Identification of Damage to Beam Structures using Modal Data

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SUMMARY
Many civil structures suffer damage from various external loads. Such damage may seriously affect their performance and often gives rise to structural failure. It is important to identify the position and degree of structural damage in order to determine structures' safety and reliability. As is known, modal characteristics of any structure are closely related to stiffness and mass of each member of a structure. This means that we can probably know the positions and degree of damage to structures from changes of the mode shape and natural frequency before and after damage. This paper is mainly concerned with damage identification techniques using modal data for beam structures. We specifically examined a measure against lack of rotational components in damaged mode shape, improvement in accuracy of damage identification using additional masses and effect of change in masses before and after damage on damage identification accuracy.

Keywords: Damage identification, Modal analysis, Beam structure

1. INTRODUCTION

A structure that has been damaged by excessive loading, such as earthquakes, and fatigue in its in-service period, may suffer some serious accident such as collapse if left untreated. For this reason, early detection of damage to such a structure and subsequent repair and reinforcement are extremely important for the safety of structures.

In order to properly safeguard a road bridge, first, it is necessary to know the current status of soundness of each part of the bridge. This can be mainly done by way of close visual inspection. However, such inspection is not completely reliable because it may depend on subjective judgment of each inspector. In addition to close visual inspection, inspections using acoustic emission, ultrasound or X-rays, have been developed. Though these methods are excellent to find the individual damage to local parts, we can not easily know the total amount of damaged parts which are distributed throughout the structures.

Recently, a new inspection method based on modal analysis of structures has been widely reported. Ren et al. (2002) proposed an inspection method for beam structures to estimate both location where stiffness degradation occurs and its degree from difference of modal data of structures before and after damage. Though Ren's method basically needs rotational displacement data of damaged structure, it is not actually easy to obtain such data.

In this study, in order to examine the practicality and accuracy of Ren's method, we carried out damage identification for beam structures in case of lack of data of rotational displacement. In addition, the effect of additional mass to the damaged structure on the accuracy of Ren's method was investigated, and applicability of Ren's method was also examined for the case that mass changed before and after structure was damaged.

2. ANALYTICAL MODEL

2.1. Analytical model of beam structure

The analytical model is a simple beam as shown in Fig. 2.1.

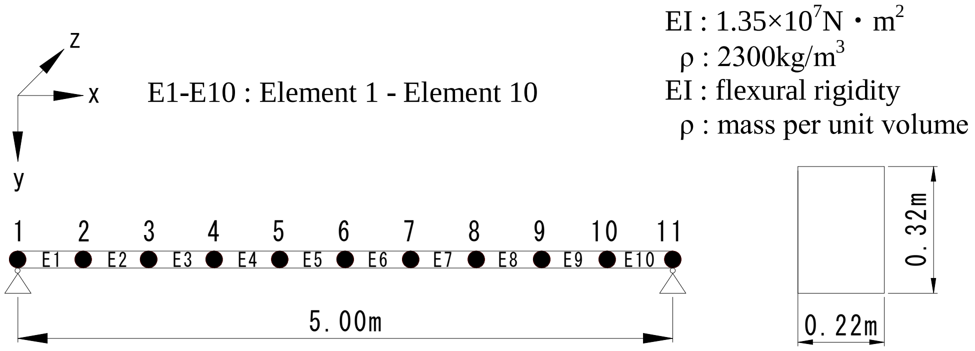

Figure 2.1. Analytical model and section

2.2. Reduction of characteristic matrix

To create a full mass matrix $[M]$ the LM (Lumped mass) method was used. The full mass matrix $[M]$ is a diagonal matrix. Full stiffness matrix $[K]$ is as shown in Eqn. 2.1.

$$[K] = \sum_{e=1}^{Ne} [k_e] \tag{2.1}$$

where $[k_e]$:element stiffness matrix, Ne: total number of elements.

