Application of the energy index of seismic wave for the prediction of structural earthquake damages

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SUMMARY:
We propose a new energy index of ground motions, named "Incident Energy Density (IED)", is defined as the integration of squared velocity time history of incident (up-going) seismic wave multiplied by the ground impedance (product of density and S wave velocity). In order to show the applicability of IED for the prediction of structural damages in comparison with PGA and PGV, non-linear response analysis of single-degree-of-freedom system and 2-dimensional nonlinear dynamic response analysis of quay wall are performed. In the result, IED has more correlation with and less variation in the ductility factors, the plastic strain energy and residual deformation than PGA and PGV. It is concluded that, the degree of the damage during earthquakes can be estimated with good accuracy by IED.

Keywords: Incident energy density, Prediction of structural earthquake damage, Response of single-degree-of-freedom system, residual deformation, quay wall

1. INTRODUCTION

When earthquake damages are estimated throughout an area such as a city, the area is divided into many rectangular segments which are usually hundreds meters on sides. The strong ground motions are predicted for the each segment, on which the estimation of structural damages is based (Onishi and Sawada, 2008). The estimated result is used to plan seismic retrofits in order to decrease the damages, temporary rehabilitations just after the earthquake and reconstructions in the long term.

Detailed seismic response analysis is hardly used for estimating earthquake damages because a large number of structures should be examined. Accordingly, the damages are usually estimated by a statistical relationship between an index of the earthquake ground motions and structural damages, which is obtained from data in past disastrous earthquakes (for example, Yamaguchi and Yamasaki, 1999). Peak ground acceleration (PGA), peak ground velocity (PGV) and response spectra are often used for the indices of earthquake ground motions. It is well known that these values are not enough to represent the intensities of the whole time histories of the ground motions because they only indicate peak values. When the structural damages caused by earthquakes are predicted by such a simple method, we should carefully select the index of ground motion, which has good correlation with damages.

2. ENERGY INDEX OF SEISMIC WAVE

There are many researches in the energy of seismic wave (Aki and Richards, 2002, Kato and Akiyama, 1975, Nozu and Iai, 2001). Most of them concluded that the energy of seismic wave has close relationship with earthquake damages of structures. In this study, we define a new energy index, named "Incident Energy Density (IED)", as

$$E = \rho V_s \int_0^\infty \dot{u}^2(t)dt\,, \tag{2.1}$$

where E is IED , ρ and V_s are the density and shear wave velocity of the ground (ρV_s is the impedance), and $\dot{u}(t)$ the velocity time history of incident (up-going) seismic wave (= $\sqrt{\dot{u}_x^{\,2} + \dot{u}_y^{\,2} + \dot{u}_z^{\,2}}$). Note that half amplitude of observed velocity should be used for $\dot{u}(t)$ if it is recorded on ground surface. It has been shown that IED has less variation than PGA and PGV in attenuation relationship with the moment magnitude and equivalent hypocentral distance (Hirai and Sawada, 2008).

Goto et al. (2011) showed that Normalized Energy Density (NED), which is defined as the average power of up-going wave multiplied by the impedance, is conserved in the layers for a 2D SH problem. Note that IED defined in this study is identical to NED in case that Fourier amplitude of the incident wave is constant. In other words, IED may adequately consider the difference of ground properties, where the seismic wave is observed, using the characteristics of NED.

3. NON-LINEAR RESPONSES OF SINGLE-DEGREE-OF-FREEDOM SYSTEM

Structural damages during earthquakes are evaluated by the simulations of non-linear responses of single-degree-of-freedom systems in this study. When the input acceleration $\ddot{u}_0$ acts on a single-degree-of-freedom system, the equation of motion is written as

$$m(\ddot{u} + \ddot{u}_0) + c\dot{u} + ku = 0\,, \tag{3.1}$$

where m is a mass, c damping factor, k stiffness and $\ddot{u}$, $\dot{u}$, u are the response acceleration, velocity, displacement, respectively, as shown in Fig. 3.1. The restoring force is assumed as a bi-linear model in this study, as shown in Fig. 3.2. Non-linear response is calculated by the numerical integration of the equation of motion in time domain using Operator Splitting method (Nakajima et al., 1990).

