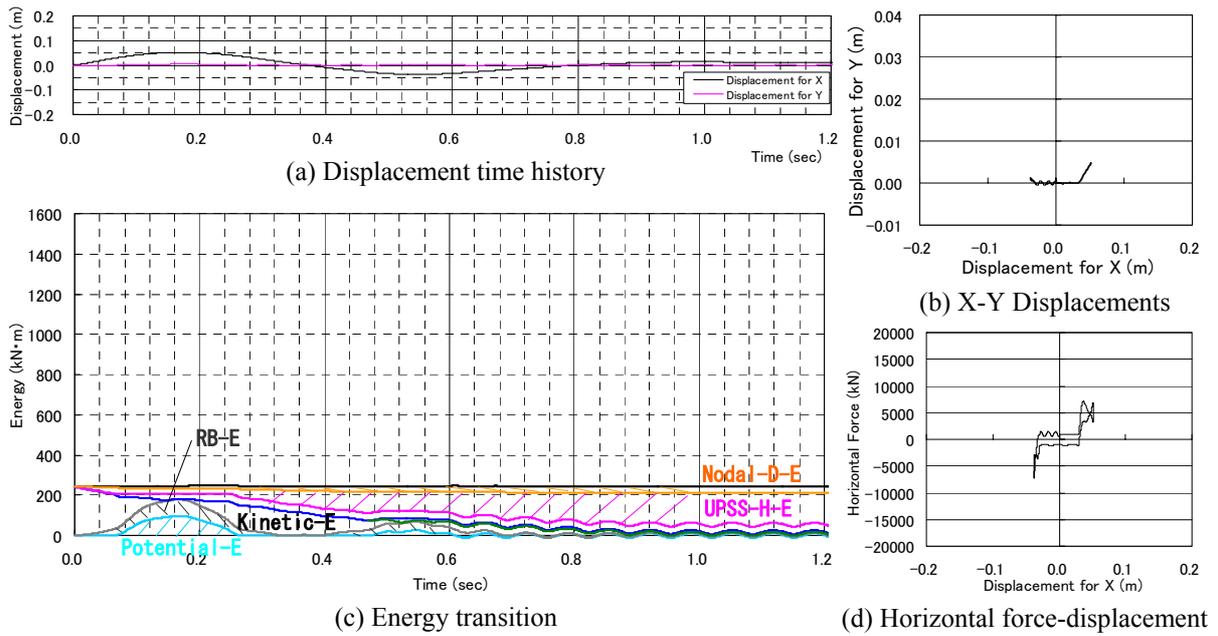


Figure 11. Results of 15-degree slope case ( $V_0 = 50$  kine,  $\mu = 0.05$ ,  $h = 3\%$ )