In order to obtain the reduced mass matrix $[\overline{M}]$, only components in the y-axis direction are used. Other components are eliminated from the full mass matrix $[M]$. Reduced stiffness matrix $[\overline{K}]$ is obtained from full stiffness matrix $[K]$ by using Guyan's static condensation method as is shown below.

$$\{x_a\} = -[K_a]^{-1}[K_{ab}]\{x_b\} \tag{2.2}$$

$$[\overline{K}]\{x_b\} = \{f_b\} \tag{2.3}$$

$$[\overline{K}] = [K_b] - [K_{ab}]^T [K_a]^{-1} [K_{ab}] \tag{2.4}$$

where, $\{x_a\}$:z-axis component of displacement vector, $\{x_b\}$:y-axis component of displacement vector, K_{ij} :Component of full stiffness matrix.

2.3. Modal data of damaged structures

Modal data of damaged structures for damage equation is obtained from Eqn. 2.5 consisting of the mass matrix and stiffness matrix which were reduced.

$$\left([\overline{K}] - \Omega_r^2 [\overline{M}]\right)\{\overline{\Phi}_r\} = \{0\} \tag{2.5}$$

where, r : mode number, Ω_r : r th natural circular frequency, and $\{\overline{\Phi}_r\}$: r th modal vector in y-direction.

On the other hand, the procedure for obtaining modal data of sound structures is as follows;

1) modal data in y-direction is obtained from Eqn. 2.5.
2) modal data in other direction is derived from Eqn. 2.3.

3. DAMAGE IDENTIFICATION IN CASE OF LACK OF MODAL DATA IN DIRECTION OF ROTATION FOR DAMAGED STRUCTURES

3.1. Element damage index and damage equation

3.1.1. Element damage index

Damage is defined as a reduction of stiffness of a beam structure. The element damage index $\Delta\alpha_e$ is defined as Eqn. 3.1.

$$\Delta\alpha_e = \frac{(EI)_e - (EI)_{De}}{(EI)_e} \tag{3.1}$$

where, $(EI)_e$: flexural rigidity before damage, $(EI)_{De}$: flexural rigidity after damage.

3.1.2. Damage equation

In this study, we used two damage equations, Dan's equation and Ren's equation. Eqn. 3.2 is the damage equation proposed by Dan et al. (1997).

$$\left[\{\Phi_{Dre}\}^T [K_e]\{\Phi_{Dre}\}\right]\{\Delta\alpha_e\} = \left\{\{\Phi_{Dr}\}^T [K]\{\Phi_{Dr}\} - \Omega_{Dr}^2\right\} \tag{3.2}$$

The damage equation that was proposed by Ren et al. (2002) is shown in Eqn. 3.3.

$$\left[\{\Phi_{Dre}\}^T [K_e]\{\Phi_{le}\}\right]\{\Delta\alpha_e\} = \left\{\left(1 - \frac{\Omega_{Dr}^2}{\Omega_l^2}\right)\{\Phi_{Dr}\}^T [K]\{\Phi_l\}\right\} \tag{3.3}$$

where, $\Delta\alpha_e$: damage index of element *e*, D : after damage, Ω : circular frequency-specific, $\{\Phi\}$: mode shape, $[K]$: stiffness matrix, r and l : mode number, e : element e.

3.2. Pseudo-inverse matrix

All coefficient matrices of the above-mentioned damage equations are not square matrices. When the coefficient matrices are not square, we can not solve the damage equation. Therefore, in that case, we must use pseudo-inverse matrices as shown in Eqn. 3.4.

$$[A]^+ = \left([A]^T [A]\right)^{-1} [A]^T \tag{3.4}$$

where, $[A]$: matrix which is not square.

3.3. Algorithm for damage identification

Fig. 3.1 shows the flowchart of procedure for damage identification based on Guyan's condensation method.