The equation of energy equilibrium is derived from Eqn. 3.1 by multiplying $\dot{u}dt$ by both sides and by integrating both sides:

$$\int_0^t m\ddot{u}\dot{u}dt + \int_0^t c\dot{u}\dot{u}dt + \int_0^t ku\dot{u}dt = -\int_0^t m\ddot{u}_0\dot{u}dt\,, \tag{3.2}$$

where, the right side of Eqn. 3.2 is the total input energy W_{in}, the first term of the left side the kinetic energy W_k at the time t, the second term the energy W_h expended by damping and the third the strain energy W_e which is a sum of elastic strain energy and plastic strain energy W_p. W_e is consistent with W_p at the end of shaking because the elastic strain energy becomes zero. Examples of input motion, calculated time histories of the energy values and the resulted hysteresis between horizontal deformation and restoring force are shown in Fig. 3.3. The structural parameters used in the calculation are shown in Table. 3.1.

The responses of single-degree-of-freedom system are calculated using the structural parameters in Table 3.1 against 754 strong ground motions (NS and EW components of 377 records) whose PGA are more than 300gal picked up from seismic records of K-NET and KiK-net strong motion seismograph networks after starting the observation in 1996. The band-pass filter from 0.1Hz to 10Hz is applied. The ductility factor (= u_{max}/u_y) and the plastic strain energy are chosen as the indices of structural damages. PGA, PGV and IED are examined in the representative indices of input ground motion. Note that the ground condition is not considered in this section. The density and shear wave velocity of the soil is assumed to be respectively 2000kg/m^3 and 300m/s for calculating IED.

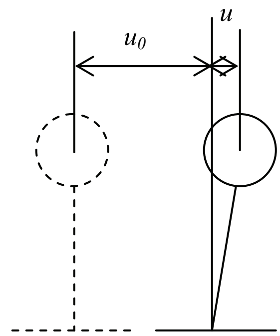


Figure 3.1. Model of single-degree-of-freedom system

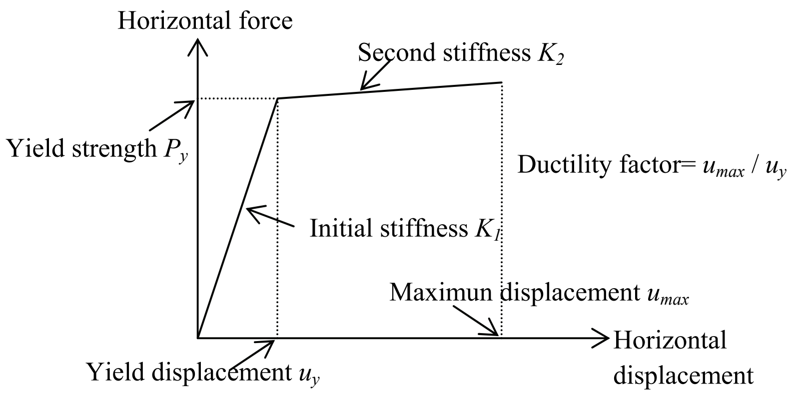
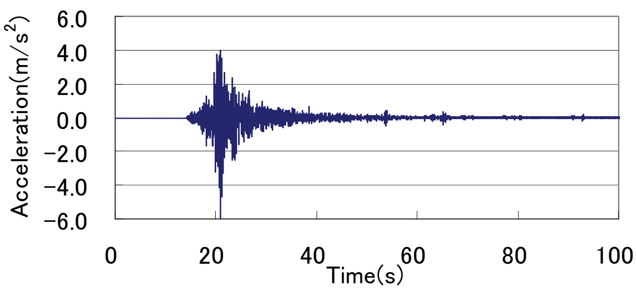
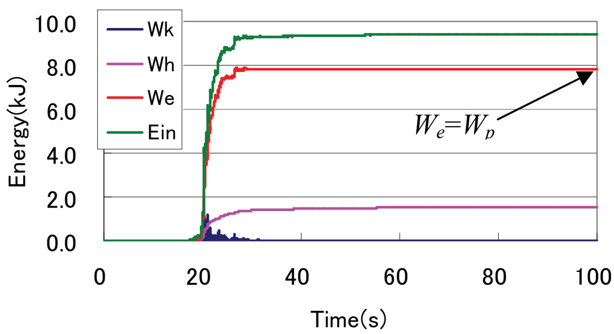