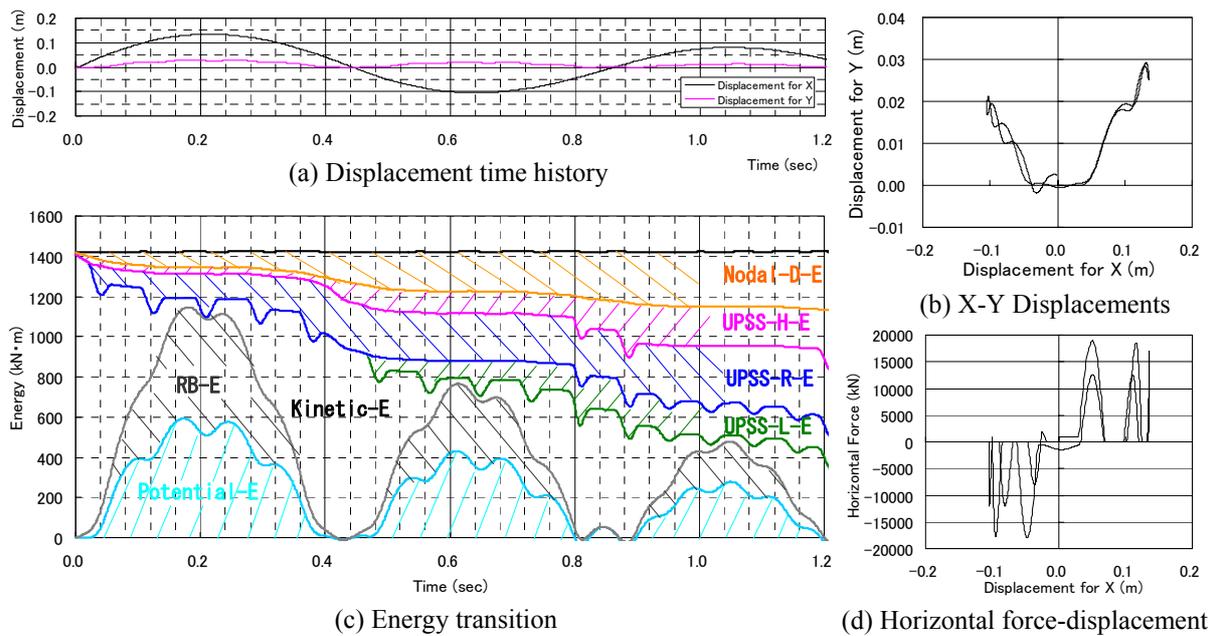


Figure 12. Results of 15-degree slope case ( $V_0 = 120$  kine,  $\mu = 0.05$ ,  $h = 3\%$ )

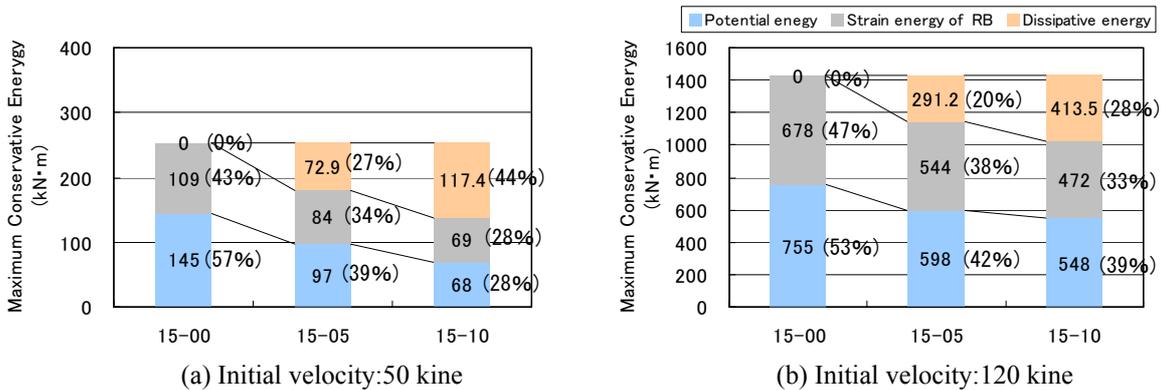


Figure 13. Maximum conservative energy (15-degree slope)

The values of two types of conservative energy, namely the gravitational potential energy (Potential-E) and the strain energy of the rubber bearings (RB-E), at the time of positive peak displacement are shown in Fig. 10., accompanied by the ratios of the two values to the initial kinetic energy. The figure shows those values for three cases of the friction coefficients (05-00, 05-05 and 05-10) to indicate the influence of the friction on those values. It is clearly shown that the sum of the conservative energy values does not reduce for the case without friction, while the sum decreases as the friction coefficient increases due to energy dissipation. In the 5% friction coefficient case (05-05), for example, the ratios of the conservative energy values are 67% and 80% for the initial velocity of 50 kine and 120 kine, respectively.

#### 4.2.2. Results of 15-degree slope cases

The analysis results of 15-degree cases (15-00, 15-05, 15-10) with the initial velocities of 50 kine and 120 kine are shown in Figs. 11. and 12., respectively. In the same manner as in the preceding section, the conservative energy values at the time of positive peak displacement for all cases are shown in Fig. 13. For the 50 kine initial velocity case, significant difference in the response is not observed in Fig. 11 compared with the 5-degree cases already presented in Fig. 8., except for a small increase in the horizontal force. However, in the 120 kine initial velocity case shown in Fig. 12., notable difference in the displacement trajectories is found; in particular, a negative displacement in the vertical component within the horizontal sliding section in Fig. 12(b). is a consequence of a jumping behaviour of the superstructure during the down-sliding on the left slope to induce downward normal contact force to the horizontal sliding surface.

Furthermore, pulse-like fluctuating behaviour of the energy values, which is not observed in 5-degree cases, can be seen in the energy transition plot shown in Fig. 12(c). This phenomenon is also attributed to the temporary jumping behaviour of the superstructure, indicated by periodical sharp rises of additional energy of approximately 100-200 kNm in UPSS-R-E or UPSS-L-E (and UPSS-H-E) depending on the position of the superstructure. The reason of this difference can be explained by the significant increase of the amount of work done on the sliding surface due to the normal force defined by Eqn. 6. in this case, although the temporary energy increase associated with the jump is not considerable in the previous cases. However, even in this case of a larger slope angle and with a larger initial velocity, the steady dissipation of the initial kinetic energy throughout the process of dynamic behavior of the system is verified in this example, involving temporary energy storage as the strain energy and gravitational potential energy.