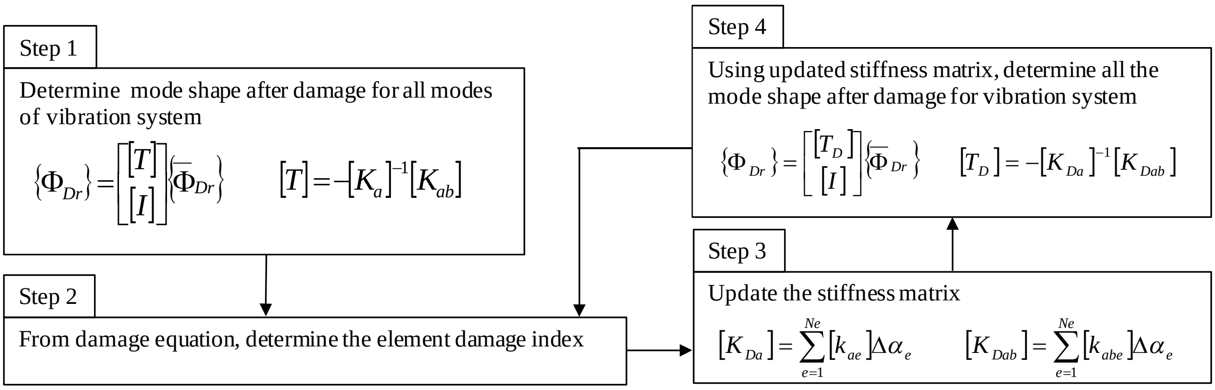

Figure 3.1. Flow chart of one cycle in damage identification using Guyan's method

3.4. Damage scenarios

We set five scenarios, D1-D5, for checking the accuracy of the above-mentioned damage identification. Table 3.1 shows the number and degree of damage of each damaged element.

Table 3.1. Damage Scenarios

Scenarios	D1	D2		D3			D4			D5						
Damage element numbers	5	5	7	2	5	8	1	5	8	2	3	4	5	6	7	8
Element damage index $\Delta\alpha_e$(%)	40	20	40	20	40	10	20	40	10	20	30	40	40	40	20	10

3.5. Damage identification results

Fig. 3.2 shows the damage identification results for five damage scenarios as shown in Table 3.1. Here, damage identification results obtained using Dan's equation and Ren's equation respectively are illustrated with element damage indexes for each damage scenario. The maximum mode numbers of damaged and sound structures are 5 and 9 respectively. The summary of the results obtained from Fig. 3.2 is as follows.

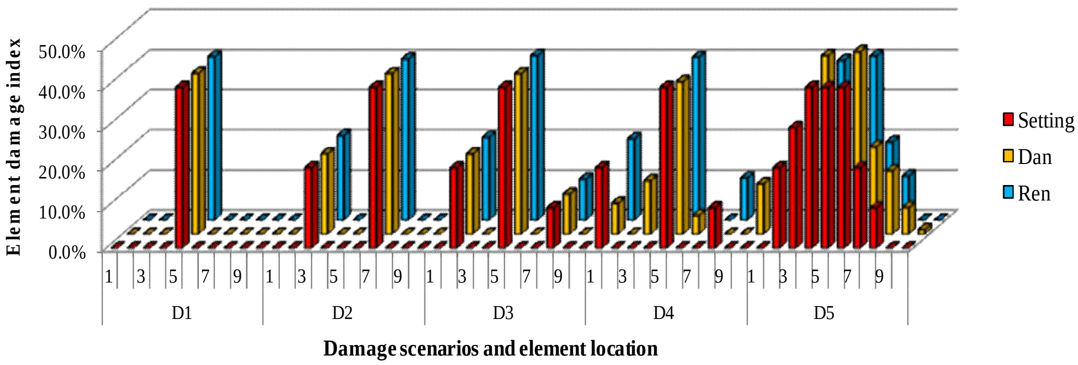

Figure 3.2. Damage identification results for scenarios D1-D5

1) Even if data component in direction of rotation of mode shape after damage is not available, using the damage identification algorithm based on Guyan's static condensation method shown in Fig. 3.1., damage can be identified accurately.
2) Accuracy of damage identification using Ren's equation is high for all scenarios. In the case that Dan's equation is used, accuracy of damage identification for scenarios D4 and D5 is low.
3) It may be said that Ren's damage equation is better than Dan's one, because use of the former brings highly accurate damage identification results for almost all scenarios even if data of high mode is unavailable.