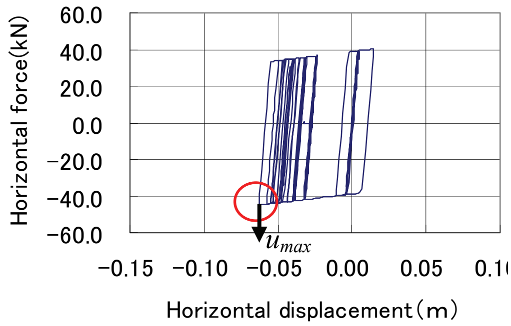
Figure 3.2. Bi-linear model



(a) Input motion



(b) Time histories of energies



(c) Hysteresis curve

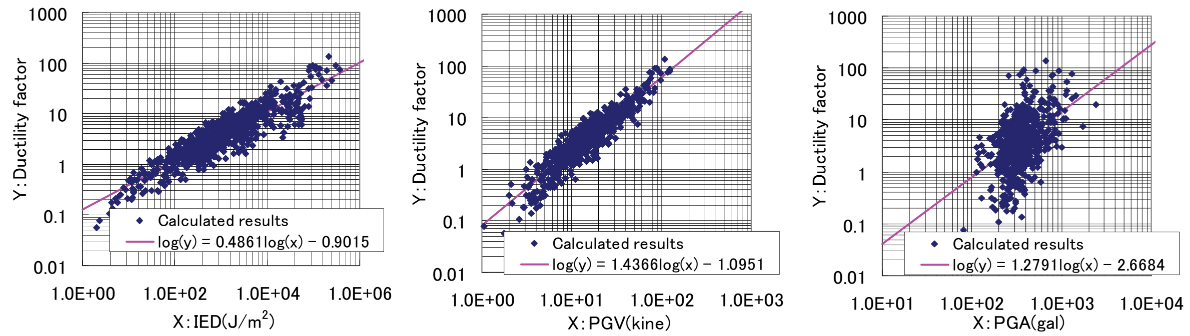
Figure 3.3. An example of response analysis of single-degree-of-freedom system

Table 3.1. Structural parameters for the single-degree-of-freedom system

Mass m (t)	Damping factor c (kN·s/m)	Yield strength P_y (kN)	Yield displacement u_y (m)	Initial stiffness K_1 (kN/m)	Second stiffness K_2 (kN/m)
40	25	39.2	0.004	9800.0	98.0

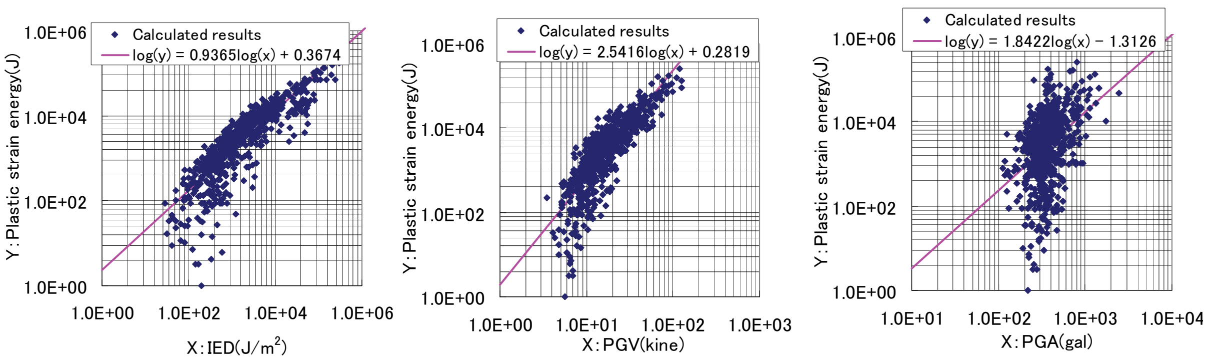
The relationship between the ductility factor and indices of ground motion are shown in Fig. 3.4. The regression line is also shown in the figure. The logarithmic standard deviations from the regression line are shown in the caption. It is shown that the correlation between PGA and the ductility factor is low. PGV and IED are comparable in correlation with the ductility factor.

The relationship between plastic strain energy and indices of strong ground motion are shown in Fig. 3.5 with the regression line and the logarithmic standard deviations. The same trends of correlations are seen in Fig. 3.5 and Fig. 3.4.