The conservative energy at the time of positive peak displacement shown in Fig. 13. indicates that the ratios of the conservative energy to the initial energy for 5% friction coefficient, for example, in the cases of 50 kine and 120 kine initial velocities are 73% and 80%, respectively.

## 5. INVESTIGATION ON INFLUENCE OF SLOPE ANGLE

The influence of the slope angle on the dynamic behaviour of the model with UPSS and rubber bearing are discussed based on the result of the analysis.

### 5.1. Energy Transformation

Figures 14. and 15. show the proportion of the various types of energy in the total amount of energy at four representative times during the period of a cycle  $T$  of the UPSS response, namely  $T/4$ ,  $T/2$ ,  $3T/4$  and  $T$ , corresponding to the positive peak, zero, negative peak and zero displacements, respectively. Results of the cases with the initial velocities of 50 kine and 120 kine are plotted in Fig. 14. and 15., respectively. In each figure, results of 5-degree and 15-degree slope cases are shown. Regardless of friction coefficients, the gravitational potential energy for 15-degree cases is greater than 5-degree cases, since the vertical displacements in 15-degree case is consistently greater than those in 5-degree cases. On the other hand, the strain energy of the rubber bearing, which is primarily determined by the horizontal displacement, is smaller in the 15-degree slope cases than the result of the 5-degree cases.

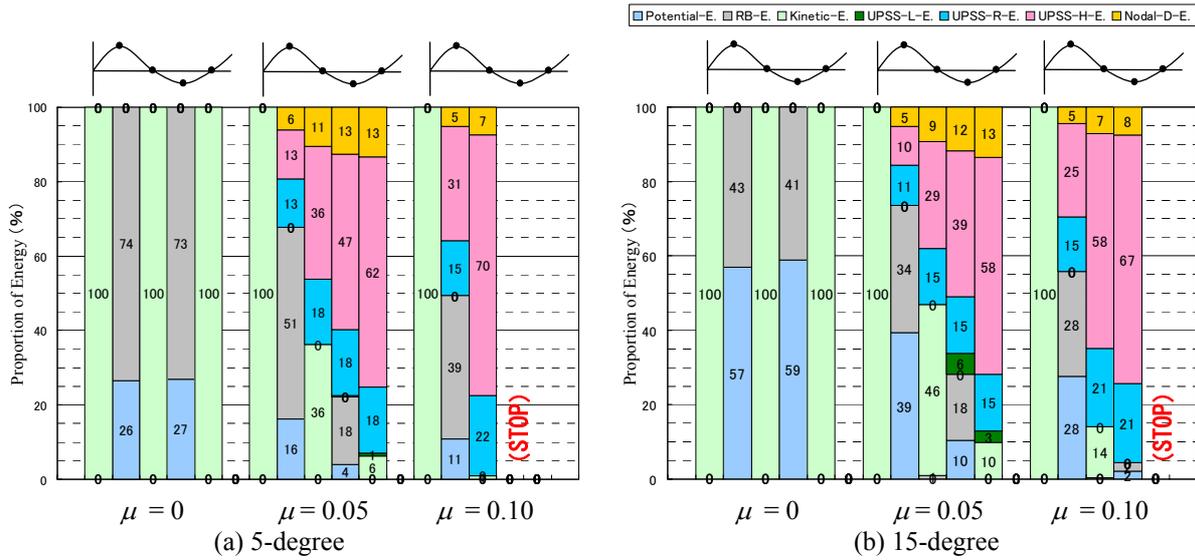


Figure 14. Transition of energy proportion ( $V_0 = 50$  kine)