4. EFFECT OF ADDED MASS ON DAMAGE IDENTIFICATION ACCURACY USING ADDED MASS

4.1. Damage equation considering added mass

We modified Ren's damage equation considering added mass as shown in Eqn. 4.1. Required input data are stiffness matrix of sound structures, modal data of structures before and after damage and increment $[\Delta M]$ of full mass matrix due to added mass.

$$\left[\{\Phi_{Dle}\}^T[K_e]\{\Phi_{re}\}\right]\{\Delta\alpha_e\} = \left\{\left(1-\frac{\Omega_{Dl}^2}{\Omega_r^2}\right)\{\Phi_{Dl}\}^T[K]\{\Phi_r\} - \Omega_{Dl}^2\{\Phi_{Dl}\}^T[\Delta M]\{\Phi_r\}\right\} \tag{4.1}$$

4.2. Added mass

We set two patterns about extra mass as follows.
Case1: Mass of 100kg is added to node No.6.
Case2: Masses of 150kg and100kg are added to node Nos.5 and 7 respectively.

4.3. Damage identification results in case of using added mass

Fig. 4.1 shows effect of added mass on the results of damage identification for scenarios D1-D5 shown in Table 3.1, where maximum mode numbers before and after damage are 9 and 7 respectively.

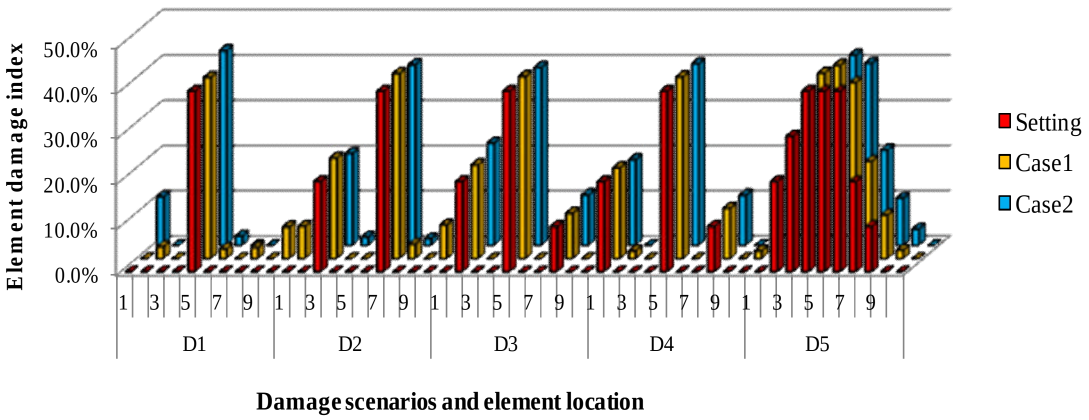

Figure 4.1. Damage identification results in case of using added mass

Though only two added mass scenarios were considered, it was found that the effect of added mass on the damage identification accuracy is small.

5. DAMAGE IDENTIFICATION IN CASE THAT MASS CHANGES BEFORE AND AFTER DAMAGE TO STRUCTURE

5.1. Damage scenarios

We considered three damage scenarios, D6-D8. Rate of decrease in mass and damage index of each element are shown in Table 5.1.