(a) IED(0.213) (b) PGV(0.192) (c) PGA(0.437)

Figure 3.4. Relations between indices of strong ground motions and ductility factor (logarithmic standard deviations in parentheses)



(a) IED(0.433) (b) PGV(0.455) (c) PGA(0.770)

Figure 3.5. Relations between indices of strong ground motions and plastic strain energy (logarithmic standard deviations in parentheses)

Parametric study on the response of single-degree-of-freedom system is conducted using 10 cases of structural parameters. The parameters and results are shown in Table 3.2. Main parameter in the study is yield seismic intensity, calculated by

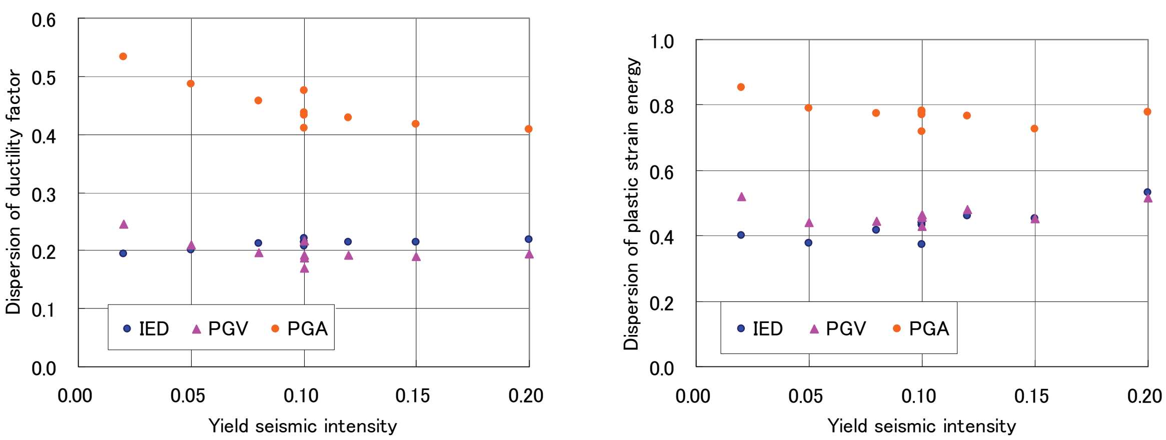
$$S_y = \frac{P_y}{mg}, \tag{3.4}$$

where g is the gravity acceleration and P_y the yield restoring force, shown in Fig. 3.2. The damping factor and initial stiffness are also varied.

Fig. 3.6(a) and (b) show the result of parametric study listed in Table. 3.2. The vertical axes are logarithmic standard deviations of ductility factor and plastic strain energy, whereas the horizontal axes are yield seismic intensities. The dispersions of ductility factor and plastic strain energy against IED, PGV and PGA have a similar trend, as follows:
(1) The dispersions against IED are roughly similar to those against PGV, and are detailly smaller when yield seismic intensity is small.
(2) The dispersions against PGA are larger than those against PGV and IED.
(3) Comparison between case1, 6 and 7 shows that, the dispersions against every index decrease as initial stiffness becomes larger.
(4) Comparison between case1 and 5 shows that, effect of damping factor on the dispersions against every index is small.

Table 3.2. Result of the parametric study of the single-degree-of-freedom system

case	Mass m	Damping factor c	Yield strength P_y	Yield displacement d_y	Initial stiffness K_1	Second stiffness K_2	Natural frequancy f	Yield seismic intensity	Relations between seismic indexes and calculated results					
									Ductility factor			Plastic strain energy(Ductility factor>1)		
	(t)	(kN・s/m)	(kN)	(m)	(kN/m)	(kN/m)	(Hz)		IED	PGV	PGA	IED	PGV	PGA
1	40	25.04	39.2	0.0040	9800.0	98.0	2.49	0.10	0.213	0.192	0.437	0.433	0.455	0.770
2	50	28.00	39.2	0.0040	9800.0	98.0	2.23	0.08	0.213	0.196	0.456	0.416	0.445	0.774
3	80	35.42	39.2	0.0040	9800.0	98.0	1.76	0.05	0.201	0.209	0.487	0.379	0.442	0.790
4	200	56.00	39.2	0.0040	9800.0	98.0	1.11	0.02	0.194	0.245	0.534	0.400	0.519	0.853
5	40	50.09	39.2	0.0040	9800.0	98.0	2.49	0.10	0.213	0.188	0.433	0.439	0.464	0.777
6	40	35.42	39.2	0.0020	19600.0	196.0	3.52	0.10	0.207	0.170	0.410	0.373	0.427	0.719
7	40	17.71	39.2	0.0080	4900.0	49.0	1.76	0.10	0.220	0.216	0.474	0.440	0.466	0.782
8	40	25.03	47.0	0.0048	9800.0	98.0	2.49	0.12	0.214	0.191	0.429	0.459	0.481	0.768
9	40	25.04	58.8	0.0060	9800.0	98.0	2.49	0.15	0.213	0.190	0.418	0.452	0.454	0.725
10	40	25.04	78.4	0.0080	9800.0	98.0	2.49	0.20	0.219	0.194	0.407	0.532	0.516	0.779