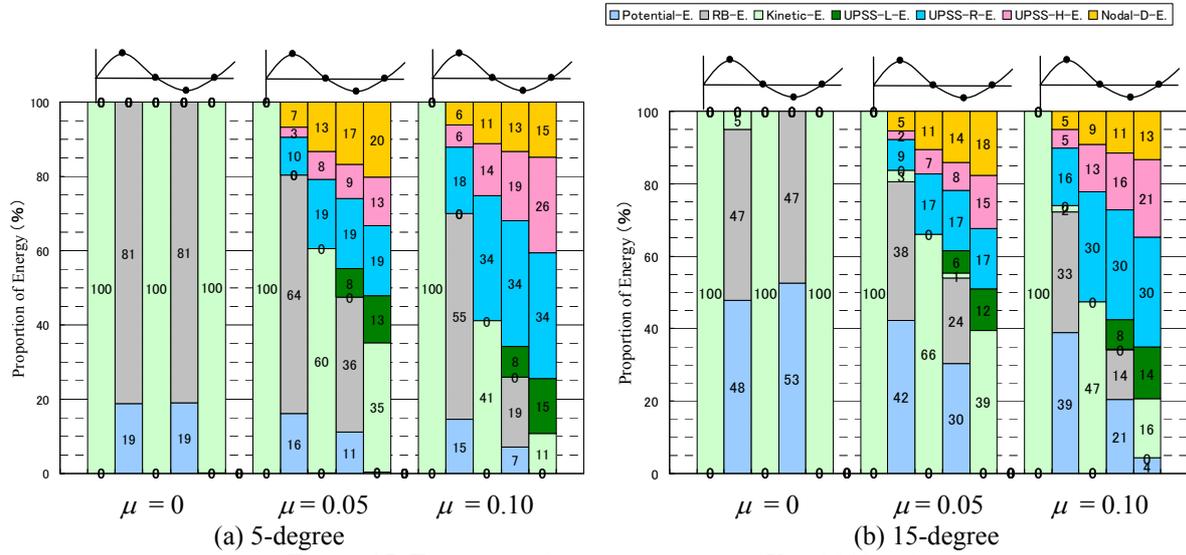


Figure 15. Transition of energy proportion ( $V_0 = 120$  kine)

All the energies are dissipated and the motion comes to halt at  $T/2$  in the 5-degree slope case with 0.1 friction coefficient and 50 kine initial velocity, and the halt takes place at  $3T/4$  in the 15-degree case. The results for the initial velocity of 120 kine shown in Fig. 15. are similar to the previous case, with the difference of the decreased proportion of UPSS-H-E and the increased proportion of UPSS-R-E for non-zero frictions. This is due to an increased action of the superstructure on the right slope caused by a greater displacement and velocity amplitude of the motion. This result suggests that the amount of energy conserved in the UPSS as gravitational energy, and the amount of energy transferred to other bridge piers represented by rubber bearing strain energy, can be controlled by adjusting the slope angle of the UPSS.

## 5.2. Energy Dissipation

The amount of dissipated energy represented by the sum of UPSS-L-E, UPSS-R-E, UPSS-H-E and Nodal-D-E in non-zero friction cases is found to progressively increases as the cyclic movement proceeds, as shown in Figs. 14. and 15.

The amount of dissipated energy during a cycle can be evaluated by the subtraction of the kinetic energy at time  $T$  from the initial energy calculated as the kinetic energy due to the initial velocity. The

**Table 3.** Remaining Kinetic Energies after 1 cycle (%)

Slope Angle		5 degrees		15 degrees	
Friction ( $\mu$ )		0.05	0.10	0.05	0.10
$V_0$	50 kine	6	0	10	0
	120 kine	35	11	39	16

ratios of the remaining kinetic energy at a cycle to the initial kinetic energy are shown in Table 3. It can easily be recognized by the table that the friction coefficient rather than the slope angle significantly affects the energy dissipation. The ratios of the remaining kinetic energy is also affected by the initial velocity, partly because the energy dissipation by friction is not proportional to the response, and separation of the superstructure from the sliding surfaces causing a loss of friction force is more likely to happen in the cases of the higher initial velocity.

## 6. CONCLUSION

The dynamic behaviour of UPSS was evaluated from a viewpoint of energy. The energy which appears in this behaviour was divided roughly into the conservative energy, including gravitational potential energy and strain energy, and the dissipative energy, such as the energy dissipation due to friction and viscous damping. In the case study, the initial kinetic energy generated by initial velocity is confirmed to be steadily reduced by repeating storage and dissipation during cyclic response.

On the other hand, since the direction of the velocity is rapidly altered at the time of collision on the different sliding surfaces, significant impact force and jumping behaviour are likely to be induced for the cases of steep slope angles and a high velocity response. These undesirable effects should be avoided in practical application of UPSS by selecting relevant slope angles and slider/sliding surface materials, which are left for future study.

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