Table 5.1. Scenarios Of Rate Of Decrease In Mass And Damage Index Of Each Element

Scenarios		1	2	3	4	5	6	7	8	9	10
D6	Rate of decrease in mass	1.0	1.0	0.9	1.0	1.0	1.0	0.8	1.0	1.0	1.0
	Damage index of element	1.0	1.0	0.8	1.0	1.0	1.0	0.6	1.0	1.0	1.0
D7	Rate of decrease in mass	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
	Damage index of element	1.0	1.0	0.8	1.0	1.0	1.0	0.6	1.0	1.0	1.0
D8	Rate of decrease in mass	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1
	Damage index of element	1.0	0.8	1.0	1.0	0.6	1.0	1.0	0.9	1.0	1.0

5.2. Damage identification results when using the damage equation under the assumption of invariant mass before and after damage

Fig. 5.1 shows damage identification results under the assumption that mass does not change before and after damage for the damage scenarios D6-D8.

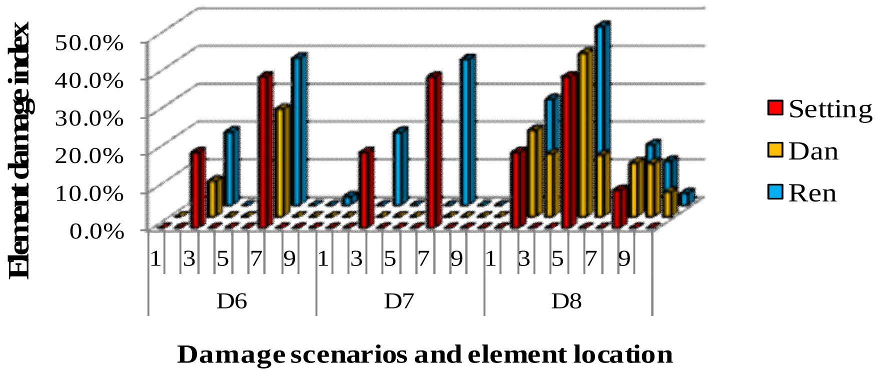

Figure 5.1. Damage identification results for scenarios D6-D8

From Fig. 5.1, it can be seen that in the case where mass of structures increases before and after damage, such as that of scenario D8, identification based on the assumption that mass is constant may bring about a questionable result as all elements have been damaged.

5.3. Damage identification using equivalent mass

Related to added mass, the mass sensitivity method was proposed by Seto (1987). The mass sensitivity method is a method which estimates the equivalent mass from the difference in natural frequency before and after mass is added. The use of equivalent mass has an advantage in which change of mass before and after damage has no effect on the damage identification result. This mass is called ”equivalent mass”. When there is a change in the mass before and after damage, damage identification can be performed without any influence of the change by using the equivalent mass.

5.3.1. Equivalent mass

Equivalent mass e_{rj} of r th mode of N-degree-freedom system is defined as Eqn. 5.1.

$$e_{rj} = m_1\left(\frac{x_1}{x_j}\right)^2 + m_2\left(\frac{x_2}{x_j}\right)^2 + \cdots + m_j\left(\frac{x_j}{x_j}\right)^2 + \cdots m_N\left(\frac{x_N}{x_j}\right)^2 \tag{5.1}$$

5.3.2. Mass-sensitivity method

The mass-sensitivity method is a method for finding the equivalent mass using change of natural frequency by adding a known mass to a prescribed part of the target structure. This method is used to model a single-degree-of-freedom system with the added mass at specified points for calculation of equivalent mass. The principle of this method is shown in Fig. 5.2.

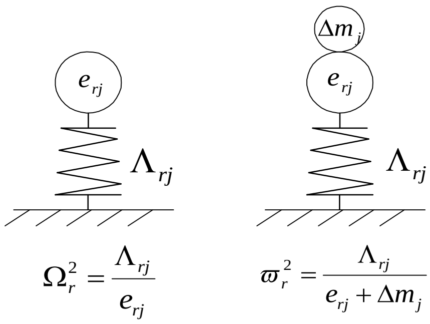

Figure 5.2. Principle of the mass sensitivity method

In this case, the equivalent mass of r th mode at point j is represented by the following equation.

$$e_{rj} = \Delta m_{rj} \frac{\omega_{rj}^2}{\Omega_r^2 - \omega_{rj}^2} \tag{5.2}$$

where, Δm_{rj} is known mass added to point j of r th mode, Ω_r represents specific circular frequency of vibration of the original system, and ω_{rj} shows specific circular frequency of the system with added mass.