(a) Dispersion of ductility factor (b) Dispersion of plastic strain energy

Figure 3.6. Comparison of dispersion of responses against yield seismic intensity

4. 2-DIMENSIONAL DYNAMIC RESPONSE ANALYSIS OF A QUAY WALL

In order to examine indices of input ground motion, earthquake structural damages of a quay wall are simulated by two-dimensional nonlinear dynamic analysis. Computer program FLIP (Finite Element Analysis Program for Liquefaction Process)(Iai et al., 1990) is used for the analysis, which can consider the liquefaction of soils and is often used in seismic design for port structures in Japan.

The dynamic response analyses are performed against 33 strong ground motions chosen randomly from 754 components used in the section 3. The quay wall, which consists of a caisson structure, is modelled by FEM, as shown in Fig. 4.1. Note that liquefiable replaced sand under the caisson and reclaimed sand behind the caisson exist whose equivalent N-values are 10. The engineering base layer is set at -20m from sea level, just under the gravel layer. Three cases of S wave velocities in the engineering base layer (V_s = 300m/s, 600m/s and 900m/s) are set, whereas the density of engineering base layer is 2000kg/m^3 in all cases. Totally 99 cases of dynamic response analyses are done for 33 input ground motions and three different V_s of engineering base layer. Note that IEDs have different values, whereas PGAs and PGVs are identical, in the three cases with different V_s of the engineering base layer.

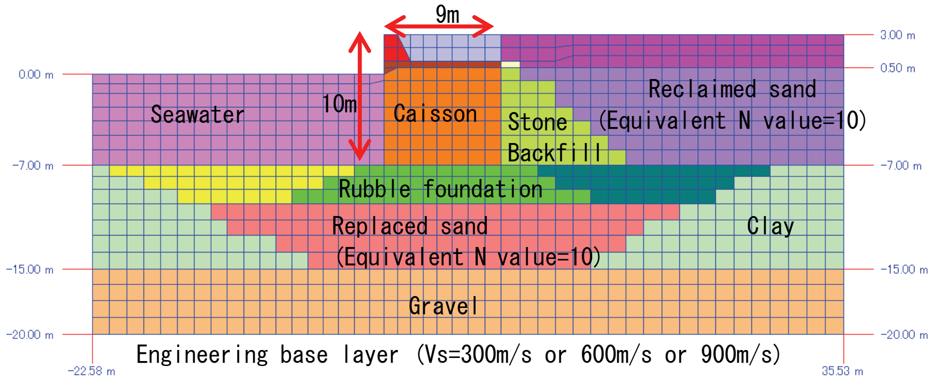
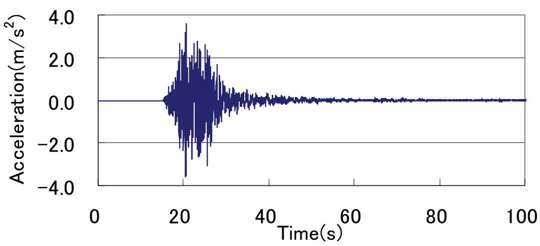
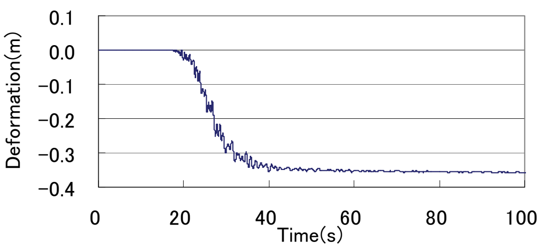