The equivalent mass for added mass was plotted. Equivalent mass corresponds to the point where added mass equal to zero on the curve which represents the relationship between both masses.

5.3.3. The damage equation using equivalent mass

Damage equations used in the damage identification method with equivalent mass are almost the same as damage equations that were presented by Dan et al (1997). When circular frequencies and mode shapes of 1st-m th mode after damage are available, the damage equation using equivalent mass can be written as in Eqn. 5.3.

$$\left[\{\Phi_{Dre}\}^T [k_e] \{\Phi_{Dre}\}\right]\{\Delta\alpha_e\} = \left\{\{\Phi_{Dr}\}^T [K] \{\Phi_{Dr}\} - \Lambda_{Drj}\right\} \tag{5.3}$$

where, $\Lambda_{Drj} = \Omega^2{}_{Dr} e_{Drj} = \{\Phi_{Dr}\}^T [\mathrm{K}_D] \{\Phi_{Dr}\}$ (5.4)

j : point of equivalent mass.

5.3.4. Identification of equivalent mass by mass-sensitivity method

Here, we present results of equivalent mass identification using the mass-sensitivity method. There are two types of material properties for each beam element as shown in Fig. 1 and Table 5.2.

Table 5.2. Material Characteristic Values

Characteristics pattern		1	2	3	4	5	6	7	8	9	10
Material 1	Mass per unit volume(kg/m³)	2300	2300	2300	2300	2300	2300	2300	2300	2300	2300
	Flexural rigidity (×10⁷N・m²)	1.35	1.08	1.35	1.35	0.81	1.35	1.35	1.22	1.35	1.35
Material 2	Mass per unit volume(kg/m³)	2300	2300	2070	2300	2300	2300	1840	2300	2300	2300
	Flexural rigidity (×10⁷N・m²)	1.35	1.35	1.08	1.35	1.35	1.35	0.81	1.35	1.35	1.35

Table 5.3. and Table 5.4. show the equivalent mass for each pattern.

Table 5.3. Identification Results Of Equivalent Mass For Material 1

Mode	Reference point j	Equivalent mass(kg)		Error(%)
		Identification value	Exact value	
First	Node 6	395.46	395.47	0.00
Second	Node 3	452.90	453.13	0.05
Third	Node 6	392.38	392.55	0.04
Fourth	Node 5	454.72	455.21	0.11
Fifth	Node 2	383.20	385.60	0.62

Table 5.4. Identification Results Of Equivalent Mass For Material 2

Mode	Reference point j	Equivalent mass(kg)		Error(%)
		Identification value	Exact value	
First	Node 6	391.74	391.74	0.00
Second	Node 3	417.87	418.06	0.04
Third	Node 6	401.30	401.47	0.04
Fourth	Node 5	456.41	456.87	0.10
Fifth	Node 2	363.80	366.15	0.64

5.3.5. Damage identification results using equivalent mass

The damage identifications were carried out using the damage equation considering equivalent mass for the scenario D6-D8 as described in Section 5.1.

Table 5.5. Identification Results Of Equivalent Mass

Mode	Reference point j	Equivalent mass(kg)		
		D6	D7	D8
First	Node 6	391.74	368.10	435.01
Second	Node 3	417.87	384.73	498.24
Third	Node 6	401.30	366.43	431.66
Fourth	Node 5	456.41	410.51	500.29
Fifth	Node 2	363.80	330.47	421.99

Table 5.5. shows identification results of equivalent mass for scenario D6-D8, using mass sensitivity method. The results of damage identification using equivalent mass shown in Table 5.5. in case that mass changes before and after damage can be seen in Fig. 5.3.

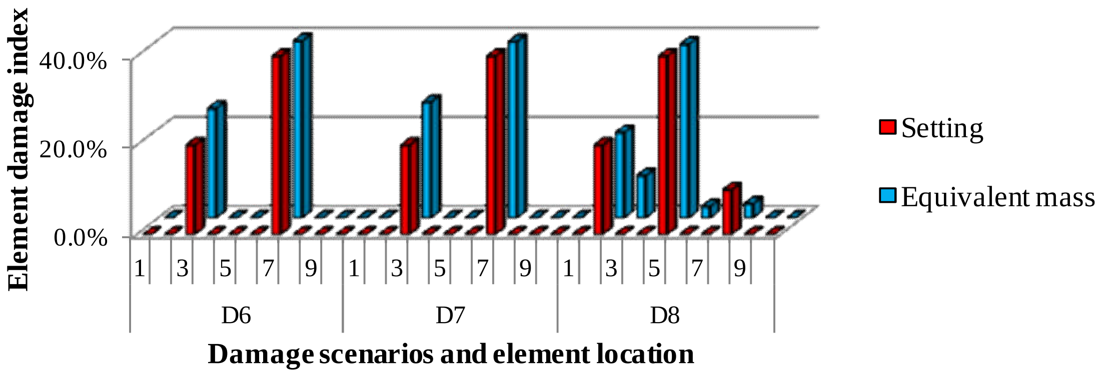

Figure 5.3. Damage identification using equivalent mass

5.3.6. Effect of errors contained in the equivalent mass on results of damage identification

Table 5.6. shows the identification error value for exact values of equivalent mass which were identified in scenario D6～D8. Exact value was determined from mass matrix and mode shape. Also, identification value was derived from the added mass and natural frequency by mass-sensitive

methods. From Table 5.6., it is evident that equivalent mass estimated by the mass-sensitivity method may contain some error.

Table 5.6. Error Of Equivalent Mass

Mode	Error(%)		
	D6	D7	D8
First	0.00	0.00	0.00
Second	0.04	0.05	0.04
Third	0.04	0.05	0.03
Force	0.10	0.13	0.09
Fives	0.64	0.80	0.51

Therefore, in order to know the effect of error included in the equivalent mass on identification result, we carried out damage identification using the exact value of equivalent mass with added error. Error values were set to be 0.0%, ± 0.5%, ± 1.0%, ± 3.0%, ±5.0%, respectively. In addition, error was represented by a random number in the range of ±3.0% to ± 0.5%, and the target scenario was only D6. As shown in Fig. 5.4, it can be seen that though when error contained in the equivalent mass is greater than or equal to 3%, it is almost impossible to identify damage, when the error is small , damage can be identified with high accuracy. For this reason, when performing damage identification using the equivalent mass, it is necessary to obtain more accurate equivalent mass from mass-sensitive methods.