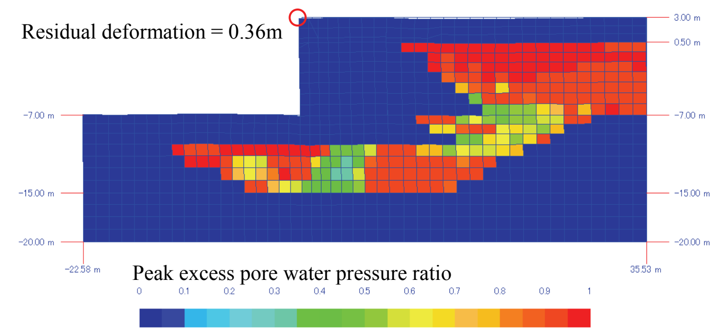
Figure 4.1. FEM model of quay wall



(a) Input motion



(b) Time history of deformation at the top of quay wall



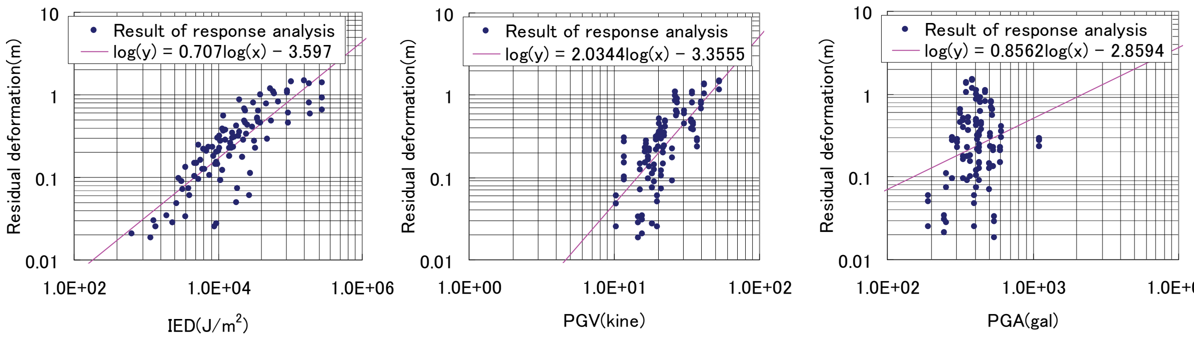
(c) Residual deformation and peak excess pore water pressure ratio

Figure 4.2. Example of dynamic response analysis

The result of dynamic response analysis is shown in Fig. 4.2, as an example. The panel (c) shows the residual deformation of quay wall after shaking and peak excess pore water pressure ratio, which ranges from 0.0 to 1.0 according to the liquefaction level. It is obvious that liquefaction occur in the replaced sand and the reclaimed sand. The residual deformation of quay wall is usually resulted by un-uniform settlement and increase of earth pressure, induced by the soil liquefaction. The horizontal residual deformation at the top of quay wall is chosen as the index of structural damages because the serviceability of the quay wall after earthquake should be judged by the residual deformation.

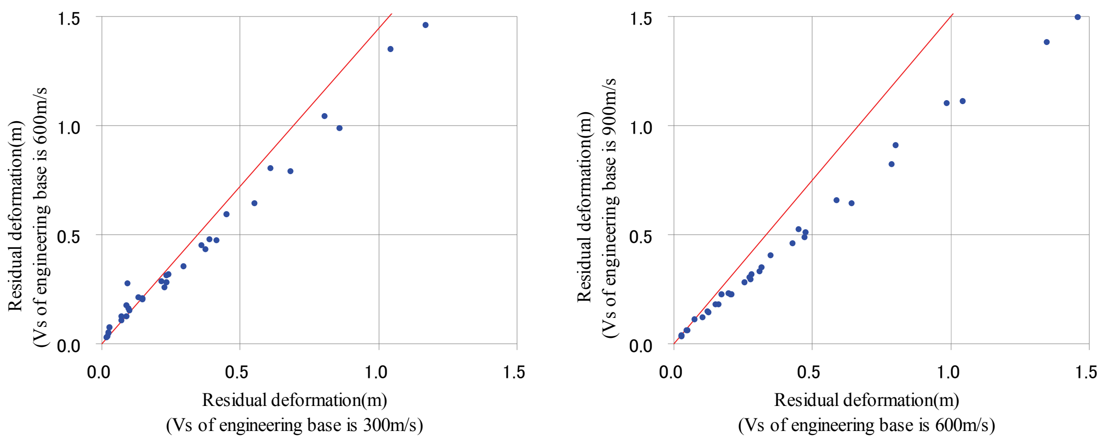
The relationships between the residual deformation and indices of strong ground motion are shown in Fig. 4.3 with the regression lines and the values of logarithmic standard deviations. The standard deviation of IED is the smallest and that of PGA is the largest.