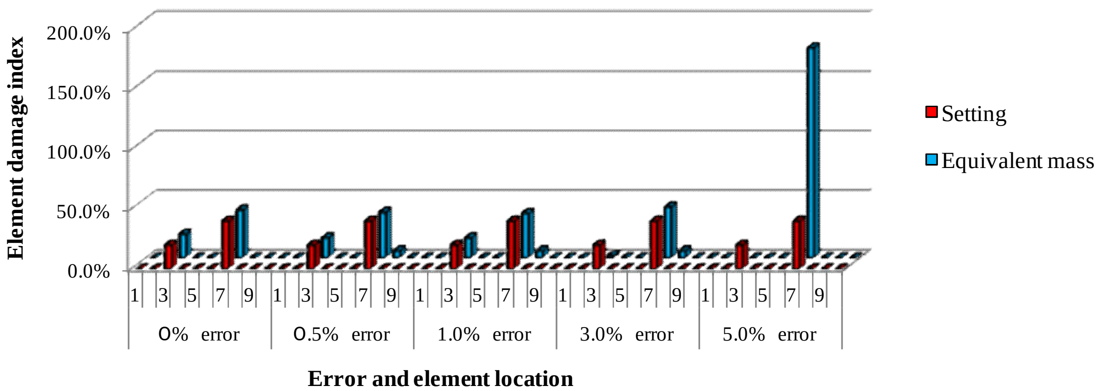

Figure 5.4. Results of damage identification that contain error in the equivalent mass

6. CONCLUSIONS

This study presented a damage identification technique using modal analysis for beam structures. To verify the effectiveness of this technique analytically, three cases were studied: mode shape expansion, damage identification using additional masses, and damage identification in case that mass of structure changes before and after damage.

Conclusions obtained from this study are as follows.

1. When rotational components in damaged mode shape are not available, structural damage can be identified using Guyan's static condensation method.
2. The damage equation proposed by Ren et al.(2002) is superior to the one proposed by Dan et al.(1997) for the practical damage identification technique.
3. Structural damage can be identified successfully, even in the case of damage identification using additional masses.
4. When mass increases before and after damage, identification based on the assumption that mass is constant may show a questionable result as all elements have been damaged.
5. It is necessary to improve the accuracy of the mass sensitivity method in order to more precisely identify structural damage using equivalent mass.

REFERENCES

Guyan, R.J.(1965). Reduction of Stiffness and Mass Matrices, *AIAA Journal, Vol.3, No.2 p.380.*

Hassiotis, S., and Jeong, G.D. (1995). Identification of stiffness reductions using natural frequencies, *Journal of Engineering Mechanics,ASCE, Vol.121,No.10,pp.1106-1113.*

Ren, W., and Roeck, G, D. (2002). Structural damage identification using modal data.I,*Simulation verification, Journal of Structural Enginee- ring, ASCE,Vol.128,No.1, pp.87-95.*

Ren, W., and Roeck, G, D. (2002). Structural damage identification using modal data.Ⅱ,*Test verification, Journal of Structural Engineeri- ng,ASCE,Vol.128,No.1, pp.96-104.*

Yuan, H., Hirao, K., Sawada, T., and Nariyuki, Y. (1994). Modal analysis inspection of damage detection for structure with nonproportionally damping, *The 9th Japan Earthquake Engineering Symposium.*

Yuan, H., Hirao, K., Sawada, T., and Nariyuki, Y. (1994). Modal analysis method for stiffness degradation identification of non-proportionally damped structure, *Struct. Eng./Earthquake Eng. Vol.11, No.2, pp.123-126.*

Yuan, H., Hirao, K., Sawada, T., and Nariyuki, Y. (1995). A Study on Reducing the Measured Modes in Modal Analysis Inspection for Damage Assessment of a Structure, *Journal of Structural Engineering, JSCE,Vol.41A.*

Dan, H., Hirao, K., Nariyuki, Y., and Sakabe,Y. (1997). Calculation Method of stiffness degradation of a structure by modal analysis, *The University of Tokushima, Faculty of Engineering, Research Report, No. 42.*

Perera, R., Huerta, C., and Orquin, M, J. (2008). Identification of damage in RC beams using indexes based on local modal stiffness, *Construction and Building Materials,Vol.22,No.8,pp.1656-1667.*

Shi, Y. Z., Law, S. S., and Zhang, M, L., (2002). Improved damage quantification from element modal strain energy change, *Journal of Engineering Mechanics,ASCE,Vol.128,No.5,pp.521-529.*

Seto, K., Ookuma, M., Yamashita, S., and Nagamatsu, A., (1987). Equivalent mass identification method of multi-degree-of-freedom system, *The Japan Society of Mechanical Engineers (Part C), No. 485, Volume 53.*