Effect of V_s in the engineering base layer on residual deformation of quay wall is shown in Fig. 4.4. The horizontal axis is the residual deformation in case with 300m/s of V_s in the engineering base layer and vertical axis is that with 600m/s in Fig. 4.4(a), whereas horizontal one is that with 600m/s and vertical with 900m/s in Fig. 4.4(b). The red line in the figure indicates that the horizontal value is identical to the vertical value. The residual deformations in case with V_s=600m/s are larger than those with V_s=300m/s and those with V_s=900m/s are larger than those with V_s=600m/s. This is the reason why the dispersion of IED is smaller than that of PGV in Fig. 4.3, whereas those are similar in Fig. 3.4 and 3.5. It is shown from the result that IED can properly consider the condition of ground where the ground motion is observed.



(a) IED (0.262) (b)PGV(0.321) (c)PGA(0.454)

Figure 4.3. Relations between indices of strong ground motions and residual deformation of quay wall (logarithmic standard deviations in parentheses)



(a) Comparison between cases with V_s=300 & 600m/s (b) Comparison between cases with V_s=600 & 900m/s

Figure 4.4. The effect of Vs in engineering base layer on residual deformation of quay wall

CONCLUSION

We propose a new energy index of ground motion, named “Incident Energy Density (IED)”, which is defined as the integration of squared velocity time history of incident (up-going) seismic wave multiplied by the impedance of ground (product of density and S wave velocity). In order to show the applicability of IED for predicting structural damages in comparison with PGA and PGV, non-linear response analyses of single-degree-of-freedom system and 2-dimensional dynamic response analyses are conducted.

The results of non-linear responses of single-degree-of-freedom system show that, the dispersions of ductility factor and plastic strain energy against IED, which do not consider the ground condition, are roughly similar to those against PGV, and are smaller than those against PGA. In the detailed case ductility factor is larger as yield seismic intensity decrease, the dispersions against IED is smaller than those against PGV. It is shown by the results that, IED is the applicable index to estimate structure damage by earthquake.

The results of 2-dimensional nonlinear dynamic response analyses of a quay wall show that, the dispersion of residual deformation of quay wall against IED is less than those against PGV and PGA because IED can properly consider the condition of ground.

It is concluded that, the degree of the damage during earthquakes can be estimated with good accuracy by IED.

REFERENCES

Onishi, Y. and Sawada, S. (2008). Strong motion to be considered for Uemachi Fault. *The 36th Symposium of Eatrhquake Ground Motion* ,83-90. (in Japanese)

Yamaguchi, N. and Yamazaki, F. (1999). Estimation of strong ground motion in the 1995 Hyogoken-nanbu earthquake based on building damage data. *Structual Eng./Earthquake Eng. (proc. of JSCE),* **612/I-49**,325-336. (in Japanese)

Aki, K. and Richards, P.G. (2002). Quantative seismology second edition, Kokonshoin (in Japanese)

Kato, B. and Akiyama, H. (1975). Energy input and damages in structures subjected to severe earthquakes. *Journal of Structural and Construction Engineering* **235**,9-18. (in Japanese)

Nozu, A. and Iai, S. (2001). A study on strong motion indices for quick damage estimation of quay wall. *Proceedings of 28th Technical meeting of Kanto Branch of Japan Society of Civil Engineers*,18-19. (in Japanese)

Hrai,T. and Sawada, S. (2008). Energy variation of earthquake ground motion in time frequency domain using wavelet transform. *The 14th World Conference on Earthquake Engineering*, No.02-0094.

Goto, H., Sawada, S. and Hirai, T. (2011). Conserved quantity of elastic waves in multi-layered media: 2D SH case – Nomalized Energy Density. *Wave Motion* **48**,602-612.

Iai, S., Matsunaga, Y. and Kameoka, T. (1990). Strain space plasticity model for cyclic mobility. *Report of the Port and Harbour Research Institute* **29:4**,27-56.